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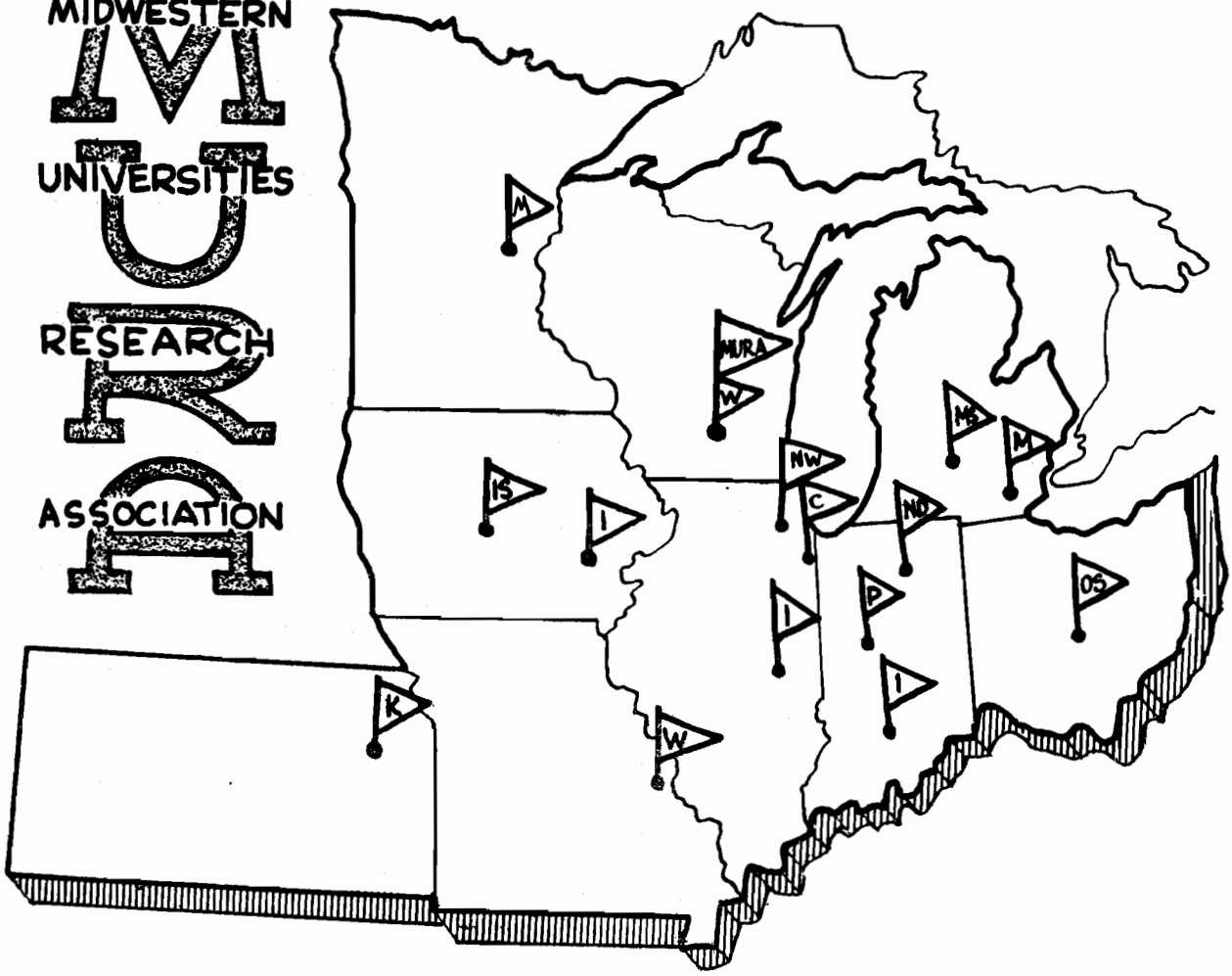

PARTICLE-PARTICLE COLLISIONS IN A STACKED BEAM

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ABSTRACT

The effect of electron-electron collisions on the longitudinal motion of electrons in the stacked beam of the two-way model is studied. It is shown that the energy loss due to collisions is very small compared to the energy loss due to radiation, and thus that the use of Liouville's theorem in μ-space is an excellent approximation.

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I. INTRODUCTION

The investigations of longitudinal collective effects in particle accelerators[1] have been based on the approximation of ignoring strong particle-particle collisions. This approximation is conveniently expressed by the statement that Liouville's Theorem is assumed to be valid in μ -space.[2]

In this report it is shown that the above theorem is not appreciably destroyed by electron-electron interactions. This is accomplished by calculating the longitudinal energy change resulting from particle-particle collisions, and showing that it is very small compared to the energy associated with longitudinal collective motions.

