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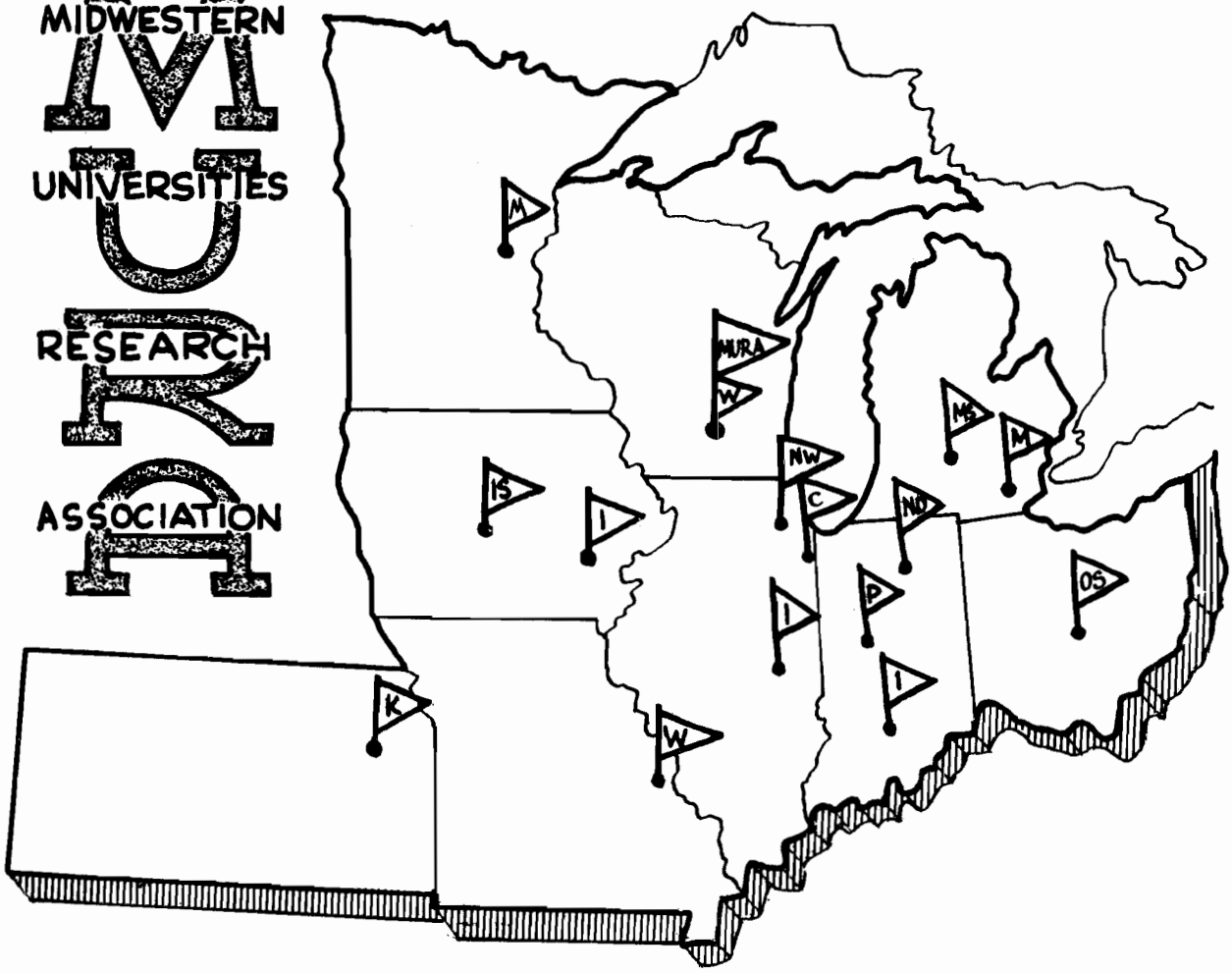

RESONANCE SURVEY IN THE ELECTRON MODEL SPIRAL SECTOR FFAG ACCELERATOR

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ABSTRACT

The relative intensity of the electron beam in the spiral sector model has been determined over a large part of the machines ν_x - ν_z stability region in order to study the effects of various resonances. The mean field index k has been varied from about 0.2 to 1.16 and the flutter has been varied from about 0.57 to 1.6. This has produced an observed variation in the radial betatron frequency, ν_x, from 1.18 to about 1.8 and in the vertical betatron frequency, ν_z, from < 0.4 to > 1.9. The beam is completely destroyed everywhere along the integral resonance ($\nu_z = 1$). The half integral resonances observed are $2\nu_z = 1$, $2\nu_z = 3$, $2\nu_x = 3$, $\nu_x + \nu_z = 2$, and $\nu_x + \nu_z = 3$, of which only $2\nu_z = 1$ and $\nu_x + \nu_z = 3$ show complete destruction of the beam along their entire length. The beam is also observed to be effected at the positions of the $\nu_x + 2\nu_z = 4$, $\nu_x + 2\nu_z = 5$, and $3\ _z = 2$ third integral non-linear resonances. Some effect of the Walkinshaw difference resonance ($\nu_x - 2\nu_z = 0$) is observed.

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A detailed measurement of the variation in intensity over a large part of the $\nu_x - \nu_y$ stability region of the Mark V spiral sector FFAG model has been made in order to study the effects of various resonances. Preliminary measurements of this type were earlier reported by Stump and Waldman.[1]

PROCEDURE

The working point of the machine was varied by changing the mean field index k and/or the flutter factor f. This was done by altering the currents in the appropriate tuning coils. The median plane field of the spiral sector model is represented approximately by the following equations:

$$B_z = B_o \left(\frac{r}{r_o}\right)^k \left[1 + f \sin\left(\frac{1}{w} \ln \frac{r}{r_o} - N\theta\right)\right]$$

$$B_r = B_\theta = 0,$$

where r is the radius from the center of the machine, k is the mean field index, f is the flutter factor, $\frac{1}{w} = N \tan \zeta$, N is the number of sectors around the machine, ζ is the spiral angle and θ is the azimuthal angle as measured from the center of the machine.

The electrons were injected into the machine at a radius of 31 cm. For this experiment the current in the k tuning coils was varied from -1.4 to +1.8 amps corresponding to varying k from about 0.2 to 1.16. The effect of different values of k on the field distribution along the median spiral of a sector is indicated in Fig. 1. The k tuning coils are wound such that no change is produced in the field at a radius of 25 cm. The average energy of the electrons increases at the injection radius and also at the detection radius as k is increased.

