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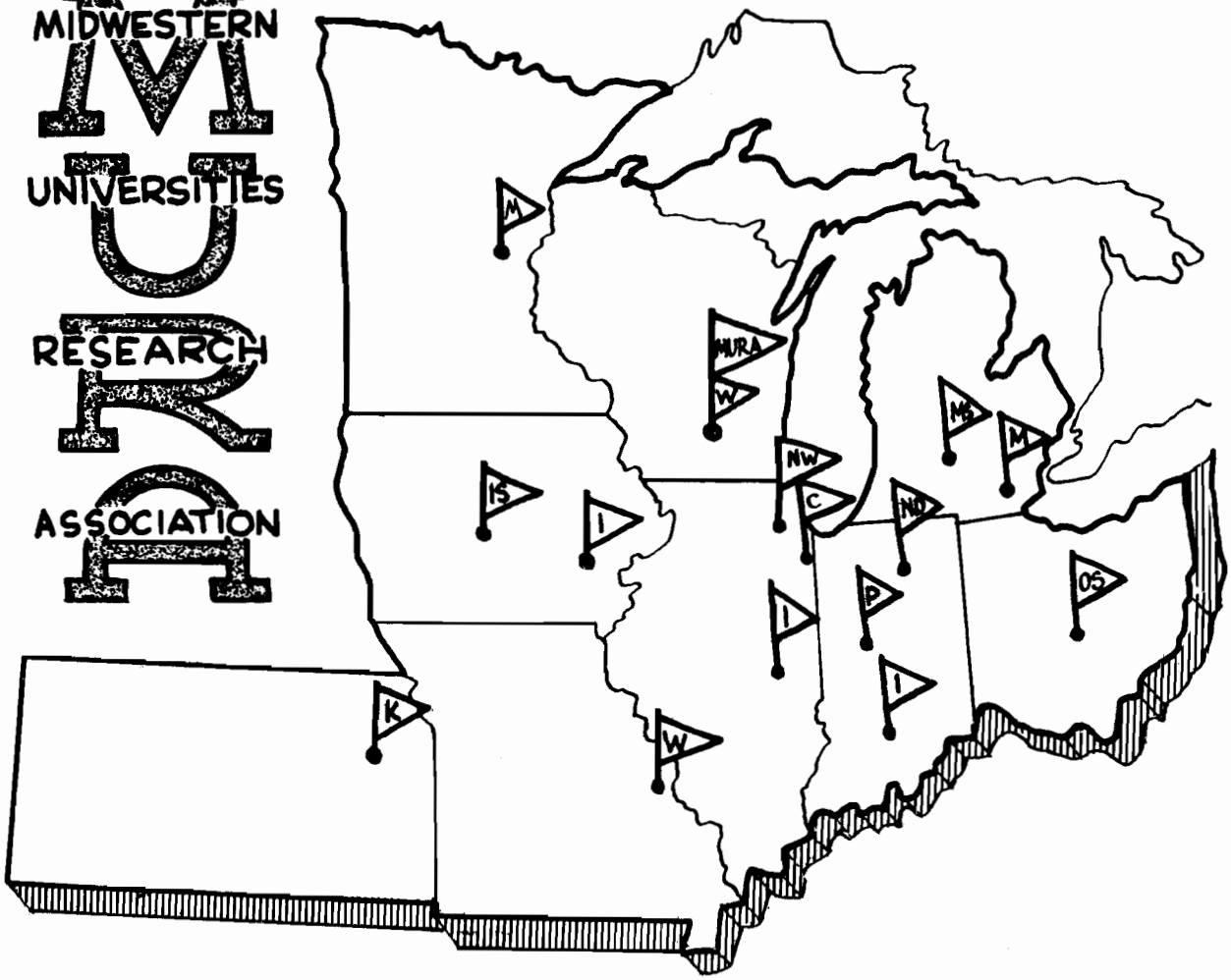

THE RADIATION ENERGY LOSS IN A FIXED FIELD ACCELERATOR

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ABSTRACT

The radiation loss per turn is calculated for the general spiral sector and radial sector FFAG accelerators.

