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IDENTIFICATION

Fixed Point Square Root, MU SQR4

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PURPOSE

To extract the square root of a non-negative fixed-point fraction F.

RESTRICTIONS

The binary point is assumed to be located immediately to the left of bit 1 of the accumulator.

METHOD

Accuracy: Maximum error = 2^{-35}.

Range: $+0 \leq F \leq 1 - 2^{-35}$, thus allowing the upper-limit equality not permitted in MU SQR2.

Procedure:

$x = \sqrt{F}$ is obtained by the iterative procedure

$$x_{n+1} = 1/2 \left[(F/x_n) + x_n\right], \quad x_o = 1 - 2^{-35}.$$

Convergence is assumed to have occurred when

$$x_{n+1} \geq x_n,$$

and x_{n+1} is taken as $\sqrt{F}$.

USAGE

Calling Sequence: Two modes of entry are available. In the first mode entry is made with the more-significant part of F in the accumulator and the less-significant part in the MQ-register, with the calling sequence:

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX SQR4, 4	α	BTSX A 4
$\alpha + 1$	error return	$\alpha + 1$	error return
$\alpha + 2$	normal return	$\alpha + 2$	normal return

where exit to the error return is made when $F \leq -0$ with F unchanged in the accumulator and MQ-register.

In the second mode of entry the more-significant part of F is in COMMON and the less-significant part of F is in COMMON + 1 (the state of the accumulator and the MQ-register is irrelevant) and the following calling sequence is used:

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX SQR4 + 2, 4	α	BTSX 2A 4
$\alpha + 1$	error return	$\alpha + 1$	error return
$\alpha + 2$	normal return	$\alpha + 2$	normal return .

The error test, unlike MU SQR2 and 3, is also made in this mode; error return is as in the first mode.

In the normal return, the routine is left with $\sqrt{F}$ in the accumulator.

Space Required:

	UA SAP	MURASS
20 words program at:	SQR4	A
3 words temporary at:	COMMON	T

CODING INFORMATION

Timing: 0.5n milliseconds, where n is the smallest integer such that

$$\sqrt{F}\left(\frac{1 - \sqrt{F}}{1 + \sqrt{F}}\right)^{2^n} < 2^{-36} .$$

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      REM SQR4 MURA FIXED POINT SQUARE ROOT                          MU   SQR4
SQR4  STO COMMON          MSP OF F STORED IN COMMON                 SQR4  0001
      STQ COMMON+1        LSP OF F STORED IN COMMON+1               SQR4  0002
      CLA SQR4+16         SET X SUB ZERO=1-2EXP(-35)                SQR4  0003
      CAS COMMON          TEST X SUB ZERO MINUS F                   SQR4  0004
      TRA SQR4+6          IF POSITIVE, CONTINUE                     SQR4  0005
      TRA 2,4             IF 0, LEAVE WITH F EXP1/2=XSUB0           SQR4  0006
      STO COMMON+2        STORE XSUBN IN COMMON+2                   SQR4  0007
      CLA COMMON          MSP OF F IN AC                            SQR4  0008
      LDQ COMMON+1        LSP OF F IN MQ                            SQR4  0009
      TMI 1,4             IF F NEGATIVE, ERROR RETURN               SQR4  0010
      DVH COMMON+2        F/XSUBN IN MQ                             SQR4  0011
      PXD 0,0             CLEAR AC                                  SQR4  0012
      LLS 35              F/XSUBN IN AC                             SQR4  0013
      ADD COMMON+2        X SUB N + F/XSUBN IN AC                   SQR4  0014
      ARS 1              X SUB (N+1) IN AC                          SQR4  0015
      CAS COMMON+2        COMPARE XSUB(N+1) WITH XSUBN              SQR4  0016
      OCT 377777777777    XSUB ZERO=1-2EXP(-35)                     SQR4  0017
      TRA 2,4             NORMAL RETURN                             SQR4  0018
      TNZ SQR4+6          INCREASE N BY 1 AND REPEAT                SQR4  0019
      TRA 2,4             RETURN IF XSUBN IS 0                      SQR4  0020
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