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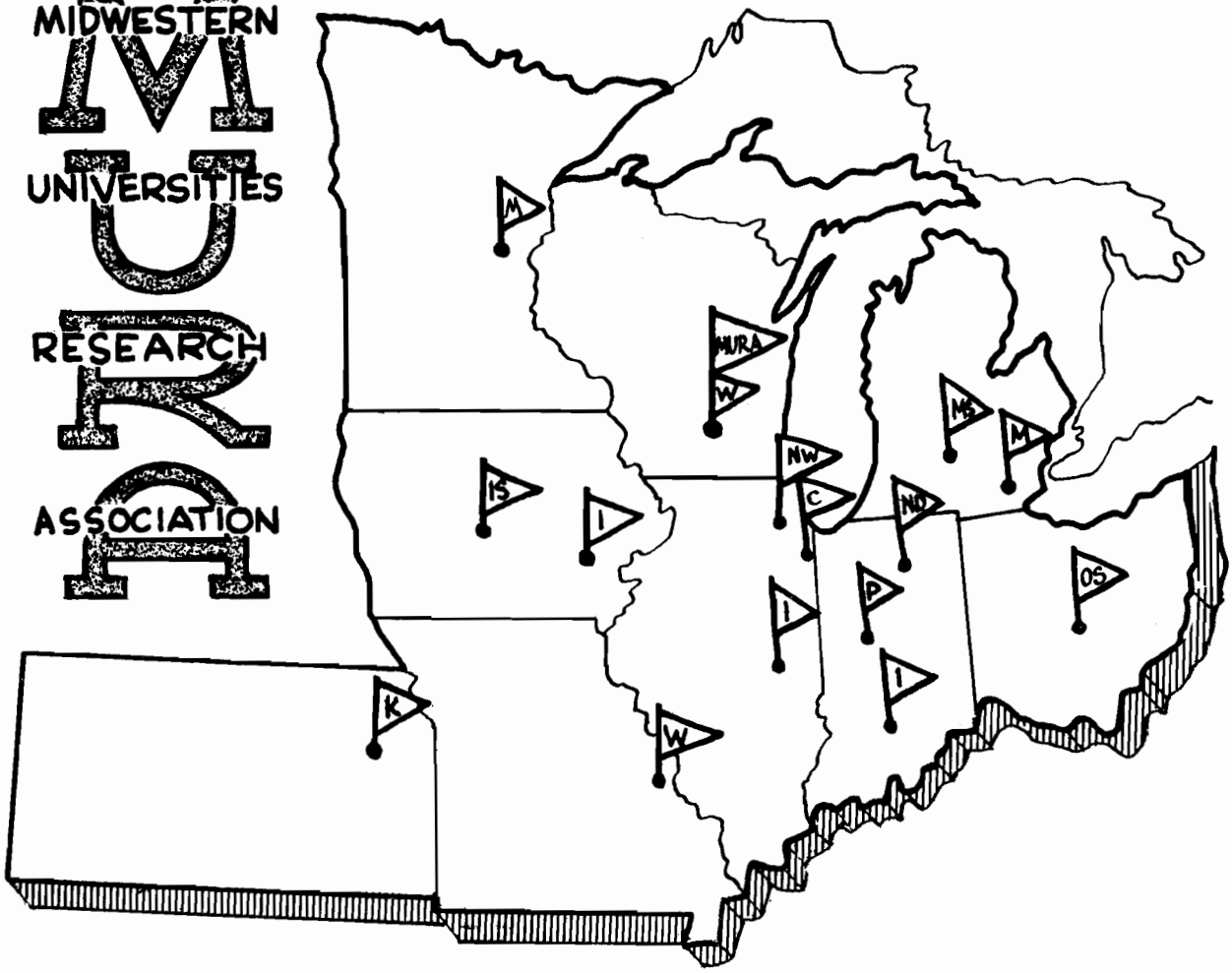


# ON CONSTRUCTING STRAIGHT SECTIONS IN A TWO-WAY FFAG ACCELERATOR

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# ON CONSTRUCTING STRAIGHT SECTIONS IN A TWO-WAY FFAG ACCELERATOR

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## ABSTRACT

This report outlines some general principles for constructing straight sections in a two-way FFAG accelerator. It is suggested that in order not to decrease the stability of the machine, the straight sections should not be so large as to appreciably affect the linear tune of the machine. It is also suggested that the effect of the straight sections on the equilibrium orbit may be controlled by the use of deliberate bumps which introduce the proper harmonics into the magnetic field.