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I. INTRODUCTION

The radiation per turn of a charged particle moving in an AG accelerator can be considerably larger than for a particle moving in a comparable non-AG accelerator. This increased radiation can be considered as due to the following two factors. Firstly, the scalloping of the equilibrium orbit in an AG machine means that the radius of curvature at some parts of the orbit can become considerably smaller than the average radius of the orbit and the radiation increases when the radius of curvature becomes smaller. Secondly, the AG machine has a larger circumference than the comparable non-AG machine and thus the particle has a longer time to radiate in each turn.

In this report we will calculate the radiation for the case of the general spiral sector machine (the Mark V). F. Mills made estimates of the AG effects on the radiation using the above qualitative considerations, and carried out the calculation for the case of a hard edge magnetic field.

II. SUMMARY OF RESULTS

In this section, we will simply list the results obtained for the radiation per turn for a general spiral sector machine whose median plane magnetic field is given by

$$H_z = -H\left(\frac{r}{r_1}\right)^K \sum_{n=-\infty}^{\infty} h_n e^{i\omega_n \varphi} \tag{2.1}$$

where $\omega_n = nN$ and

$$\varphi = \theta - \frac{1}{\omega N} \ln \frac{r}{r_1} \tag{2.2}$$

The derivations are given in Sections III and IV.

The radiation per turn for a particle mvoing in its equilibrium orbit is given by

$$\bar{W} = \frac{4\pi}{3} \frac{e^2}{R} \left(\frac{E}{mc^2}\right)^4 \left(\frac{v}{c}\right)^3 \cdot b^2 \sum_n |h_n|^2 \tag{2.3}$$

where R is the average radius of the equilibrium orbit and is given by

$$R = r_1 \left(\frac{bpc}{eHr_1}\right)^{1/k+1}, \tag{2.4}$$

and b is given by

$$b = \frac{1}{\beta}\left(\sqrt{h_0^2 + 2\beta} - h_0\right) \tag{2.5}$$

where $\beta = 4\left[(k+2)/N^2\right] \sum_{n\geq 1} |h_n|^2$.

The result for the radiation per turn given by Eq. (2.3) can be better understood if one realizes that the factor

$$\frac{4\pi}{3} \frac{e^2}{R} \left(\frac{E}{mc^2}\right)^4 \left(\frac{v}{c}\right)^3 \tag{2.6}$$

is just the radiation per turn for a particle moving in a circle of radius R. The correction factor $b^2 \sum_n |h_n|^2$ is related to the square of the circumference factor and increases with increasing circumference factor.

Eq. (2.3) for the radiation per turn is expected to have an error of about 10 to 20%.

If the particle performs betatron oscillations about the equilibrium orbit, the radiation per turn is still given with an error of 10 to 20% by Eq. (2.3) for presently considered FFAG machines. This is because the non-linear stability limits prevent the betatron oscillations from becoming so large as to appreciably change the radiation per turn from what it is on the equilibrium orbit.

III. DERIVATION OF THE RADIATION RESULTS

If a particle moves under the influence of an external electromagnetic field $\vec{E}$, $\vec{H}$, then the energy radiated per second by the particle is given by

$$W(t) = \frac{2c}{3}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{E}{mc^2}\right)^2 \left\{\left(\vec{E} + \frac{\vec{v}}{c}\cdot\vec{H}\right)^2 - \left(\vec{E}\cdot\frac{\vec{v}}{c}\right)^2\right\}. \quad (3.1)$$

For a particle moving in a magnetic field $\vec{H}$ only, the radiation is given by

$$W(t) = \frac{2c}{3}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{E}{mc^2}\right)^2\left(\frac{\vec{v}}{c}\times\vec{H}\right)^2. \quad (3.2)$$

The energy radiated per turn is then given by

$$\bar{W} = \int_{turn} dt\ W(t), \quad (3.3a)$$

$$\bar{W} = \frac{2c}{3}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{E}{mc^2}\right)^2\left(\frac{v}{c}\right)^2 \int_{turn} dt\ H^2. \quad (3.3b)$$

We will just calculate the radiation in the equilibrium orbit and ignore betatron oscillations about the equilibrium orbit. We will see later that including the betatron oscillations will not affect the results very much.