The effect of the flutter coils on the azimuthal field distribution along a constant radius of the machine at the median plane is shown in Fig. 2. As the flutter

is decreased the average value of the field, and therefore the average value of the electron energy, increases. The flutter coils were designed for a k of 0.7. Since no Δk - Δf coils were built, scaling errors are introduced which increase in magnitude as larger Δk and Δf currents are used. For this experiment the flutter current was varied from -1.7 to +1.7 amps corresponding to a flutter change from about 0.57 to about 1.6.

Since the change in k produced by the Δk coils varies with radius, it was decided to make the measurements fairly close to the injector. A radius of 37 cm was chosen as about the nearest one could get to the injection orbit and still clearly differentiate the accelerated beam from particles which were ejected from the machine at injection time. The beam intensity was estimated by the integrated pulse height from a plastic scintillation detector[1] inserted in the vacuum tank. During the intensity survey the injector filament current was kept low and constant; likewise the betatron core voltage, the gas pressure and the injection time were kept constant. The injector and inflector voltages were adjusted at each working point so as to peak up the beam. The magnet currents were cycled off and on four times after each adjustment to a new working point.

The next step after obtaining the intensity as a function of the tuning currents was to determine the values of the vertical and radial betatron frequencies corresponding to the various machine operating points. This was done (following the method described by Cole, et al[2] and Hammer, et al[3]) by rf resonance excitation of the betatron oscillations. An rf voltage of from 60 to 100 volts was applied on two small parallel plates inserted 120 degrees around the machine and at approximately the same radius as the scintillation probe. The rf frequency was then varied until

it was the proper value to enhance the amplitude of the betatron oscillations. To aid in differentiating vertical from radial knockout a push-pull type oscillator was designed by C. Nielsen and built by P. Nielsen. Vertical knockout was observed most strongly when this oscillator was connected symmetrically across the parallel plates and radial knockout was observed most strongly when both plates were fed at the same potential with respect to the ground of the vacuum tank. Vertical knockout of the beam was identified by a decrease in the intensity of the beam hitting the scintillation probe. This decrease in intensity was caused by the electrons with their increased vertical amplitude of oscillation hitting the walls of the vacuum tank or the vertical scanning probe. Radial knockout of the beam was identified by a shift in the time of arrival of the beam at the detector. The frequency of the rf oscillator corresponding to the various knockout frequencies was determined by mixing its signal with that from a Hewlett Packard Model 608D VHF Signal Generator and observing the resulting beat signal obtained on an oscilloscope. These frequency measurements could be reproduced to within about 0.2%. The frequencies at which resonance enhancement of the betatron oscillations was observed, in terms of the betatron oscillation frequencies, are of the form

$$f_{rf} = \left| p\,\nu_x \pm q\,\nu_z \pm m \right| f_o,$$

where p and q are 0, 1, or 2; m is a positive integer, and f_o is the particle revolution frequency. Two rf oscillators were used to cover a frequency range from about 12 megacycles to about 74 megacycles. The electron frequency of revolution at the detection radius (37 cm) varied from about 42 megacycles to about 58 megacycles over the tuning range covered by this experiment. A typical set of frequency measurements, assignments and results for a particular working point is given in Table I.

In order to obtain ν_x and ν_z from these measurements their approximate value must be known by other measurements.

TABLE I. Determination of ν_x and ν_z from rf resonance frequencies measured at the working point requiring no tuning coils.

Measured Frequency (Mc)	Character of Knockout	f_o (Mc)	$f_{rf}/f_{o_{av}}$	Assignment (f_{rf}/f_o)	Results
50.53	z strong	49.25	1.025	ν_z	ν_z = 1.025
47.96	z strong		.974	$2 - \nu_z$	1.026
27.48	z medium	49.24	.559	$2 - (\nu_x + \nu_z)$	$\nu_x + \nu_z$ = 2.441
21.76	z strong		.442	$(\nu_x + \nu_z) - 2$	2.442
29.74	z medium	49.40	.604	$1 - (\nu_x - \nu_z)$	$\nu_x - \nu_z$ = .396
19.66	z strong		.399	$\nu_x - \nu_z$	.399
49.27		49.27	1.00	f_o	
70.23	x very strong	49.42	1.426	ν_x	ν_x = 1.426
28.63	x very strong	49.43	.581	$2 - \nu_x$	1.419
20.80	x very strong		.422	$\nu_x - 1$	1.422
41.13	x strong	49.26	.835	$2\nu_x - 2$	$2\nu_x$ = 2.835
57.38	x medium		1.163	$4 - 2\nu_x$	2.837
46.40	x medium		.941	$3 - 2\nu_z$	$2\nu_z$ = 2.059
75.43	x medium		1.530	$5 - (2\nu_z + \nu_x)$?	
Average		49.3			ν_x = 1.420 ν_z = 1.026

RESULTS

The intensity survey as a function of the tuning currents was taken in a series of runs where one of the tuning currents was kept constant (generally that for the k

tuning coils) and the other was varied over most of its range in such a way as to determine most of the maxima and minima of intensity. A typical plot obtained by this procedure is shown in Fig. 3. This plot shows the relative beam intensity measured by the scintillation detector as a function of the current in the flutter tuning coils for the case where there was no current in the Δ k coils. The experimentally measured flutter at three points is indicated at the bottom of the graph. The vertical and radial betatron frequencies measured by the rf resonance method are indicated above the curve. Locations of certain resonances are also indicated on the graph.

The composite result of the intensity survey as a function of the tuning currents is given in Fig. 4. The darkest regions represent areas of maximum intensity, the lighter regions represent lesser intensities. Adjacent areas differ in intensity by a factor of the square root of 10 exce pt for the least intense region which represents a change in intensity of a factor of 10. The approximate measured values of k and flutter corresponding to the tuning currents are shown on the graph. The values for the flutter were obtained when there was no Δ k current and the values for the mean field index k were obtained when there was no Δ flutter current.

By measuring the betatron frequencies as a function of tuning currents it was possible to obtain interpolation graphs which could be used to transfer points from the intensity survey as a function of the tuning currents to an intensity map in terms of the betatron oscillation frequencies. The graph in Fig. 5 shows how the vertical and radial betatron frequencies depend on the tuning currents. The resonance diagram which was obtained by use of the interpolation procedure is shown in Fig. 6. The relative intensity scale indicated on the figure is the same as that for Fig. 4. The

increasing width of the resonance regions as one goes from the center of the plot is mostly a result of the increasing non-scaling of the field as the tuning currents are increased. This is in agreement with the observation that the resonance regions are broader on the diagram obtained by Stump and Waldman[1] who worked at a larger radius and therefore should have a greater non-scaling effect. An indication of the scaling effect without tuning coils is given in Fig. 7 by the curves showing measured frequency as a function of radius.

