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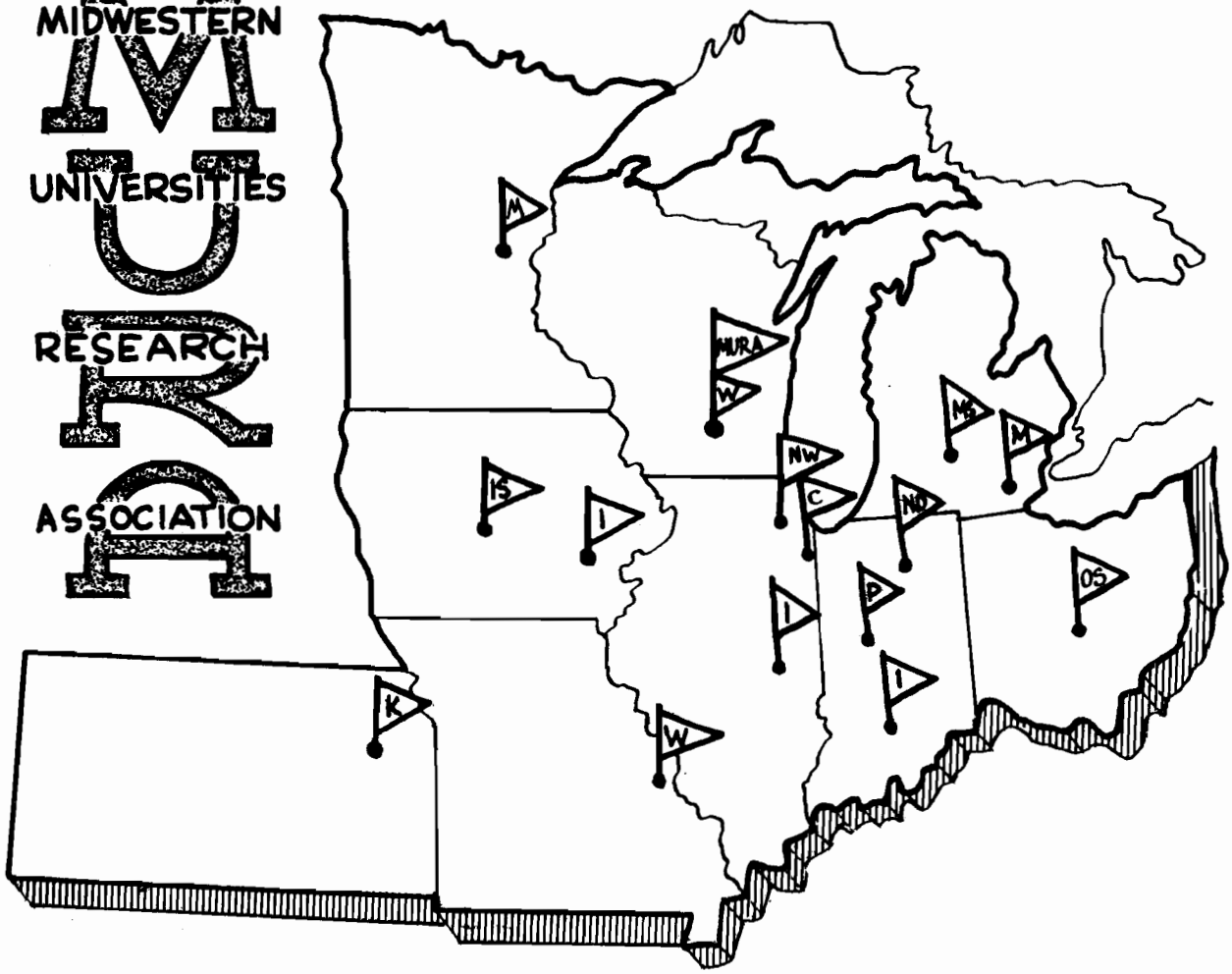

THEORETICAL DESIGN OF A LARGE TWO-WAY RADIAL SECTOR FFAG ACCELERATOR

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THEORETICAL DESIGN OF A LARGE TWO-WAY RADIAL SECTOR FFAG ACCELERATOR

George Parzen**

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ABSTRACT

Rough theoretical expressions are given for the tune, circumference factor, stability limits, and the effect of misalignments for a large two-way radial sector FFAG accelerator.

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I. INTRODUCTION

While the actual design of a big two-way radial sector machine will require a good deal of computer work, such as that being done by F. T. Cole, the design of the machine can be determined in a rough way by theoretical considerations. It is likely that the results eventually found by the computer will not differ too greatly from those found theoretically.

In this report, we will consider the tune, circumference factor, stability limits, and the effect of misalignments. We will give rough expressions for these quantities for a big two-way machine, and we will give numerical results for the particular machine having N = 64, K = 200, and $H_z = -H\left(\frac{r}{r_i}\right)^K \cos N\theta$.

Please note that the expressions are sometimes written so as to apply only to the case of a large two-way radial sector machine.

