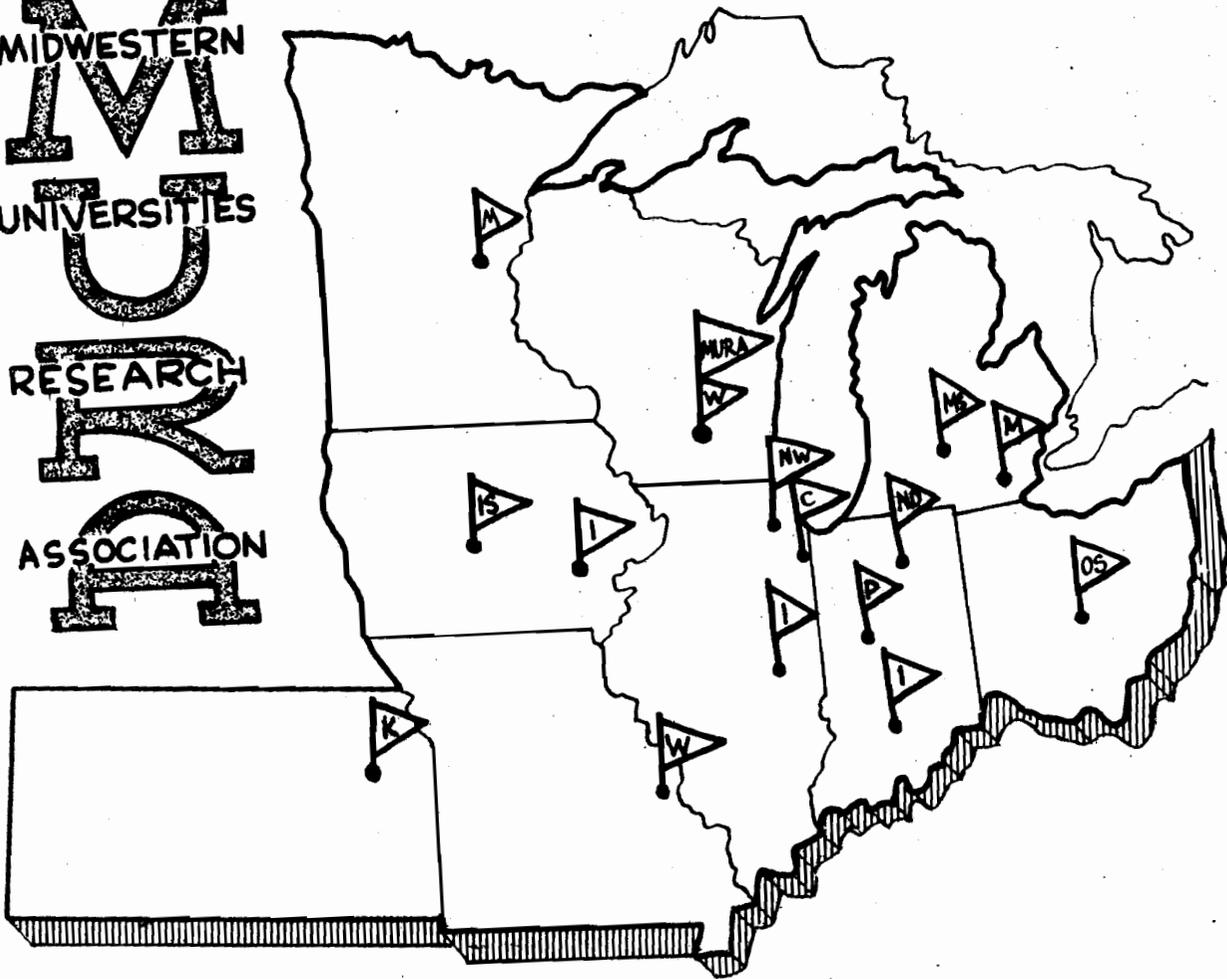




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REPORT BUDBIT  
(Program 157)  
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Internal  
(IBM Program)

BUDBIT  
(Programme 157)

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This program was inspired by letters from L. J. Laslett (q. v.) dated 30 September 1957 and 7 October 1957. Its object is to investigate the motion of protons in the magnetic field of a Budker beam.