DISCUSSION

One source of uncertainty in all of these measurements is that the amplitudes of the betatron oscillations were not sufficiently controlled. In the measurement of intensity as a function of the tuning currents the injector and inflector voltages were adjusted for each point so as to obtain the maximum intensity. For a given working point the amplitude of the radial oscillations increases as the injection energy increases. The amplitude of the vertical oscillations may increase or decrease. The maximum amplitude of the vertical oscillations was limited by the walls of the vacuum tank during the intensity survey. When the measurements were made of the frequencies of the betatron oscillations at the various working points no quantitative attempt was made to return to the same injection energy as was used in the intensity measurement other than to attain again the conditions for maximum intensity. To observe the vertical knockout clearly it was generally found desirable to reduce the vertical aperture by means of the vertical scanning probe to a point where it was just beginning to affect the beam observed at the scintillation detector. The betatron frequencies as a function of the amplitude of oscillation at the design point are presented in the curves in Fig. 8. It is seen that though the variation of the x tune

with amplitude is small, the variation of the z tune with amplitude is considerable. It is estimated that there is about a 1% to 2% indeterminancy in the values of ν_z obtained as a result of this variation of frequency with amplitude. The indeterminancy in the value of ν_x caused by this effect is probably much less.

Upon investigation of Fig. 6 it is observed that the positions of the resonance lines often do not coincide with the center of the troughs of minimum intensity. One obvious reason for this effect is the non-scaling of the fields which can cause the trough to shift one way or the other from the position determined by frequency measurements at the detection radius. Another possible cause is the interpolation method used to obtain the resonance diagram from the intensity survey. Because of the difficulty observed in interpreting knockouts near a resonance all operating points for which vertical and radial betatron oscillation frequencies were obtained were in good beam regions. It has been observed by C. H. Pruett with the radial sector model that the measured ν_x - ν_z relationship to the tuning currents often departs from a smooth relationship (such as indicated in Fig. 5) near a resonance. A further source for shifting the apparent position of the resonances is the dependence of the tune on the amplitude of oscillation as discussed above. The betatron frequencies were in general measured with a restricted vertical amplitude whereas the intensity measurements were made with the vertical amplitude restricted only by the walls of the vacuum tank. Therefore there is a spread in frequency present at any given working point which would be expected to narrow any resonance and, depending upon the position of the vertical scanning probe when measuring knockout frequencies with respect to the minimum and maximum vertical amplitudes, to shift the apparent position of the center of the resonance.

Further observation of the resonance diagram in Fig. 6 shows that the integral resonance has the strongest effect on the beam, which is what one would expect. Near an integral resonance a field inhomogeneity or misalignment which introduces inhomogeneous terms in the equations of motion causes large displacements of the equilibrium orbit. When the tune is exactly integral, no closed equilibrium orbit exists with such inhomogeneities. The only integral resonance observed with certainty here, the $\nu_z = 1$ resonance, completely destroys the beam over a fairly large stop band which however, is considerably narrower than the $\Delta\nu = \pm 0.1$ stop band widths observed for the integral resonances in the radial sector model.[2] Though the range of the flutter coils was such that one would expect to detect the location of the $\nu_z = 2$ resonance, the non-scaling of the tuning coils, especially at the relatively high tuning currents required, make observations in this region uncertain.

The half integral resonances which are driven by terms introduced chiefly by imperfections in the mean field index k are represented here by the $2\nu_z = 1$, $2\nu_z = 3$, and $2\nu_x = 3$ resonances and also by the linear sum resonances $\nu_x + \nu_z = 2$ and $\nu_x + \nu_z = 3$. The $\nu_z + \nu_x = 3$ machine (or imperfection) resonance coincides, in this machine, with the 4th integral $2\nu_x + 2\nu_z = 6$ ($2\sigma_x + 2\sigma_z = 2$) sector (or essential) resonance and may be the reason this resonance appears to have a much stronger effect than the other half integral resonances. Only the $\nu_x + \nu_z = 3$ and the $2\nu_z = 1$ half integral resonances show complete destruction over their entire length of action on the diagram. This differs from the results obtained with the radial sector model where all half integral resonances observed destroyed the beam with a stop band width of $\pm .01$. One

reason for the half integral resonances in general not showing complete destruction along their entire length may be the non-scaling character of the various field coils. Because of this non-scaling character the resonances are crossed somewhere between the injection of the electrons and their detection. This would have the effect of making the resonance regions broader and shallower. A second reason for the resonances not appearing any sharper than they do is the spread in tune of the machine at each working point due to the change in betatron frequency with amplitude of oscillation. A third effect which would cause the resonance regions to be less effective in reducing the beam intensity might be correct shimming of the field.

The effects of several third integral non-linear resonances are seen. They are the $\nu_x + 2\nu_z = 4$, $\nu_x + 2\nu_z = 5$ and $3\nu_z = 2$ resonances. It is somewhat surprising that the $\nu_x + 2\nu_z = 3$ non-linear resonance does not appear equally as strong.

The "Walkinshaw" resonance ($\nu_x - 2\nu_y = 0$) does not by itself cause instability, but couples radial and vertical motion. This coupling can drive the motion to a different resonance because of the variation of tune with amplitude, so that the Walkinshaw resonance can indirectly cause instability. There appears to be clear evidence for the Walkinshaw from the resonance diagram in Fig. 6. It is felt that the effect of this resonance may be as small as it is because of the care that was taken with the shimming of the fields. This can be investigated by placing planned bumps in the field. The Walkinshaw is the only sector resonance that can be clearly differentiated in this machine as all other sector resonances coincide with imperfection resonances because of the choice of six sectors.

It is interesting to note that the design working point (indicated by the cross in Fig. 6) is in the center of the largest region of highest intensity indicating that a good

choice was made for the design parameters. The experimental working point (that with no tuning coils) is indicated by the x on the diagram and is seen to be fairly close to the design point. The chief reason for the difference in location of the two points is that the experimental flutter is about 5% lower than the design flutter.