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## I. INTRODUCTION

This report will outline some general principles for constructing straight sections in a two-way FFAG accelerator. The problem involved is to construct straight sections which will not seriously decrease the stability of the machine and, at the same time, will not seriously increase the circumference factor of the machine.

In order not to seriously decrease the stability of the machine, the straight sections should not be so large so as to appreciably change the linear tune( $\nu_x$, $\nu_y$ ) from the value it had before the straight sections were put in. If the straight sections are so large as to appreciably affect the tune, they will decrease the stability of the machine in at least the following two ways:

1) They will increase the sensitivity to misalignments.
2) In an FFAG machine, the tune changes with the amplitude of the betatron oscillations. Straight sections will cause the tune to vary more rapidly as the amplitude changes, and thus the tune will be driven on to a bump resonance at a smaller amplitude.

Since the linear tune ( $\nu_x$ , $\nu_y$ ) is relatively insensitive to perturbations of the magnetic field, one can introduce surprisingly large straight sections without affecting the linear tune very much. However, the equilibrium orbit is relatively sensitive to magnetic field perturbations, and the effect of the straight sections is to increase the size of the oscillations of the equilibrium orbit, which means that the circumference factor of the machine is increased.

It seems likely, however, that the size of the oscillations of the equilibrium orbit can be controlled by introducing a deliberate bump in the magnetic field having

the properties which are described in Section III. If this deliberate bump can be introduced as is described in Section III, and if it does control the size of oscillations of the equilibrium orbit, then the length of the straight sections will be limited only by the requirement that the linear tune not be affected.

## II. TUNE CONSIDERATIONS

If one introduces just one straight section into a two-way machine, which straight section subtends the angle $\Delta\theta$, then in order for the tune not to be affected by the straight section, $\Delta\theta$ is restricted by

$$\Delta\theta \ll \frac{\pi}{2}\frac{N}{K}\frac{1}{C-2} \tag{2.1}$$

where C is the circumference factor of the machine.

For N = 64, K = 200, this would mean $\Delta\theta \ll .112$. For a machine radius, R, of 180 meters, this means $R\Delta\theta \ll 20.2$ meters. This would suggest that one can insert straight sections of about 2 meters without changing the tune very much. Possibly, one could have straight sections up to about 4 meters without affecting the stability of the machine too much.

If more than one straight section is introduced, the above results are roughly unchanged. If M straight sections are introduced, the magnetic field is perturbed M times as much as for one straight section. However, the resonance lines introduced by the straight sections are now M times further apart, and if one chooses the operating tune properly, the two effects will approximately cancel.

Note that one may add to the above straight sections the straight section space already available in each sector.

## III. EQUILIBRIUM ORBIT CONSIDERATIONS

So far, tune considerations permit straight sections of about 3 meters, without affecting the tune and stability of the machine very much. A 3 meter straight section will, however, considerably disturb the equilibrium orbit. The oscillations of the equilibrium orbit are increased by $\Delta x$ where

$$\Delta x \cong \frac{2}{\pi} \frac{\Delta \theta}{N} (C-2) ; \tag{3.1}$$

$\Delta x$ is in units of R. Eq. (3.1) is for one straight section and $\nu_x$ is assumed to be .25 from an integer.

For N = 64, K = 200 and for a 3 meter straight section $\Delta x$ = .0075 This would give an increase in circumference factor of $K \Delta x$ = 15%.