The program integrates the following equations using floating point arithmetic:

$$\frac{d}{d\theta} \left[ \eta(\theta) \frac{dx}{d\theta} \right] = \lambda \left[ \eta(\theta) - \frac{2}{1 + (x-x_f)^2 + (y-y_f)^2} (x-x_f) \right]$$

$$\frac{d}{d\theta} \left[ \eta(\theta) \frac{dy}{d\theta} \right] = \lambda \left[ C - \frac{2}{1 + (x-x_f)^2 + (y-y_f)^2} (y-y_f) \right]$$

where:  $\eta(\theta) = \frac{\text{Mechanical linear momentum}}{\text{Max. stable mech. linear momentum}}$

$$= \alpha + \beta \left( \frac{\theta}{2\pi} \right) + \gamma \left( \frac{\theta}{2\pi} \right)^2 + \delta \left( \frac{\theta}{2\pi} \right)^3 \leq 1$$

$$x = \frac{R-R_0}{r^*}, \quad y = \frac{z}{r^*}, \quad \lambda = \frac{R_0}{r^*}$$

$\lambda$  and  $C$  are constants;  $0 \leq C < 1$ .

During the first  $n$  integration steps:

$$x_f = A_{x1} + B_{x1}\phi + C_{x1}\phi^2 + D_{x1}\phi^3$$

$$y_f = A_{y1} + B_{y1}\phi + C_{y1}\phi^2 + D_{y1}\phi^3$$

During the next  $N_{RK} - n$  integration steps;

$$x_f = A_{x2} + B_{x2}\phi + C_{x2}\phi^2 + D_{x2}\phi^3$$

$$y_f = A_{y2} + B_{y2}\phi + C_{y2}\phi^2 + D_{y2}\phi^3$$

where  $\phi = \theta/2\pi, \text{ mod. } 1$ .

The variable  $\theta$  is carried in units of  $2\pi$  and can be represented as:  $(\theta/2\pi) = k + \theta'$  where  $k$  is an integer and  $0 \leq \theta' < 1$ . Initial values  $k_0$  and  $\theta'_0$  may be given to this variable. The program accepts an eleven digit  $k_0$ , but prints only the last five digits of  $k$ .

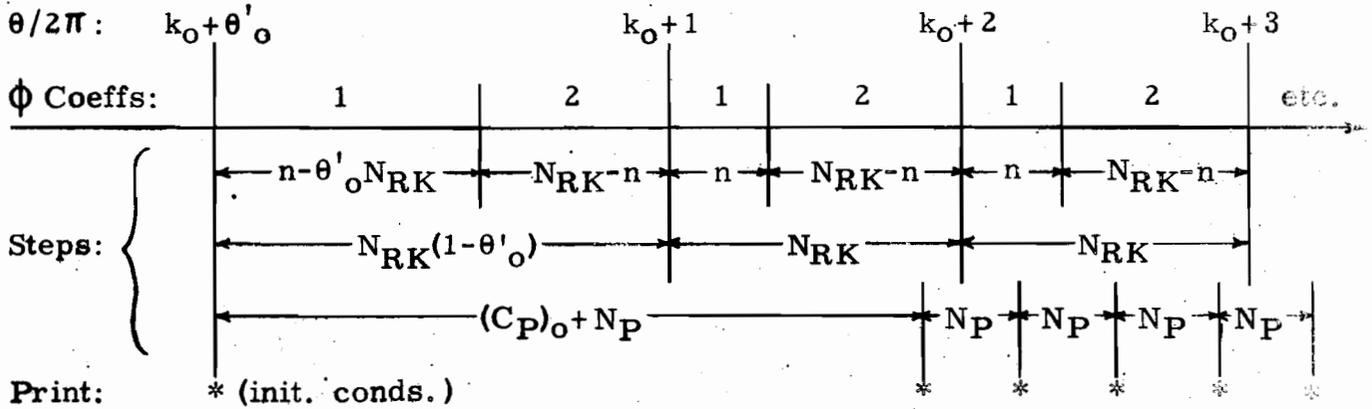
$N_{RK}$  integration steps are used on an interval  $\Delta(\theta/2\pi) = 1$ . Results are printed every  $N_p$  steps and the entire run lasts for  $N_E$  steps. The counter for  $N_p$  may be given an initial value  $(C_p)_0$ , so that the first result is printed after  $N_p + (C_p)_0$  steps.

