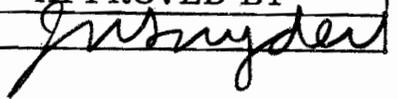


APPROVED BY


IDENTIFICATION

Gauss Quadrature (Three-Point), MU GQF1

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PURPOSE

To evaluate $I = \int_{-1/2}^{+1/2} \Phi(u) du$ in fixed point arithmetic.

RESTRICTIONS

No internal check is made on overflow, since it is left to the user to ensure that all numbers stay in the range $-(1 - 2^{-35})$ to $+(1 - 2^{-35})$.

METHOD

Procedure: The three-point Gauss quadrature formula

$$\int_{-1/2}^{+1/2} \Phi(u) du \cong R_1 \Phi(u_1) + R_2 \Phi(u_2) + R_3 \Phi(u_3)$$

is used; where u_1, u_2, u_3 are the points of subdivision of the interval, $-1/2 \leq u \leq +1/2$, and R_1, R_2, R_3 are the weights.

The integral over the interval $(-1/2, +1/2)$ is related to the integral over the arbitrary interval (a, b) as follows:

$$\int_a^b f(x) dx = (b - a) \int_{-1/2}^{+1/2} \Phi(u) du,$$

where $\Phi(u)$ is obtained from $f(x)$ by making the change in variable

$$x = (b - a) u + \frac{a + b}{2}.$$

The $\Phi(u)$'s are evaluated by a closed subroutine known as the AUXILIARY SUBROUTINE, which must be provided by the user. During each pass through MU GQF1 three entires are made into the AUXILIARY SUBROUTINE to obtain $\Phi(u_1), \Phi(u_2), \Phi(u_3)$.

Accuracy: The three-point Gauss quadrature formula is exact when $\Phi(u)$ is a polynomial of degree five or less. For $\Phi(u)$ not a polynomial of degree five or less, the error is

$$\int_{-1/2}^{+1/2} \Phi(u) du - [R_1 \Phi(u_1) + R_2 \Phi(u_2) + R_3 \Phi(u_3)] = 4.96 \times 10^{-7} \Phi^{(6)}(\eta),$$

$$(-1/2 \leq \eta \leq +1/2).$$

The roundoff error is always less than 1.5×2^{-35} and therefore is negligible in comparison with the above truncation error (assuming $\Phi^{(6)}(\eta)$ not vanishingly small).

(Cf. J. B. Scarborough, Numerical Mathematical Analysis, The John Hopkins Press, 1955, Chapter VII. Z. Kopal, Numerical Analysis, John Wiley and Sons, Inc., 1955, Chapter VII.)

USAGE

Calling Sequence:

UA SAP	
Loc.	Instruction
\mathcal{L}	TSX GQF1, 4
$\mathcal{L} + 1$	Return

MURASS	
Loc.	Instruction
\mathcal{L}	BTSX A 4
$\mathcal{L} + 1$	Return

Return is made with the value of the integral in the AC and in TGQF1 + 2.

The symbolic location of the AUXILIARY SUBROUTINE, WGQF1 (MURASS: W), must be defined by the user in the assembly of MU GQF1.

Space Required:

UA SAP	
21 words program at:	GQF1
3 words temporary at:	TGQF1

and space for the AUXILIARY SUBROUTINE.

MURASS	
	A
	T

The three words of temporary storage are not available for use by the AUXILIARY SUBROUTINE since they contain information which is to be used later by MU GQF1. However, when MU GQF1 does not have control, these three words of temporary storage are available.

Relocatable Cards: The nominal addresses in the relocatable program deck have been defined as follows:

SYMBOLIC ADDRESS	NOMINAL ADDRESS
GQF1	0
TGQF1	2000 ₈
WGQF1	3000 ₈

CODING INFORMATION

The AUXILIARY SUBROUTINE must be written as a closed subroutine: Entry to the AUXILIARY SUBROUTINE is made with u in the accumulator via

UA SAP	
Loc.	Instruction
α	TSX WGQF1, 4
$\alpha + 1$	Return

MURASS	
Loc.	Instruction
α	BTSX W 4
$\alpha + 1$	Return

and on exit $\Phi(u)$ must be in the accumulator.

Timing: $[1.5 + 3 (\text{AUXILIARY SUBROUTINE time in milliseconds})]$ milliseconds.

GQF1	REM GQF1 MURA GAUSS QUADRATURE (THREE-POINT)	MU	GQF1
	SXD TGQF1,1	GQF1	0001
	SXD TGQF1+1,4	GQF1	0002
	PXD 3,0	GQF1	0003
	STO TGQF1+2	GQF1	0004
	LXA GQF1+2,1	GQF1	0005
	CLA GQF1+18,1	GQF1	0006
	TSX WGQF1,4	GQF1	0007
	LRS 35	GQF1	0008
	MPR GQF1+21,1	GQF1	0009
	ADD TGQF1+2	GQF1	0010
	STO TGQF1+2	GQF1	0011
	TIX GQF1+5,1,1	GQF1	0012
	LXD TGQF1,1	GQF1	0013
	LXD TGQF1+1,4	GQF1	0014
	TRA 1,4	GQF1	0015
	OCT 543113757210	GQF1	0016
	OCT 000000000000	GQF1	0017
	OCT 143113757210	GQF1	0018
	OCT 107070707071	GQF1	0019
	OCT 161616161616	GQF1	0020
	OCT 107070707071	GQF1	0021

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SAVE IR1
SAVE IR4
CLEAR AC
CLEAR SUM BOX
SET IR1=3
U
COMPUTE PHI (U)
PHI (U) TO MO
MULTIPLY BY R
ADD TO SUM
LOOP RE-ENTRY, OUT AFTER 3 PASSES
RESTORE IR1
RESTORE IR4
OUT
U SUB 1
U SUB 2
U SUB 3
R SUB 1
R SUB 2
R SUB 3

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