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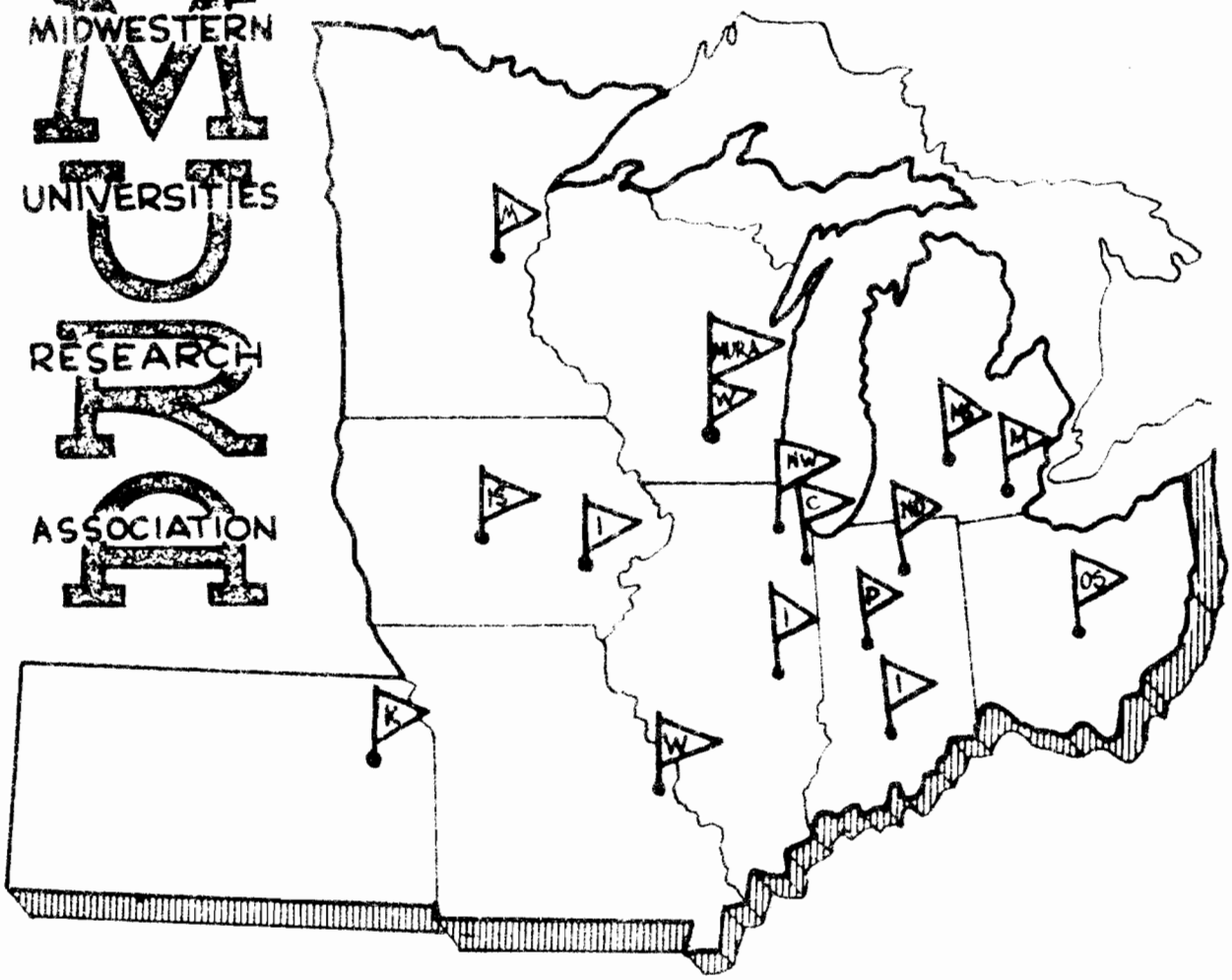

REPORT FIXED POINT NEWTON-COTES QUADRATURE, MU NC11

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APPROVED BY
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IDENTIFICATION

Fixed Point Newton-Cotes Quadrature, MU NC11

M. R. Storm - July 9, 1957

Midwestern Universities Research Association, Madison, Wisconsin

PURPOSE

To approximate the value of an integral of the square of a function for which N + 1 ordinates are known. A Newton-Cotes formula is used and the computation is done on a semi-double-precision basis.

RESTRICTIONS

This program requires the Double Precision Addition routine, MU DPA2, whose location must be defined at assembly time.

No internal checks are made on overflow, since it is left to the user to ensure that all numbers remain in the range $-(1 - 2^{-35})$ to $+(1 - 2^{-35})$.

METHOD

The Newton-Cotes formula used here is:

$$\int_{x_0}^{x_4} y\,dx = \frac{4h}{90}(7y_0 + 32y_1 + 12y_2 + 32y_3 + 7y_4)$$

where h = the integration step size and is uniquely determined by $\frac{1}{N}$. Since this routine provides $\int_{x_0}^{x_4} y^2 dx$ rather than the integral of y, each of the single-length ordinates $y_0, \cdots, y_4$ are squared before multiplying by their corresponding coefficients.

These multiplications are done in double precision where the squared ordinates are 70 bit numbers and the coefficients are 35 bit numbers. Additions are always done on the assumption that both numbers are 70 bit fractions.

Range: $|y_i|$ must be less than one
$N \geq 4$; N is always a multiple of 4.

Accuracy: An approximation to the truncation error is given by $\frac{8\,y^{(6)}\,h^7}{945}$, where $y^{(6)}$ is the value of the sixth derivative at some point between x_0 and x_4.