In Section II, we shall derive equations relating the values some physical quantities have in the laboratory system to the values the same physical quantities have in certain later used moving systems. In Section III, we shall determine the energy which, as viewed from the moving reference frame, the fastest particle loses as a consequence of collisions with other electrons. In the same Section III, we shall find that when viewed from the laboratory system, the larger part of the energy lost by the faster of the two colliding particles goes into the longitudinal energy of the slower of the two particles; the rest of the considered energy goes, as will be shown into the transverse energy of the two particles. Finally, in Section IV, we shall

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[1] C. E. Nielsen and A. M. Sessler, MURA 441
A. M. Sessler, MURA 442

[2] R. L. Mills and A. M. Sessler, MURA 433

compare the magnitudes of the losses caused by collisions with those caused by radiation, as well as discuss the implication of this calculation for the theory of collective phenomena.

II. TRANSFORMATION FORMULAS

Unprimed symbols will henceforth represent quantities in the laboratory system; primed symbols will represent the same quantities in the system moving with the fastest particle.

As a starting point, we use the relativistic equation which relates energy and momentum of a particle as viewed from the moving frame, to the energy of the same particle as viewed from the laboratory system:

$$E = \left[E' - \beta_\Sigma c\, p_x'\right] \gamma_\Sigma , \tag{1}$$

where $\beta_\Sigma = \frac{u}{c}$, $\gamma_\Sigma = \frac{1}{\sqrt{1-\beta_\Sigma^2}}$, and u is the relative velocity of the two systems of reference.

According to (1), the energy difference, as viewed from the laboratory system, between the fastest particle, which is at rest in the moving frame, and the particle moving with the velocity v_x' in the moving frame is

$$E_{max} - E \equiv \Delta E = \left[\tfrac{1}{2} m v_x'^2 - m u v_x'\right] \gamma_\Sigma , \tag{2}$$

provided $v_x/c \ll 1$, which is indeed the case in our problem.

Equation (2) can be solved for v_x', yielding

$$v_x' = u \pm \sqrt{\left(u^2 - \frac{2\Delta E}{\gamma_\Sigma m}\right)} \simeq c\, \frac{\Delta E}{\gamma_\Sigma m c^2} \tag{3}$$

For the two-way model[3]

[3] F. T. Cole, E. A. Day, R. O. Haxby, and A. M. Sessler, MURA 375

$\gamma_\Sigma = 80$

$E = 40$ Mev

$(\Delta E)_{max} = 3$ Mev,

which, using (3), implies

$$v'_{max} = 2.4 \quad 10^9 \text{cm/sec}, \tag{4}$$

as the maximum relative velocity of electrons as viewed by an observer moving with the fastest particle.

For $v_x \ll u \simeq c$ equation (2) reduces to

$$\Delta E = -muv'_x \gamma_\Sigma \tag{5}$$

We shall also need the equation in relating the transverse energy, as viewed from the moving frame, to the transverse energy, as viewed from the laboratory frame. This relation can be directly derived from equation (2), by setting $v'_x = 0$

$$\Delta E_\perp = (\Delta E'_\perp)\, \gamma_\Sigma . \tag{6}$$

Finally, we note

$$\Delta t' = \frac{\Delta t}{\gamma_\Sigma} . \tag{7}$$

III. ENERGY LOSS AS A RESULT OF COLLISIONS

In the center of mass system of two colliding electrons, the angle of deflection during a collision is given by the equation

$$\cot \frac{\theta''}{2} = \frac{2m'(v'')^2}{e^2} \quad p \tag{9}$$

Here θ'' is the angle of deflection, p is the impact parameter, and v'' is the velocity relative to the center of Mass.

Using equation (9), the result of a collision can be described by

the equation

$$v' = v'' - v''\cos\theta'' = v''(1-\cos\theta'')$$

$$= 2v''\sin^2\theta''/2 = 2v''\left[\frac{1}{1+\frac{4(m')^2\,v''^4}{e^4}\,\rho^2}\right], \qquad (10)$$

where v'' and $v''\cos\theta''$ are the velocities of the faster electron as viewed from the center of mass system, before and after the collision, respectively; v' is the velocity acquired as a result of the collision by the fastest electron with respect to a reference frame moving with the velocity equal to the velocity of the fastest electron before it has undergone the collision.