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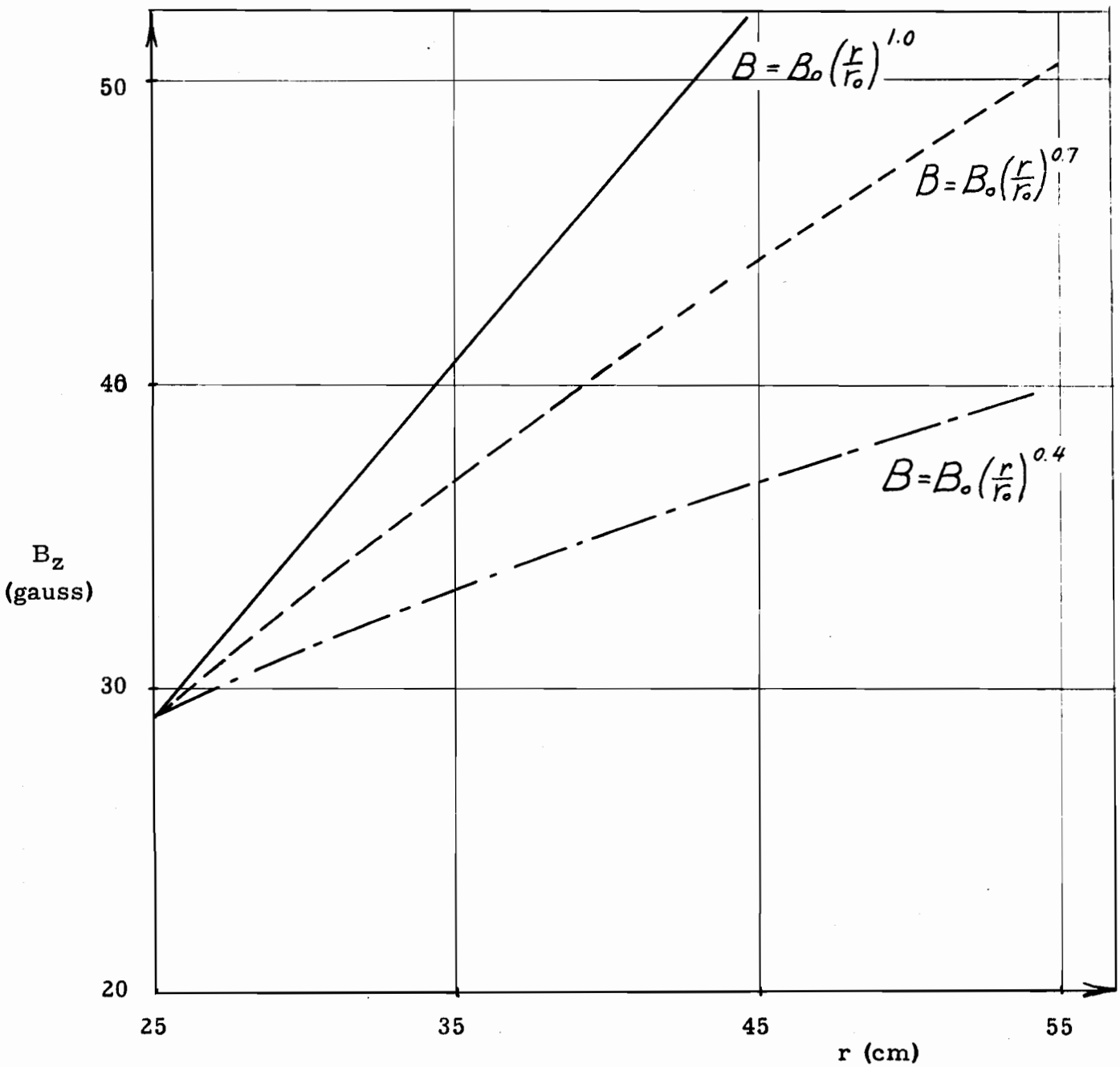