We will treat the case of the spiral sector machine whose median plane magnetic field is given by

$$H_z = -H\left(\frac{r}{r_1}\right)^K \sum_{n=-\infty}^{\infty} h_n e^{i\omega_n \varphi} \tag{3.4}$$

where $\omega_n = nN$ and

(3.5)

The equilibrium orbit for this magnetic field is by[1]

$$r = R(1+x) \tag{3.6}$$

where R is the average radius of the orbit and is given by

$$R = r_1\left(\frac{bpc}{eHr_1}\right)^{1/K+1}, \tag{3.7}$$

and

$$b = \frac{1}{\beta}\left(\sqrt{h_0^2 + 2\beta} - h_0\right), \tag{3.8}$$

where $\beta = 4\left[(K+2)/N^2\right] \sum_{n\geq 1} |h_n|^2$

$x(\theta)$, the oscillation about $r = R$, is given by

$$x = b \sum_{n\neq 0} \frac{\bar{h}_n}{\omega_n^2} e^{i\omega_n \theta}, \tag{3.9}$$

$\bar{h}_n = h_n \exp\left(-i\frac{n}{N}\ln\frac{r}{r_1}\right)$.

The above result for the equilibrium orbit has an error of about 10 to 20%, and we expect the same sort of accuracy for our results for the radiation.

It is convenient in carrying out the integration over a turn in Eq. (3.3b), to integrate over θ rather than t. Since

$$d\theta = \frac{v\, dt}{\sqrt{r^2 + r'^2}} \quad , \tag{3.10}$$

we find for the radiation per turn

$$\bar{W} = \frac{2}{3}\left(\frac{e^2}{mc^2}\right)^2 \left(\frac{E}{mc^2}\right)^2 \frac{v}{c} \int_0^{2\pi} d\theta \sqrt{r^2 + r'^2}\; H^2 \; . \tag{3.11}$$

In doing the integration, we will make the two approximations we made in MURA-258 and which we believe are valid for a wide class of magnetic fields. We will assume that on the equilibrium orbit that $\left[r'(\theta)/r(\theta)\right]^2 << 1$, or that

$$\left(\frac{dx}{d\theta}\right)^2 << 1 \tag{3.12}$$

We will also assume that the magnetic field does not vary very much over the radial extent of the equilibrium orbit; this means that

$$\begin{aligned} K x &<< 1 \\ \frac{1}{\nu} x &<< 1 \end{aligned} \tag{3.13}$$

We can then write Eq. (3.11) for the radiation per turn as

$$\bar{W} = \frac{4\pi}{3} \frac{e^2}{R} \left(\frac{E}{mc^2}\right)^4 \left(\frac{\nu}{c}\right)^3 \cdot I , \tag{3.14}$$

where

$$I = \frac{1}{2\pi} \int_0^{2\pi} d\theta \, (1+x) \, C^2(\theta) , \tag{3.15}$$

and

$$C(\theta) = \frac{eR}{pc} H_z . \tag{3.16}$$

The function $C(\theta)$ is closely related to the circumference factor. The radius of curvature, $\rho(\theta)$, is given at each point by

$$\frac{1}{\rho(\theta)} = \frac{e H_z}{p c} . \tag{3.17}$$

Thus $C(\theta) = R/\rho(\theta)$, and the circumference factor is just the maximum value of $C(\theta)$. The integral I is then roughly the square of the circumference factor. The coefficient of I in Eq. (3.14) for $\bar{W}$, is just the radiation per turn for pure circular motion.