Theoretical considerations would indicate that this increase in the size of the oscillations of the equilibrium can be removed by the use of a deliberate bump. We introduce a perturbation of the magnetic field $\Delta H_z$, which has only harmonics $\Delta H_{z,n}$ for $n \simeq \nu_x$ and these harmonics are just equal and opposite to those introduced by the straight section.

Since the equilibrium orbit is mainly affected by those harmonics introduced by the straight section for which $n \simeq \nu_x$, the above deliberate bump will, to first order, cancel the distortion of the equilibrium orbit caused by the straight section.

This "pure harmonic" bump having only harmonics for $n \simeq \nu_x$ can be obtained by going around the machine and varying the excitation of each magnet by a slight amount so as to produce a ripple in the magnetic field having the form

$$\Delta H_z \simeq -\Delta H \left(\frac{r}{r_1}\right)^K \sin \nu_x \theta \tag{3.2}$$

The size of the bump needed to cancel the distortion of the equilibrium orbit caused by the straight section is given (for one straight section) by

$$\frac{\Delta H_z}{H_z} \cong \frac{1}{\pi} \frac{\Delta \theta}{N} \tag{3.3}$$

A meter straight section would require a bump of about $\Delta H_z / H_z \simeq .0015$, or a $\Delta H_z$ of about 25 gauss at the outer radius. This result is for a single straight section. M straight sections would require an M times larger bump.

It will be necessary to carry out computer studies to see if deliberate bumps of the "pure harmonic" type can truly control the oscillations of the equilibrium orbit as the theory indicates.

## IV. CONSIDERATIONS OF THE NUMBER OF STRAIGHT SECTIONS

The number of straight sections one can insert depends on the particular location of the operating tune ( $\nu_x$ , $\nu_y$ ). We will assume that $\nu_y \simeq$ 4 and $\nu_x \simeq$ 20. The number of straight sections, M, that can be introduced is restricted by the consideration that the resonances introduced by the straight sections should not lie too close to the operating tune ( $\nu_x$ , $\nu_y$ ).

If one introduces just one straight section, then resonance lines are introduced at $\nu_x$ (or $\nu_y$ ) = .5, 1, 1.5, 2--which means the operating tune is about .25 from the resonance lines. If M straight sections are introduced, one should try to make the operating tune be about .25 M from the resonance lines, which now are

at $\nu_x$ (or $\nu_y$) = $\frac{1}{2}M, M, \frac{3}{2}M, 2M \ldots$ assuming that the straight sections have no symmetry which would eliminate certain harmonics.

A choice of M which seems favorable is M = 5, with $\nu_y$ = 3.75, $\nu_x$ = 21.25. This would give 5 straight sections and the operating tune is about 5/4 from the resonance lines.

Another choice that may be considered is M = 16, with $\nu_y$ = 3.75, $\nu_x$ = 20.25. This would give 16 straight sections and the operating tune is about 16/4 from the resonance lines. This would, however, require a large harmonic bump to restore the equilibrium orbit and it may not be possible to restore the equilibrium orbit.

It is possible to construct straight sections with the proper symmetry such that only odd harmonics are introduced into the magnetic field. The resonance lines of such straight sections lie somewhat differently, and this may be advantageous for some particular operating tunes. In this case, if we have 2 M straight sections, there are M superperiods, and the resonance lines lie at $\nu_x$ (or $\nu_y$) = $\frac{1}{2}M$, $\frac{3}{2}M$, $\frac{5}{2}M$, ....

In this case the choice of 2 M = 4 straight sections, which has been suggested by F. Cole and K. Symon, seems favorable. This would give 4 straight sections and with $\nu_y$ = 3.75, $\nu_x$ = 20.25, the operating tune is about 3/4 from the resonance lines. This distance of the operating tune from the resonance lines is not quite as good as was obtained by using the usual straight sections, but this may not be serious.

Another choice that may be considered is 2 M = 8, with $\nu_y$ = 3.75 and $\nu_x$ = 20.25. This would give 8 straight sections and the operating tune is about 7/4 from the resonance lines.