FIG. 1

EFFECT OF CHANGE IN MEAN FIELD INDEX k ON FIELD DISTRIBUTION ALONG MEDIAN SPIRAL OF A SECTOR - SPIRAL SECTOR MODEL

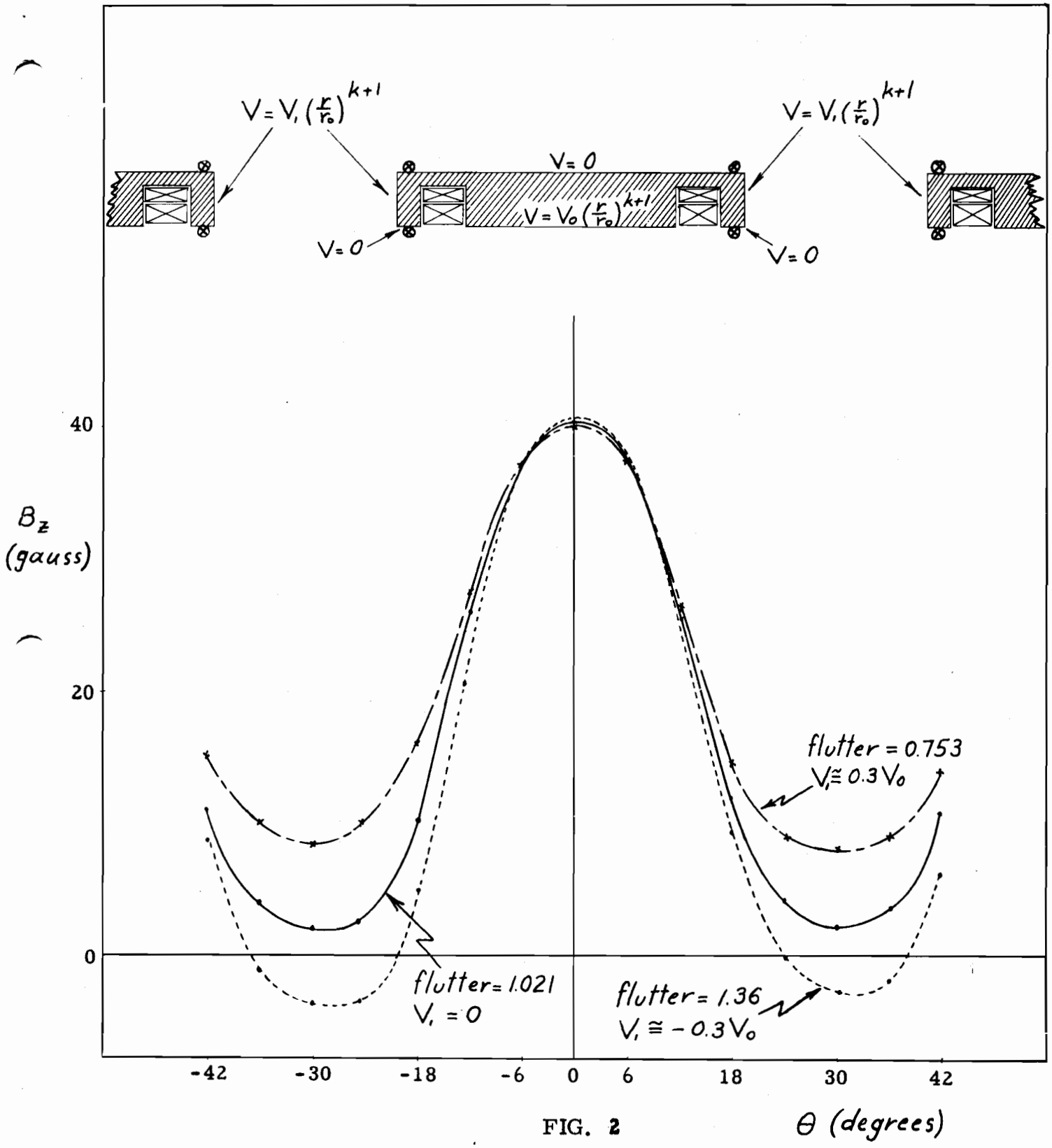


FIG. 2

EFFECT OF CHANGE IN FLUTTER ON AZIMUTHAL FIELD DISTRIBUTION - MEASURED FIELDS - SPIRAL SECTOR MODEL

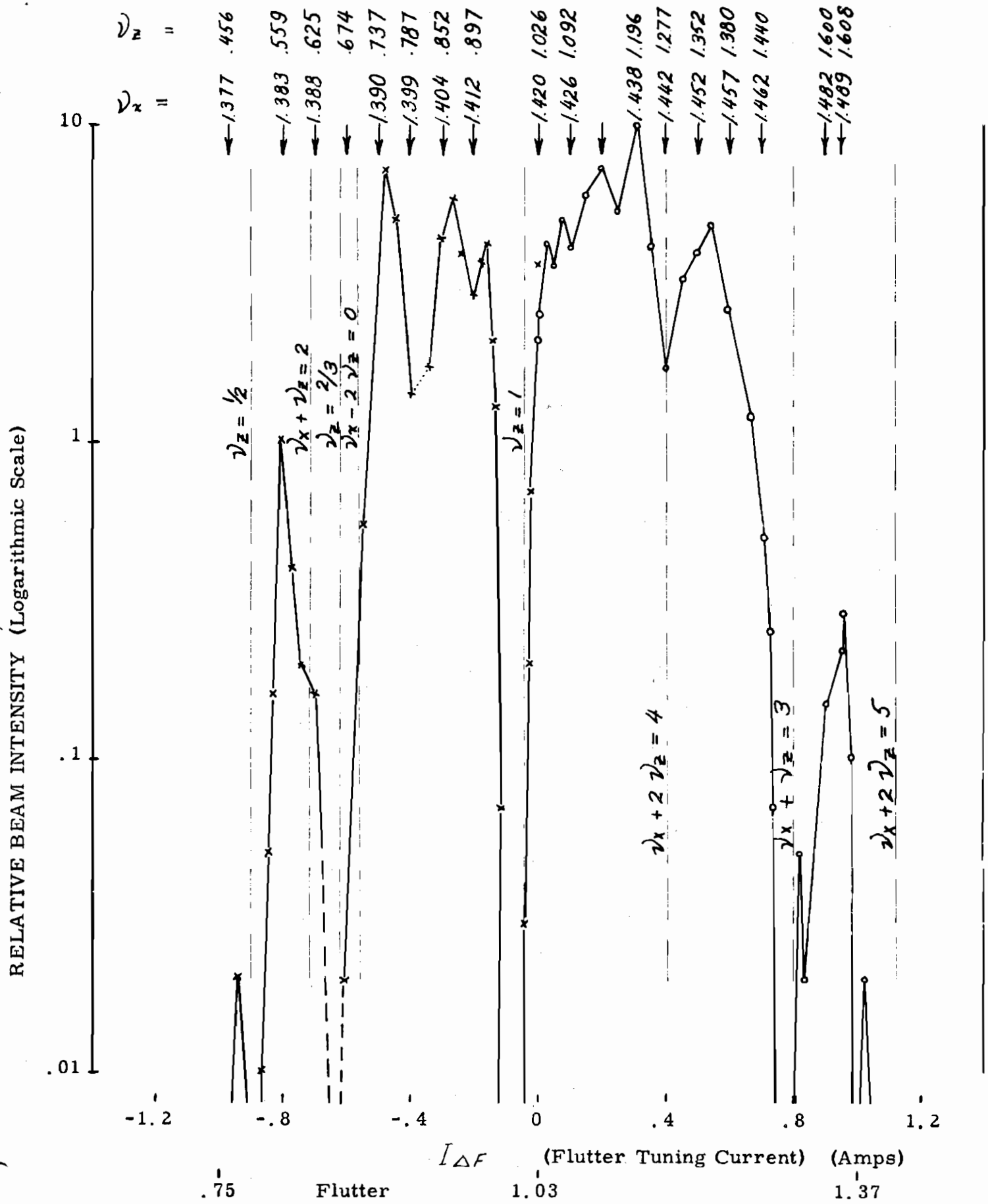


FIG. 3

BEAM INTENSITY AS A FUNCTION OF FLUTTER TUNING CURRENT WITH NO CURRENT THROUGH THE Δk COILS. THE ν_x AND ν_z MEASURED BY THE rf RESONANCE METHOD ARE INDICATED ABOVE THE CURVE. POINTS WHERE CERTAIN RESONANCES WERE CROSSED ARE ALSO INDICATED.