In view of what has been said above, the velocity the fastest electron acquires in its own reference frame as a result of collisions with other electrons during time interval dt' is

$$W' = \int_{v''_{min}}^{v''_{max}} \int_{\rho_{min}}^{\rho_{max}} 2\pi\rho d\rho \left[\frac{2v''\ n(v'')}{1+\frac{4m'^2 v''^4}{e^4}\,\rho^2}\right] v''dt'dv'' \qquad (11)$$

$$= \int_{v''_{min}}^{v''_{max}} \frac{\pi e^4\ n(v'')}{2m'^2\ v''^2} \ln\left(1+\frac{4m'^2\ v''^4}{e^4}\,\rho^2\right)\Bigg|_{\rho_{min}}^{\rho_{max}} dt'dv''$$

Using eq (3) we find that

$$n(v_x'') = \frac{dN\,(v_x'')}{d\,(v_x'')} = \frac{dN\,(\Delta E)}{d\,(\Delta E)} \quad \frac{d(\Delta E)}{d(v_x'')}$$

$$= 2\,\frac{[N]}{\Delta E}\quad \frac{\gamma_s}{c}\,mc^2 = 2\,[N]\,\frac{80}{6}\,\frac{1}{c} \qquad (12)$$

where $n(v_x'')$ is the number of electrons whose velocity in the center of mass system lies between v_x'' and $v_x'' + dv_x''$, $N(v_x'')$ is the number of

electrons whose velocity is equal or smaller than v''_x, $[N]$ is the total number of electrons per cm^3; the factor 2 comes from the fact that the velocity when viewed by an observer moving with the fastest electron is twice the velocity as viewed from the center of mass of the two colliding particles.

Combining equations (11) and (12) we obtain

$$W' = [N]\frac{\pi 80}{6 \times C} \frac{e^4 dt'}{m'^2} \int_{v''_{min}}^{v''_{max}} \frac{dv''}{v''^2} \ln\left(1 + \frac{4m'^2 v''^4}{e^4} \rho^2\right) \Bigg|_{\rho_{min}}^{\rho_{max}} \tag{13}$$

$$= [N]\frac{\pi \times 80}{6 \times C} \frac{e^4}{m'^2} dt' \ln\left(\frac{4m'^2}{e^4} \rho^2_{max}\right) \frac{1}{v''_{max}} \ln v''_{max}$$

We assume

$$N = 1.6 \times 10^9 \qquad N' = 1.6 \times 10^9 \gamma_\Sigma \tag{13a}$$

$$\rho_{max} = 2cm$$

dt' = time of revolution = $\frac{4.2 \times 10^{-8}}{80}$

v''_{max} = ½ v'max which according to equation (4) yields

$$v''_{max} = 1.2\ 10^9 \text{ cm/sec}$$

Substituting the above values into the eq (13), yields

w' = 80x5 cm/sec, or 4,480 cm/sec per turn, for the velocity (14) acquired by the fastest electron during one turn.

According to eq (5), this corresponds to

$$\Delta E = muv'_x \gamma_\Sigma = m_o c^2 \frac{v'_x}{c} \gamma_\Sigma \tag{15}$$

$$= \frac{80 \times 56}{2 \times 3 \times 10^{10}} \text{ Mev} = 0.92 \times 80 \times 10^{-3} \text{ ev} = 7.36 10^{-2} \text{ ev}$$

for the longitudinal energy lost during one turn by the fastest

electron, as viewed from the laboratory system. We now proceed to determine the fraction of the longitudinal energy lost by the fastest particle which is transformed into the transverse energy of the two colliding particles.

Using eq. (9), we find this energy to be:

$$W' = 2\int_{V_{min}}^{V_{max}} \int_{p_{min}}^{p_{max}} 2\pi p\,dp\, v'dt'\; n(v'')\tfrac{1}{2} m'v''^2 \sin^2\theta\, dv'$$

$$= 2\iint 2\pi p\,dp\, v'dt' n(v'')\tfrac{1}{2} m'v''^2 \times$$

$$\times\; 4\left[\sin^2\tfrac{\theta}{2}\cot\tfrac{\theta}{2}\right]^2 dv''$$

$$= 2\int_{V_{min}}^{V_{max}} \int_{p_{min}}^{p_{max}} 2\pi p\,dp\, v''dt' n(v'') \times$$

$$\times\, 2m'v'^2\left[\frac{2m'\frac{v'^2}{e^2}p}{1+4\frac{m'^2v''^4}{e^4}p^2}\right]^2 dv''$$

$$= 2\int_{V_{min}}^{V_{max}} n(v'')4\pi\, dt'\; mv''^3 \times$$

$$\times\; \frac{1}{4\frac{m'^2v''^4}{e^4}}\; \ln\left[\frac{1+4m'^2v'^4p^2}{e^4}\right]\Bigg|_{p_{min}}^{p_{max}}$$

