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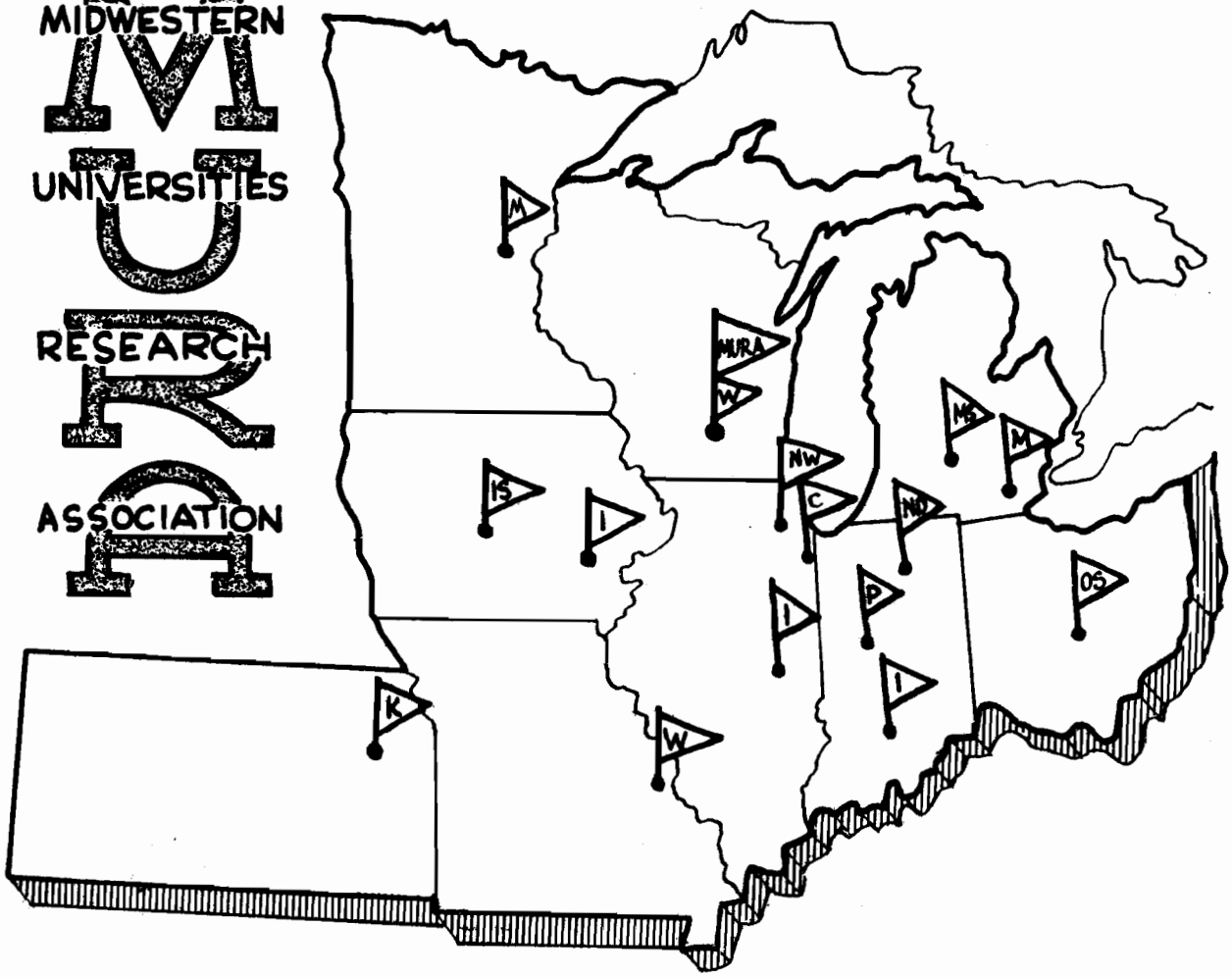

COMPARISON OF 10 AND 15 BEV TWO-WAY FFAG ACCELERATOR DESIGNS

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REPORT **NUMBER** 430

MURA-430
Internal

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COMPARISON OF 10 AND 15 BEV TWO-WAY FFAG ACCELERATOR DESIGNS

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August 27, 1958

ABSTRACT

Symmetric radial sector FFAG accelerators of 10 and 15 Bev with the same N and k are compared. Except for transition energy problems, the 10 Bev design is only slightly better than the 15 Bev design in terms of space charge limits, etc. It appears that such a 10 Bev accelerator installation would cost two-thirds the cost of the 15 Bev installation.

*AEC Research and Development Report. Research supported by the Atomic Energy Commission, Contract No. AEC AT(11-1) 384.