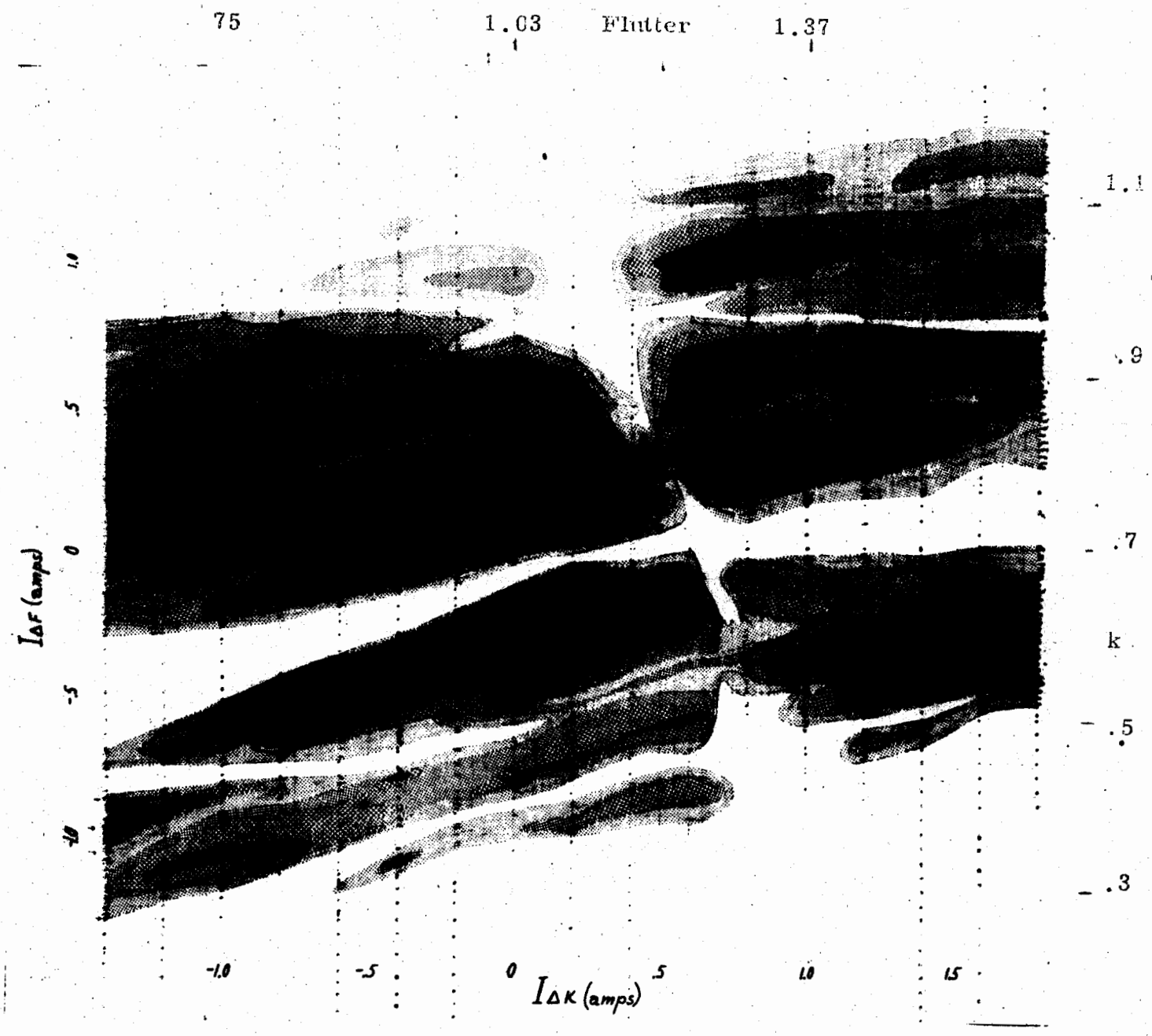


FIG. 4 Intensity Survey. Values of k indicated along the y axis were measured with no flutter tuning current. Values of flutter indicated along the abscissa were measured with no k tuning current. Areas of different beam intensity are represented by different shading. The darkest regions are the most intense. Lighter regions are less intense and differ in intensity by factors of the $\sqrt{10}$ except for the lightest region which represents an intensity change of a factor of 10.