Using equations (12) and relations (13a), we find

$$W' = 8\pi \frac{4.2 \times 10^{-8}}{80} m' 2 \times 1.6 \times 109 \times 80 \times \frac{80}{6} \frac{1}{c} \times$$

$$\times \int_{v_{min}}^{v_{max}} \left\{ \frac{v''^3 \rho_{max}^2}{1 + \frac{4m'^2 v''^4 \rho_{max}^2}{e^4}} + \right.$$

$$\left. + \frac{\frac{1}{4m'^2 v''^4}}{e^4} \ln \left[1 + \frac{4m'^2 v''^4}{e^4} \rho_{max}^2 \right] \right\} dv''$$

-the same term with ρ_{max} replaced by ρ_{min}

$$= 1.48\ 10^{-6}\ m' \left[\frac{e^4}{m'^2 \rho_{max}^2} \right] \ln \left[1 + \frac{4m'^2 v''^4}{e^4} \rho_{max}^2 \right] \Bigg|_{v_{min}}^{v_{max}}$$

$$+ \frac{e^4}{m'^2 \rho_{max}} \int \frac{1}{v} \ln \left[1 + \frac{4m' v'^4 \rho_{max}^2}{e^4} \right] dv$$

- term in ρ_{min}

Careful inspection of above equation or the actual evaluation of the terms comprising this equation shows that compared with the first term all the other terms are negligibly small.

This means

$$W' = 1.48(10)^{-6} m' \frac{e^4}{m'^2 \rho_{max}^2} \ln \left[1 + \frac{4m'^2 v''^4 \rho^2}{e^4} \right] \Bigg|_{v_{min}}^{v_{max}}$$

$$= 1.48(10)^{-6} \frac{530(10)^{-40}}{9.1(10)^{-28} \times 4} \ln\left[1 + \frac{4\times(9\cdot 10^{-28})^2(1.7\ 10^{9})^4}{530 \quad 10^{-40}} \quad 4\right] = \qquad (17)$$

$$= 6.3\times 10^{-6}\ (e^{v}) \quad \text{per turn}$$

According to eq (6) this corresponds to

$$W = 5.5\times 10^{-4} \text{ ev per turn,} \qquad (18)$$

as viewed from the laboratory system; where W is the longitudinal energy of the fastest particle transfered during one turn through collisions into the transverse energy at the colliding particles. As a check of the method used above, we shall now show that as viewed from the laboratory system the total energy is conserved during a collision.

Using eq (5), we find that the difference between the longitudinal energy lost by the faster of the two colliding particles and the longitudinal energy acquired by the slower of the two particles is

$$\Delta W = (mu_1 v_x^{1} \gamma_1 - mu_2 v_x^{1} \gamma_2) \qquad (19)$$

$$= mv''\ 2 \sin^2\frac{\theta}{2}(u_1\gamma_1 - u_2\gamma_2)$$

$$= 2mv'' \sin^2\frac{\theta}{2}(u_1\gamma_1 - u_1\gamma_2 + u_1\gamma_2 - u_2\gamma_2)$$

For small angles, $\sin \theta/2 \simeq \frac{\theta}{2}$ (this approximation is valid for the majority of collisions considered, with

$$(\gamma_1 - \gamma_2)u_1 \gg (u_1 - u_2)\gamma_1$$

eq. (19) reduces to

$$\Delta w = \frac{1}{2} mv'' \theta^2 c\, \Delta\gamma \qquad (20)$$

According to equation (7) and (16), the transverse energy acquired by the two particles during the same collision is

$$\Delta W_2 = \frac{1}{2} m v''^2 \sin^2\theta \, (\gamma_1 + \gamma_2)$$

$$\simeq \frac{1}{2} m v''^2 \theta^2 \cdot 2 \bar{\gamma}$$

and according to equation (3) this is

$$\Delta W_2 = \frac{1}{2} m v'' C \, \theta^2 \Delta\gamma \tag{21}$$

Comparing equations (20) and (21), we see that the energy is indeed conserved during a collision.

IV. DISCUSSION

According to Parzen[4/] the radiation loss in a similar accelerator is 3 ev per turn per electron. In the last section we calculated that the longitudinal energy which is being transformed by collisions into transverse energy is 5×10^{-4} ev per turn, for the fastest particle. Comparing the two values, one must conclude that when considering energy losses in the beam one can ordinarily neglect the energy losses caused by collisions.

According to eq (14), as viewed from the moving reference frame, the fastest-particle losses per turn are less than 5×10^3 cm/sec of its original $3.2 \; 10^9$ cm/sec velocity. This clearly indicates that the perturbation to the longitudinal motion as caused by collisions can be considered negligible and hence also indicates that as far as the longitudinal motion is concerned, Liouville's Theorem in μ -space is valid even after taking into account the effect of

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[4/] G. Parzen, MURA-376

collisions. The above statement is demonstrated more readily in the laboratory system. Here, according to eq (15), the energy loss is less than 0.1 ev per turn, for the fastest particle, the original longitudinal energy of this particle, however, is 40 Mev.

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