I. INTRODUCTION

It has been suggested that consideration be given to colliding beam accelerators of lower energy, for example 10 Bev, as an alternative to the MURA design.[1] Below is a comparison of many of the relevant features of a 10 and a 15 Bev Ohkawa-type symmetric radial sector accelerator, together with a collection of the pertinent relationships. It may be hoped that these standards and formulae might be applied to other accelerator designs as a means of comparison.

The accelerator parameters N, k, ν_x, ν_z for 10 Bev were chosen identical to those for 15 Bev in Reference 1. Table I lists parameters of both examples. Choosing the same N and k is partly justified by recalling the 1955 MURA proposal and other MURA discussions[2, 3, 4] in which essentially these numbers form the basis for a 10 Bev radial sector design. It appears that the field gradient is still reasonable, although now 29 kilogauss per meter (maximum) as compared to 22.5 in the Two-Way Model. The motivation for maintaining high k is very strong since it so strongly affects the cost and since it keeps the transition energy "outside" the machine.

II. PERFORMANCE AND DESIGN COMPARISON

A. Maximum Stacked Current Density

Except for factors close to unity arising from the detailed shape of the injected beam,

$$J_{max} = I_o \frac{k(\nu\Delta\nu)\, a\, \gamma^2}{R^3} \qquad \frac{FB}{P} = 6 \times 10^{11} \frac{a_2 \gamma^2}{R^3} \tag{1}$$

for the k, ν_z of Table I and $\Delta\nu$ of 1/4. Here F is the azimuthal rf compression factor near injection (.4), P the injection energy resolution (10^{-3}), B the phase space density filling safety factor (.25), and I_o the usual 3×10^7 amperes.

Table II gives numerical values for a final beam radius, a_2, of 2 cm for both cases. Here it is assumed that the current density is determined only by the space charge limit at injection and the momentum compaction at full energy.

B. Space Charge Limit at Stacking Energy

Assuming the beam unneutralized and no image effects, the space charge limited current at the stacking energy is

$$I_{max} = I_o (\nu\Delta\nu) \left(\frac{a_2}{R}\right)^2 \gamma^3 \cong 3 \times 10^7 \left(\frac{a_2}{R}\right)^2 \gamma^3 \qquad (2)$$

C. Injection Space Charge Limit

Assuming both beams injected to the same initial amplitude, a_1 = 15 cm (perhaps justified since the vertical stability limits appear very large) the injection space charge limited current is given by

$$I = I_o (\nu\Delta\nu) (\beta\gamma)^3 \left(\frac{a_1}{R}\right)^2 = 8.1 \times 10^5 \left(\frac{a_1}{R}\right)^2 \qquad (3)$$

for 50 Mev injection.

D. Other Criteria for Beam Size

The preceding sections assume the available apertures the same for both machine designs. For the beam to damp to the same final radius, a_2,

the injection radius is different. Very closely,

$$\frac{a_1}{a_2} = \sqrt{\frac{P_2}{P_1}} \tag{4}$$

If the beam size, a_1, is determined by stability limits, then a_1/R should be held constant, and a_1 for the 10 Bev example should be taken as 10 cm rather than 14. In this instance the injection space charge limit is 0.57 amperes for both examples, and J max_2 for 10 Bev will be about 75 amperes/cm^2 rather than 90 as determined in Part A.

E. Image Effects

The space charge limited current at maximum energy including effects of electrostatic and magnetic images (assumed over the entire circumference for simplicity) is given by

$$I_{image} = \frac{I_o \, \nu_z \, \Delta\nu_z \, \gamma_2}{R^2 \left[\frac{1 - \beta_2^2}{a_2^2} + \left(\frac{\alpha_E}{2t^2} + \frac{\alpha_B \beta_2^2}{2g^2} \right) \right]} \tag{5}$$

The quantities α_E and α_B are close to unity[5] and t and g are the conductor and iron semiapertures, respectively, at the stacking energy. The most important term in the denominator is $\alpha_E/2\,t^2$. For both examples, t = 3 cm, g = 7.5 cm was used. In this situation γ_2/R^2 is the relevant quantity proportional to I_{image}.

For both examples image effects are sufficiently serious to require some sort of cancellation.

F. Multiple Gas Scattering

The mean square multiple scattering angle is given by

$$\left\langle \theta_s^2 \right\rangle = \frac{2.8\ N\ Z^2}{E^2} \tag{6}$$

where N is in moles, E the particle energy in Mev, and Z the atomic number of the scattering gas. The beam life is given as

$$T = 4.9\ \frac{E^2 \left\langle \theta^2 \right\rangle}{P} \tag{7}$$

where E is in Bev, P in mm H_g of nitrogen, and θ the allowable scattering angle before beam destruction. Since $\theta \propto \nu_z/R$,

$$T \propto \frac{\nu_z^2\ E^2}{P\ R^2} \tag{8}$$

Thus, the beam life is the same for the 10 and 15 Bev examples since E/R is approximately the same.