(Cf. Milne, Numerical Calculus. The Princeton University Press, New Jersey, 1955, Chapter IV, pp. 122-124.)

USAGE

Calling Sequence:

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX NCI1, 4	α	B TSX A 4
$\alpha + 1$	(prefix) B, O, N	$\alpha + 1$	C D B O N
$\alpha + 2$	return	$\alpha + 2$	return

where prefix = D determines the number of words between successive ordinate points. N is the number of subdivisions in the entire integration process and B is the address of the first ordinate. N + 1 ordinate points separated by D must be stored in memory or incorrect results will occur.

Upon exit from this routine the double precision integral is in the AC and MQ and is also stored in COMMON + 2 and COMMON + 3.

Space Required:	UA SAP	MURASS
57 words program at	NCI1	A
9 words temporary at	COMMON	T

The symbolic location of MU DPA2 must be defined by the user in the assembly of MU NCI1.

For an assembly by MURASS the locator Z must be defined as the starting location of MU DPA2.

Relocatable Cards: The nominal addresses in the relocatable program deck have been defined as follows:

Symbolic address	Nominal address
NCI1	0
DPA2	1000_8
COMMON	2000_8

CODING INFORMATION

Timing: (4.14 N + .85) milliseconds which includes the time spent in the auxiliary subroutine, DPA2.

```
      REM NCI1 NEWTON-COTES QUADRATURE                                  MU   NCI1
NCI1  SXD COMMON+4,1        STORE INDEX REGISTERS                      NCI1  0001
      SXD COMMON+5,2                                                   NCI1  0002
      SXD COMMON+6,4                                                   NCI1  0003
      CAL 1,4               PARAMETER WORD                             NCI1  0004
      PDX 0,1               N TO X1                                    NCI1  0005
      STA NCI1+20           PLACE FWA OF Y                             NCI1  0006
      STA NCI1+21                                                      NCI1  0007
      ARS 15                DELTA TO DECREMENT POSITION                NCI1  0008
      COM                   (DELTA+1) COMPLEMENTED                     NCI1  0009
      STD NCI1+45           STORE IN TXI INSTRUCTION                   NCI1  0010
      PXD 5,1               N TO AC                                    NCI1  0011
      STO COMMON+7          STORE IN MEMORY ALSO                       NCI1  0012
      LDQ NCI1+56           2 TO MQ                                    NCI1  0013
      LLS 35                CLEAR MQ AND PLACE 2 IN AC                 NCI1  0014
      DVH COMMON+7          FORM 2H=2/N                                NCI1  0015
      STQ COMMON+7          STORE 2H                                   NCI1  0016
      LXA NCI1+4,2          J=0 TO X2                                  NCI1  0017
      HTR COMMON+2,0,24576 CLEAR SUM1                                  NCI1  0018
      HTR COMMON+3,0,24576 CLEAR SUM2                                  NCI1  0019
      LXA NCI1+10,4         I=5 TO X4                                  NCI1  0020
      LDQ **,2              Y SUB J                                    NCI1  0021
      MPY **,2              SQUARED                                    NCI1  0022
      STO COMMON+8          STORE MSP                                  NCI1  0023
      MPY NCI1+56,4         LSP TIMES A SUB I                          NCI1  0024
      LRS 35                POSITION PRODUCT                           NCI1  0025
      STO COMMON            STORE FOR DPA2                             NCI1  0026
      STQ COMMON+1                                                     NCI1  0027
      LDQ COMMON+8          MSP                                        NCI1  0028
      MPY NCI1+56,4         MULTIPLY BY A SUB I                        NCI1  0029
      SXD COMMON+8,4        SAVE X4                                    NCI1  0030
      TSX DPA2,4                                                       NCI1  0031
      MPY COMMON+7          MULTIPLY LSP BY 2H                         NCI1  0032
      LRS 35                                                           NCI1  0033
      STQ COMMON+1          STORE FOR DPA2                             NCI1  0034
      LDQ COMMON            MSP                                        NCI1  0035
      STO COMMON                                                       NCI1  0036
      MPY COMMON+7          MULTIPLY MSP BY 2H                         NCI1  0037
      TSX DPA2,4                                                       NCI1  0038
      CLA COMMON+2          SUM1                                       NCI1  0039
      LDQ COMMON+3          SUM2                                       NCI1  0040
      TSX DPA2,4                                                       NCI1  0041
      LXD COMMON+8,4        RESTORE X4                                 NCI1  0042
      STQ COMMON+3          STORE NEW SUM2                             NCI1  0043
      STO COMMON+2          STORE NEW SUM1                             NCI1  0044
      TNX NCI1+46,4,1       REDUCE I                                   NCI1  0045
      TXI NCI1+20,2,**      ADD DELTA TO X2                            NCI1  0046
      TIX NCI1+19,1,4       N-4 REPLACES N                             NCI1  0047
      LXD COMMON+4,1        RESTORE INDEX REGISTERS                    NCI1  0048
      LXD COMMON+5,2                                                   NCI1  0049
      LXD COMMON+6,4                                                   NCI1  0050
      TRA 2,4               EXIT                                       NCI1  0051
      OCT 047644764476      7/45                                       NCI1  0052
      OCT 266026602660      32/45                                      NCI1  0053
```

OCT 104210421042	12/45	NCI1	0054
OCT 266026802660	32/45	NCI1	0055
OCT 04764476447[illegible]	7/45	NCI1	0056
OCT 000002000000	INTEGER 2 SCALED	NCI1	0057