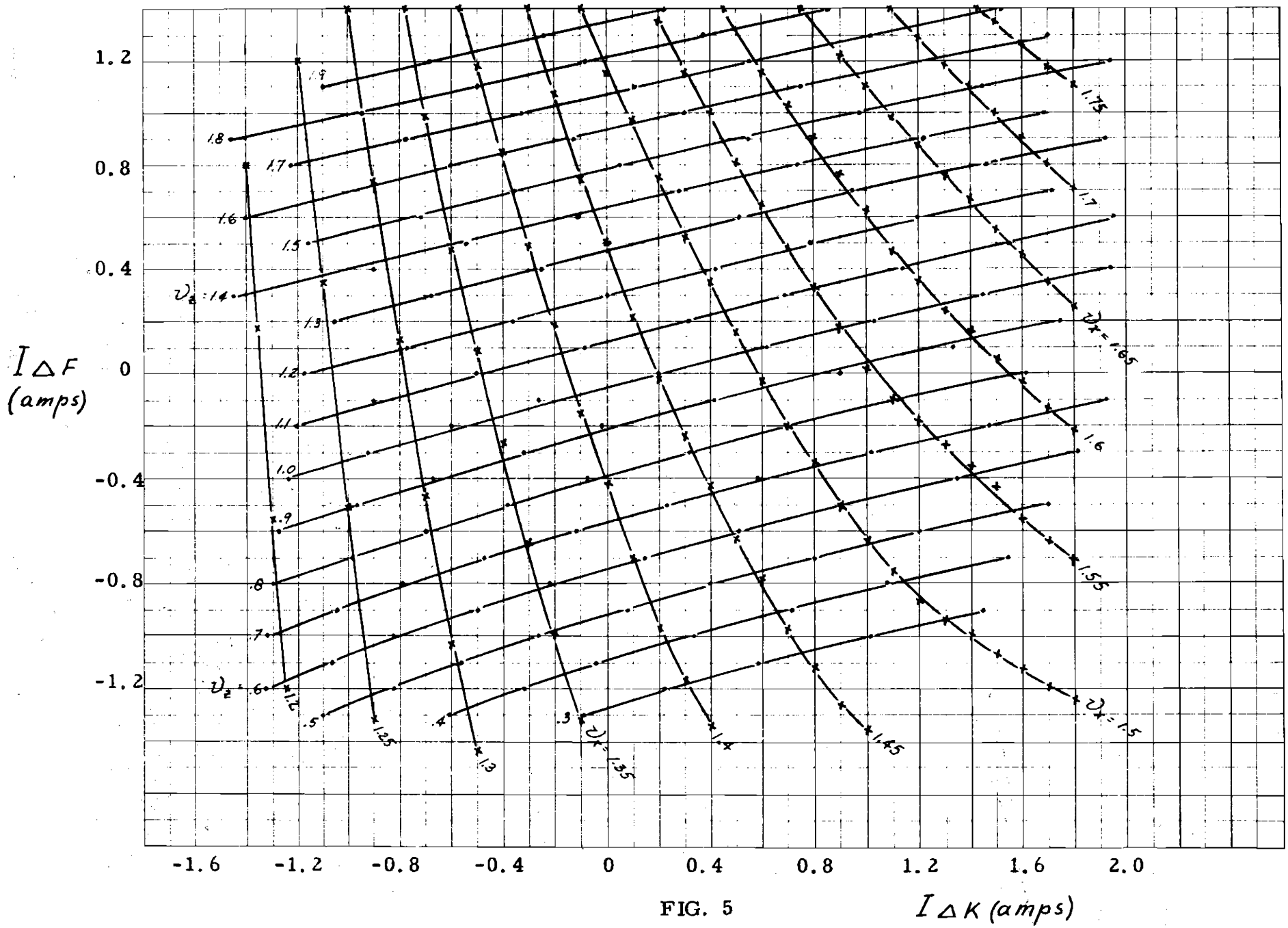
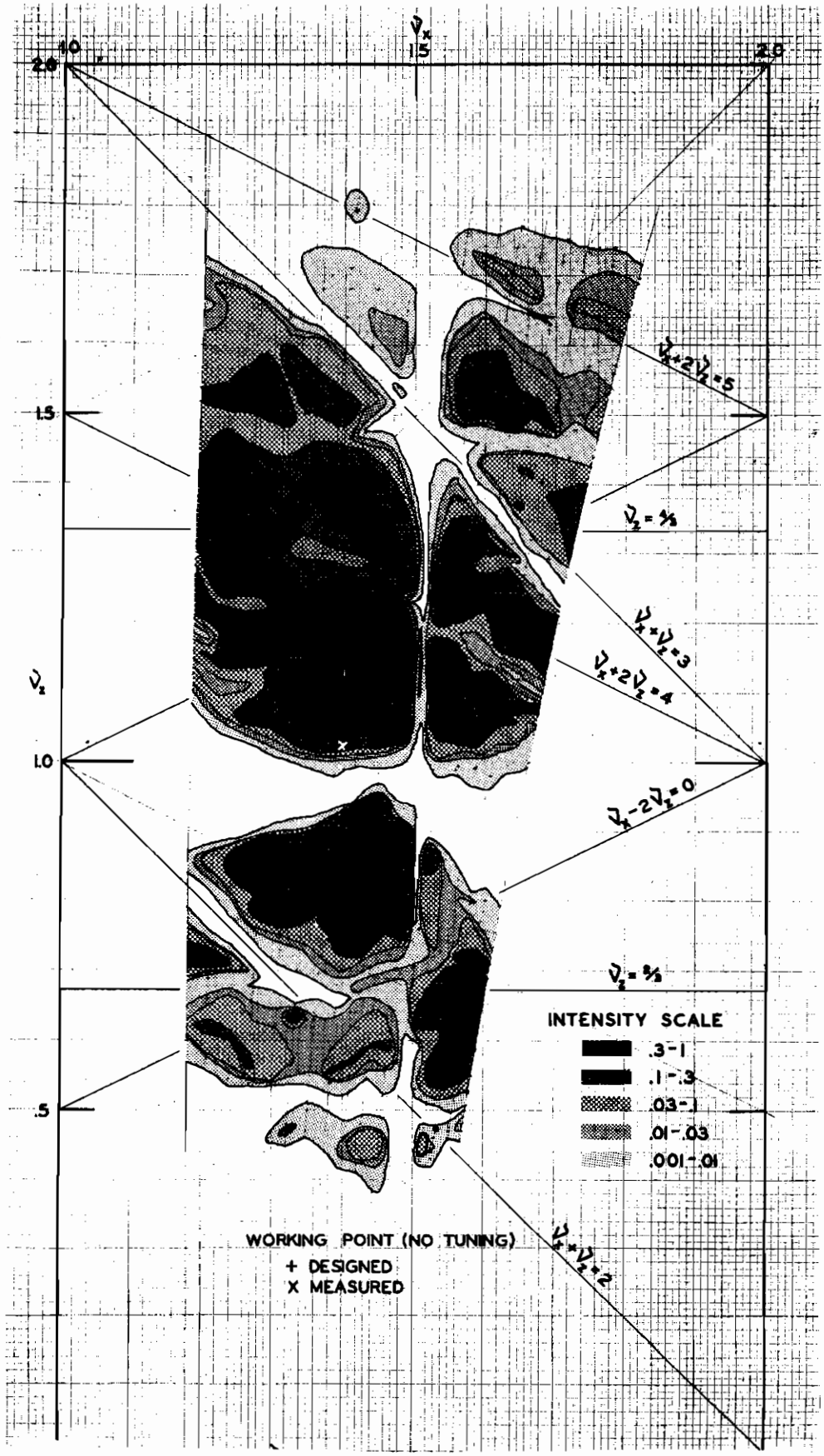