II. TUNE AND CIRCUMFERENCE FACTOR

The tune is given by[1, 2]

$$\nu_x^2 = 2K, \tag{2.1}$$

$$\nu_y^2 = \frac{N^2}{K} \tag{2.2}$$

The circumference factor is given by[1, 2]

$$C = \sqrt{2}\,\nu_y + 2, \tag{2.3}$$

$$C = \sqrt{\frac{2N^2}{K}} + 2,$$

Numerical Results

For the machine with N = 64, K = 200, we find

$$\nu_x = 20$$

$$\nu_y = 3.13$$

$$C = 6.5$$

III. MISALIGNMENTS

It seems that the severest restriction on the size of misalignments stems from the displacement of the equilibrium orbit in the vertical direction by the misalignment.

We will describe the misalignment in two ways. We can describe it in terms of an error in the median plane magnetic field, or we can describe it in terms of displacements of the magnets.

Let ΔH be the largest error in the radial component of the magnetic field on the equilibrium orbit, and let H be the largest value of the magnetic field on the equilibrium orbit. Then the field error is described by

$$\delta = \frac{\Delta H}{H} , \tag{3.1}$$

If δ is due to a vertical misalignment, ΔZ, of a magnet then ΔZ and δ are related by

$$\delta = K \frac{\Delta Z}{R} \tag{3.2}$$

where R is the radius of the equilibrium orbit.

The allowable field error depends on how much displacement of the equilibrium orbit we allow. Suppose we assume that we will allow a displacement of 1/10 of the

vertical half-gap. Then the r.m.s. allowable field error in each magnet is given by

$$\delta = \frac{1}{10} \frac{\sqrt{2N}}{C} \frac{\nu_x}{4} \frac{\text{half-gap}}{R} \tag{3.3}$$

where C is the circumference factor. The allowable r.m.s. radial misalignment of each magnet is then given by

$$\Delta Z = R \frac{\delta}{K} \tag{3.4}$$

Numerical Example

If N = 64, K = 200, C = 5.5 and R = 150 meters, half-gap = 10 cms.

$$\delta = 1.06 \times 10^{-4}$$

$$\frac{\Delta Z}{R} = .53 \times 10^{-6}$$

$$\Delta = 7.5 \times 10^{-3} \text{ cms}$$

It is believed that the above results for δ and Δr should be good to about a factor of two.

IV. STABILITY LIMITS

The range of tune that one can achieve in a large two-way radial sector machine is severely limited. It is probable that the

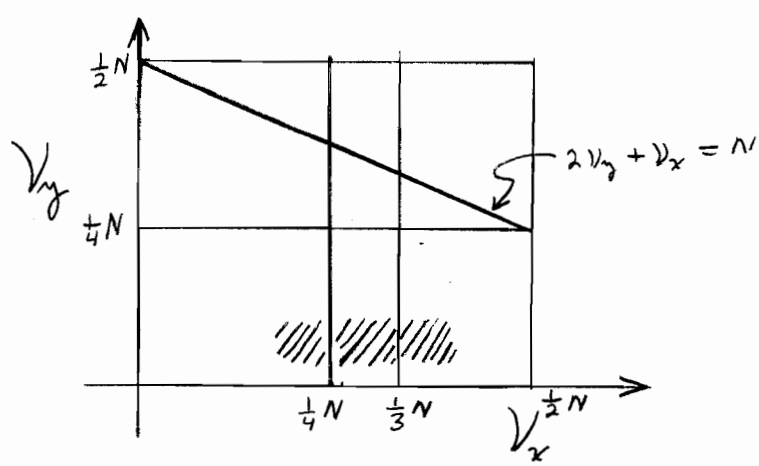

Fig. 1

tune will lie in the shaded region shown in the tune diagram in Fig. 1.

If this is the case, then the radial stability limit will most likely be due to the $\nu_x = \frac{1}{3} N$ resonance, unless one happens to be exceptionally close to the $\nu_x = \frac{1}{4} N$ resonance. The vertical stability limit will most likely be due to the $2\nu_y + \nu_x = N$ resonance.

The Radial Stability Limit

We will assume that the radial stability limit is due to the $\nu_x = \frac{1}{3} N$ resonance. Then the radial stability limit, A_x, is given by[3, 4]

$$A_x = \frac{2}{N^2} \frac{\left| \left(\frac{\nu_x}{N}\right)^2 - \frac{1}{9} \right|}{M} \tag{4.1a}$$

where

$$M = \left(\frac{K}{2N^2}\right)^{3/2} \tag{4.1b}$$

A_x is the stability limit in units of R, the orbit radius. Eq. (4.1b) for M is valid only for the two-way radial sector machine.

Numerical Results

For N = 64, K = 200 and assuming that $\nu_x = .28$, we find that

$$M = 11.4 \times 10^{-3} \tag{4.2a}$$

$$A_x = 1.31 \times 10^{-3} \tag{4.2b}$$

The Vertical Stability Limit

We assume that the vertical stability limit is due to the $\nu_x + 2\nu_y = N$ resonance. If the tune lies roughly in the shaded region of Fig. 1, then the vertical stability limit is given by[5, 4]

$$A_y = \frac{1}{N^2} \frac{.15}{M} \tag{4.3a}$$

where

$$M = \left(\frac{K}{2N^2}\right)^{3/2} \tag{4.3b}$$

There is roughly no tune dependence in Eq. (4.3) for A_y, because the linear tune is assumed to be rather far from the resonance line

Numerical Example

For N = 64, K = 200

$$A_y = 3.65 \times 10^{-3} \tag{4.4}$$

It is believed that the above results for the stability limits are good to about a factor of two. They are probably too large by a factor of two.

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