G. Beam-Beam Bump

In symmetric, two-way accelerators, the vertical defocusing arising from the space charge and current effects of one beam on another at the points of intersection is a design constraint. The detuning effect is proportional to ν_{BB}^2, where

$$\nu_{BB}^2 = \frac{.4INR}{B\rho\ a_2 \tan\theta} \tag{9}$$

For the parameters chosen, the effect is the same for both 10 and 15 Bev designs.

H. Other Factors

The time to accelerate to full energy is less for 10 Bev than 15 Bev for the same volts per meter. This decreases the time required to attain a given stacked beam current in the 10 Bev case.

The charge injected per pulse is proportional to the injected current times the circumference, so that the time-average current, considering the accelerator operating as a one-way machine, is simply proportional to I_1.

III. STRUCTURE AND COST

A. Iron

From Reference 1, the iron volume is represented approximately by

$$V \propto \frac{R^3}{k^2} \tag{10}$$

Holding k constant indicates an R^3 dependence, however if the vertical gap and copper window are held constant between designs, the volume ratio will be less than the ratio of R^3.

B. Copper and Power

Holding the aperture and peak field invariant, the required ampere-turns are the same for both designs. The length of each winding is proportional to the linear plan dimensions of each magnet, so that the ratio of copper volume and power between the two designs is the ratio of the radii.

C. Vacuum Design

The cost of the vacuum system will be determined by the total area of vacuum tank. Since the vertical aperture is constant and much less than

the radial aperture, the area is proportional to R^2.

D. RF System

For the same voltage gain per meter, the number of rf cavities will be proportional to radius. The ferrite volume of each cavity will be reduced approximately linearly with radius, assuming the same cross section area. Thus the ferrite and its rf and biasing power should vary as R^2.

E. Buildings

The accelerator tunnel will vary in circumference and radial aperture as R, so that its costs should be roughly proportional to R^2. Other buildings (target, control, laboratory, etc.) will probably not depend on the accelerator energy.

F. Invariants

Costs of shielding, controls, injection gear, experimental apparatus, computing facilities, models, architecture and engineering, and staff should be independent or at most only slowly varying functions of machine energy for accelerators of this magnitude.

G. Costs

The budget of Reference 1 was broken into percentages. In Table III the percentages appropriate for the 10 Bev design are obtained by dividing the 15 Bev figures by the 15 Bev/10Bev ratio, assuming iron costs proportional to iron weight, etc. The total for the 10 Bev design should then indicate what fraction of the total cost of a 15 Bev installation would be required for a 10 Bev installation.

IV. CONCLUSIONS

It is striking how close to linear the machine cost appears. A reduction in k would increase the radial aperture and increase somewhat iron, copper, vacuum, power, rf, and building costs. It is worth noting that the time to complete a large machine of this type is a very slowly varying function of energy, and the number of Ph. D. physicists and senior engineers required probably independent of energy, unless goals are reduced below a few Bev.

It is tempting to some to think in terms of dramatic savings in iron and consequently in total accelerator cost resulting from a reduced design energy. For the two-way Ohkawa synchrotron (which is as extravagant of iron as any accelerator considered for this energy) this saving is not overwhelming. Perhaps it is appropriate to close with a theorem first enunciated by Haxby: starting from a given design, costs go up as the cube of the energy but go down linearly.

TABLE I

	10 Bev	15 Bev
N	66	66
k	209	209
ν_x	24	24
ν_z	4.25	4.25
R	124 meters	180 meters
B_{max}	18 kilogauss	18 kilogauss
γ	12	17
G injection	40 cm	40 cm
G max energy	15 cm	15 cm

TABLE II

Parameter	Formula	Value	
		10 Bev	15 Bev
J_{max_2}	(1)	$90 \frac{\text{amperes}}{\text{cm}^2}$	$60 \frac{\text{amperes}}{\text{cm}^2}$
I_{max_2}	(2)	1.35×10^3 amperes	1.8×10^3 amperes
I_1	(3)	1.18 amperes	0.57 amperes
a_1/a_2	(4)	6	7
γ_2/R^2		7.8×10^{-8}	5.25×10^{-8}

TABLE III

COST

Item	Dependence on Accelerator Radius (constant k, N)	15 Bev		Factor	10 Bev	
		Quantity	% of Cost		Quantity	% of Cost
Iron	$< R^3$	62,500 tons	33.6	25/62.5	25,000 tons	13.5
Copper	R	3,000 tons	6.7	2/3	2,000 tons	4.5
Vacuum	$\sim R^2$		7.0	1/2		3.5
Power Supplies	R	45 MW	1.8	2/3	30 MW	1.2
rf System	$\sim R^2$		6.4	1/2		3.2
Tunnel	$< R^2$		10.5	2/3		7.0
Invariant Items			34.0	1		34.0
TOTALS			100.0			66.9

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