FIG. 5

RELATION OF BETATRON FREQUENCIES TO TUNING CURRENTS



Resonance Survey - Spiral Sector Model

Figure 6

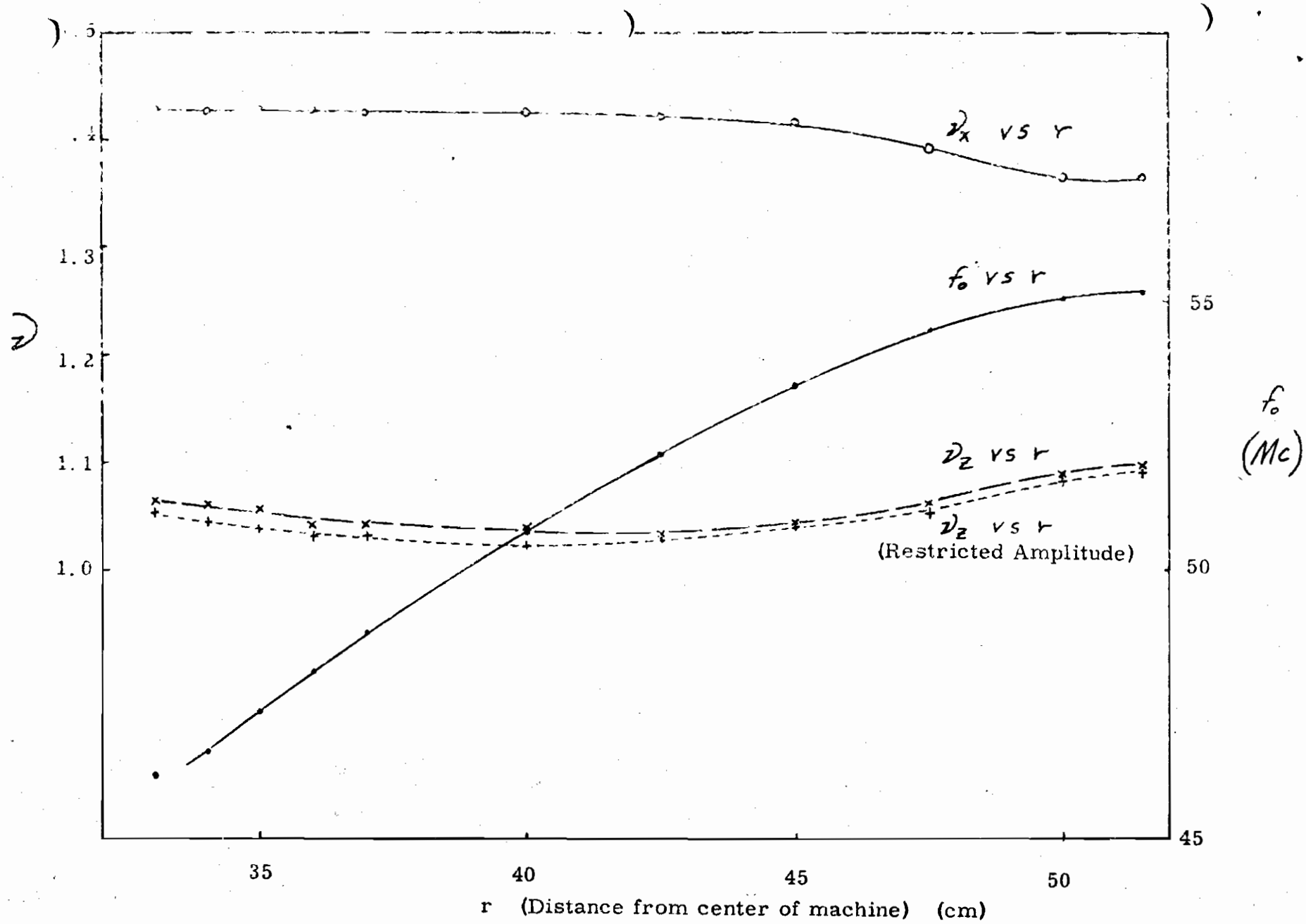


FIG. 7

MEASURED BETATRON AND REVOLUTION FREQUENCIES AS A FUNCTION OF r. NO TUNING CURRENTS

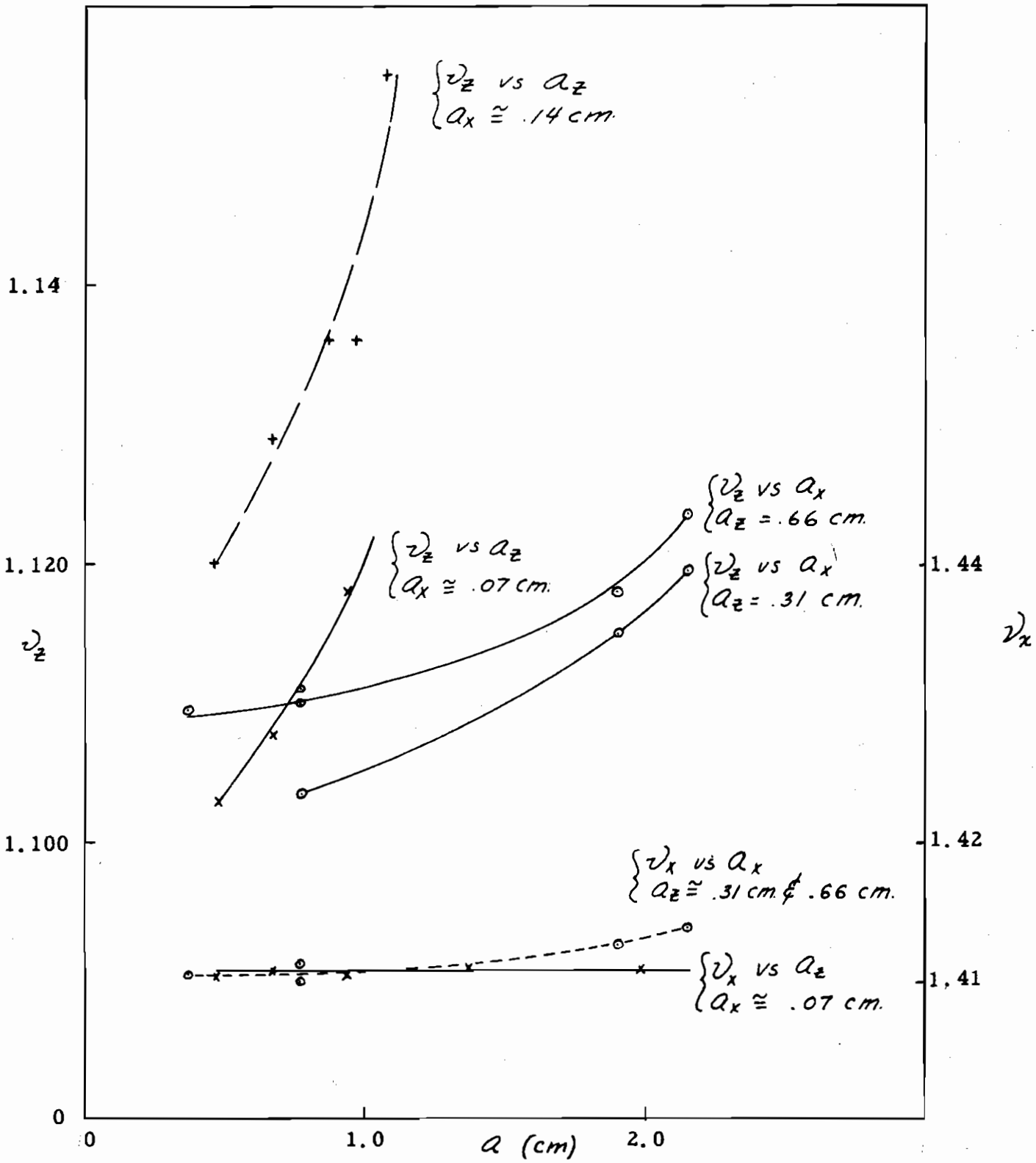


FIG. 8

MEASURED VARIATION OF TUNE WITH AMPLITUDE OF OSCILLATION AT DESIGN POINT- SPIRAL SECTOR MODEL