To compute I, we put in for H_z the spiral sector magnetic field and find

$$I = \frac{b^2}{2\pi} \int_0^{2\pi} d\theta \left\{ \sum_n (1+x)^{k+1-in/w} h_n e^{i\omega_n \theta} \right\}^2 . \tag{3.18}$$

Now as $K x << 1$, $x/\nu << 1$, we can write I as

$$I = \frac{b^2}{2\pi}\int_0^{2\pi} d\theta \left\{ \sum_n \left(1 + (K+1-in/\nu)x\right) h_n e^{i\omega_n \theta} \right\}^2 . \tag{3.19}$$

We can drop the $(K+1-in/\nu)x$ term with an error of about 10 to 20%. Thus

$$I = b^2 \sum_n |h_n|^2 . \tag{3.20}$$

The radiation per turn is then given by

$$\bar{W} = \frac{4\pi}{3}\frac{e^2}{R}\left(\frac{E}{mc^2}\right)^4\left(\frac{\nu}{c}\right)^3 \cdot b^2 \sum_n |h_n|^2 . \tag{3.21}$$

One might point out that $b^2 \sum_n |h_n|^2$ is related to the square of the circumference factor, (see MURA-258) and increases with increasing circumference factor.

As an illustration of the application of Eq. (3.21), let us consider the MURA model machine which is a 50 Mev radial sector electron machine. For one-way operation of this machine

$$\frac{1}{\nu} = 0, \quad K = 9.3, \quad N = 16, \quad h_1/h_0 = 3.88$$

At 50 Mev, the average radius of the equilibrium orbit is 198 cm. The above data for the machine was obtained from[2] F. Cole and A. M. Sessler.

We will calculate the radiation per turn in the 50 Mev orbit. The radiation

from a circular orbit of 198 cm is given by

$$\bar{W}_{circle} = \frac{4\pi}{3} \frac{e^2}{R} \left(\frac{E}{mc^2}\right)^4 \left(\frac{v}{c}\right)^3$$
$$= .30 \text{ electron volts / turn} \tag{3.22}$$

The correction factor $b^2 \sum_n |h_n|^2$ is 10 in this case. Thus, the radiation per turn is

$$\bar{W} = 3.0 \text{ electron volts/turn} \tag{3.23}$$

IV. EFFECT OF OSCILLATIONS ABOUT THE EQUILIBRIUM ORBIT

In section III, we calculated the radiation per turn in the equilibrium orbit. In general, the particle will have betatron oscillations about the equilibrium orbit, and in this section we will estimate the effect of these oscillations on the radiation.

According to Eq. (3.3b), the radiation per turn depends on the integral $\int dt\, H^2$ over one turn. In Section III, we evaluated this integral over the equilibrium orbit. Now if the betatron oscillations are so small that the magnetic field on the oscillation orbit does not change very much from its value on the equilibrium orbit, then the radiation per turn for the oscillation orbit will be about the same as for the equilibrium orbit. This turns out to be case for the FFAG machines being considered at present.

We will now estimate the change in the magnetic field in oscillations about the equilibrium orbit. Let u and y be the horizontal and vertical oscillations about the equilibrium orbit measure in units of R. We will

suppose that $\mu \simeq \eta$. Using the results in MURA-273 for the expansion of the magnetic field about the equilibrium orbit, we find that the percentage change in the magnetic field is of order

$$K\mu, \frac{1}{w}\mu, N\mu \tag{4.1}$$

whichever is the largest.

If μ would get very large Eq. (4.1) would give a large change in the magnetic field. However, μ cannot exceed the stability limits of the machine. It has been found, that μ is roughly of the order of x , the radial extent of the equilibrium orbit for FFAG machines being considered at present. Thus, by Eqs. (3.12) and (3.13), the percentage change in the magnetic field is small, and we can expect the results of Section III for the radiation to hold when betatron oscillations are present with an error of 10 to 20%.

REFERENCES

1. G. Parzen - MURA-258

2. F. T. Cole and A. M. Sessler - MURA-373