BUDBIT AGENDUM (PROGRAMME 157)

1. All parameters will be zero unless set otherwise (except  $\eta (dx/d\theta)_{\max} = x_{\max} = \eta (dy/d\theta)_{\max} = y_{\max} = 10$ ). Do not enter such zeros.
2. All values are held from run to run. In a series, enter only those parameters which change on subsequent sheets.
3. For backward integration, replace  $N_{RK}$  with  $-N_{RK}$ .
4. To eliminate computation of the x-equation (SS4 up), check box:
5. To eliminate computation of the y-equation (SS5 up), check box:
6. For application of the search feature (SS6 up), check box:

Parameter	Address	Value		Parameter	Address	Value	
ID No.	101			$N_p$	105		
$N_E$	102			$(C_p)_o$	106		
$N_{RK}$	103			$k_o$	107		
n	104						
				END INTEGERS			
		N	EXP			N	EXP
$\theta_o$	110			$A_{x1}$	126		
				$B_{x1}$	127		
$\eta(dx/d\theta)_o$	112			$C_{x1}$	128		
$\chi_o$	113			$D_{x1}$	129		
$\eta(dy/d\theta)_o$	114			$A_{x2}$	130		
$\psi_o$	115			$B_{x2}$	131		
$\eta(dx/d\theta)_{MAX}$	116	+		$C_{x2}$	132		
$\chi_{MAX}$	117	+		$D_{x2}$	133		
$\eta(dy/d\theta)_{MAX}$	118	+		$A_{y1}$	134		
$\psi_{MAX}$	119	+		$B_{y1}$	135		
$\lambda$	120			$C_{y1}$	136		
C	121			$D_{y1}$	137		
$\alpha$	122			$A_{y2}$	138		
$\beta$	123			$B_{y2}$	139		
$\gamma$	124			$C_{y2}$	140		
$\delta$	125			$D_{y2}$	141		
				END FP NOS			

The number of integration steps between printing may be visualized with the help of the following diagram plotted along a  $\theta/2\pi$ -axis:



The print format of initial conditions is as follows:

00157	ID				
00157	$N_E$	$N_{RK}$	$n$	$N_p$	$(C_p)_0$
00157	$\eta(dx/d\theta)_{max}$	$x_{max}$	$\eta(dy/d\theta)_{max}$	$y_{max}$	
00157	$\lambda$	$C$			
00157	$\alpha$	$\beta$	$\gamma$	$\delta$	
00157	$A_{x1}$	$B_{x1}$	$C_{x1}$	$D_{x1}$	
00157	$A_{x2}$	$B_{x2}$	$C_{x2}$	$D_{x2}$	
00157	$A_{y1}$	$B_{y1}$	$C_{y1}$	$D_{y1}$	
00157	$A_{y2}$	$B_{y2}$	$C_{y2}$	$D_{y2}$	
			SS4	SS5	SS6

In order to save computation time, the sense switches 4 and 5 are provided. If SS4 is up, the entire x-equation and the quantities  $x$ ,  $x_p$ , and  $dx/d\theta$  will be ignored by the computer. Similarly, if SS5 is up, the y-quantities will be ignored.

The standard Search Feature is provided for a series. If sense switch 6 is up, the first run of a series which reaches  $N_E$  steps will terminate the series.

The average time per run is  $13 + (.04 + .6/N_p)N_E$  seconds.

A sample agenda sheet is attached. All parameters will be regarded as zero unless specifically set, except the overflow constants  $\eta(dx/d\theta)_{max} = x_{max} = \eta(dy/d\theta)_{max} = y_{max} = 10$ . All parameters are retained from run-to-run in a series. The agenda for a series should be carefully stapled together. Only those quantities which change should be entered on subsequent sheets. The program accepts an eleven digit ID number, where

$$ID = (\text{HUMAN ID}) 10^8 + \text{RUN ID.}$$

