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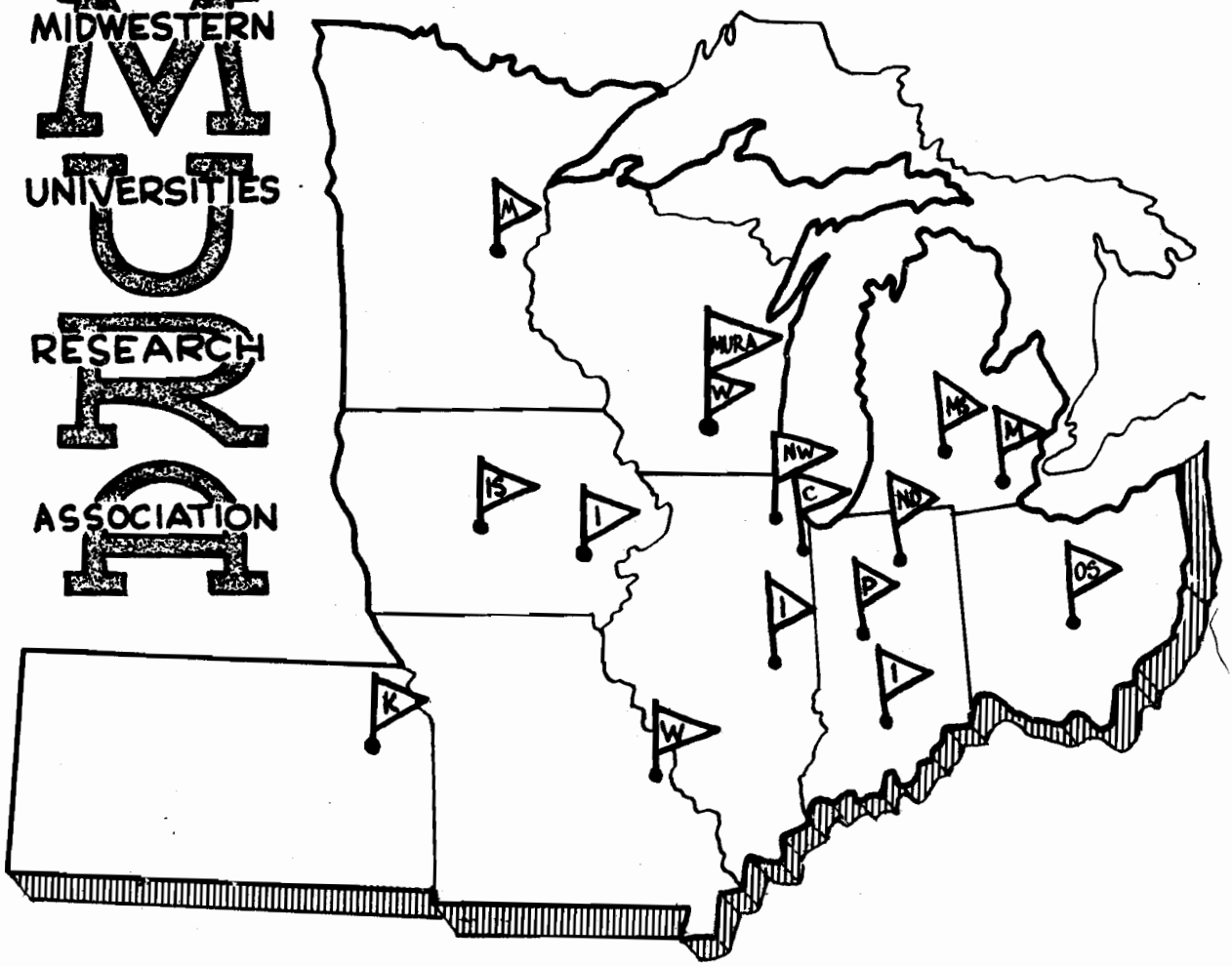

COMPUTATIONAL EVIDENCE FOR A SPIRALLY-RIDGED FFAG RESONANCE NEAR $3\sigma_x + 2\sigma_y = 2\pi$

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COMPUTATIONAL EVIDENCE FOR A SPIRALLY-RIDGED FFAG RESONANCE NEAR $3\sigma_x + 2\sigma_y = 2\pi$

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ABSTRACT

Computational evidence of a $3\sigma_x + 2\sigma_y = 2\pi$ resonance is presented for a spirally-ridged fixed-field alternating-gradient accelerator with $N = 40$, a median-plane field proportional to $1 + 1.1708 \sin 2\pi\xi - 0.2991 \cos 2(2\pi\xi)$, $44.08 \leq k \leq 46.3$, and $1/w = 360 + 1.25(k-46.0)/0.3$. The computations were performed with the aid of the WELL-TEMPERED FIVE computational program. The resonance, although not strong, appeared to be definitely present and to exhibit a width approximately proportional to the cube of the radial amplitude.

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** The work reported here was performed at Madison, with the MURA I.B.M.-704 computer, while the author was on leave of absence from Iowa State College, Ames, Iowa.

I. MOTIVATION

In the course of orientation work preparatory to studies of the $\sigma_x = \sigma_y$ resonance, the results of which have been reported elsewhere, evidence was obtained for resonant effects attributable to $3\sigma_x + 2\sigma_y = 2\pi$. Since attention had not customarily been given previously to resonances of as high order as this, it was felt worth while to obtain further computational information concerning the $3\sigma_x + 2\sigma_y = 2\pi$ resonance. It is the purpose of this report to present the results of this study.

The nature of the resonant effects with which we are concerned is similar to that found for other coupling resonances, exhibiting an exponential growth of axial amplitude when the radial amplitude exceeds a certain threshold value with the threshold amplitude the smaller the closer the operating point is to the resonance in question.

II. THE COMPUTATIONS

The computations were performed by means of the I.B.M. computer at Madison, using the WELL-TEMPERED FIVE computational program (MURA-226, Int.). The computations pertained to a spirally-ridged FFAG structure with N=40, a median-plane field proportional to $1 + 1.1708 \sin 2\pi\xi - 0.2991 \cos 2(2\pi\xi)$*, $44.08 \leqslant k \leqslant 46.3$, and $1/w = 360 + 1.25\,(k-46.0)/0.3$. Initial orientation runs were undertaken to determine the small-amplitude betatron oscillation frequencies for the machine parameters used and the location of the fixed point. Due to the narrowness of the resonance, one might wish for somewhat greater absolute accuracy in the resultant values of $3\,(\sigma_x/\pi) + 2\,(\sigma_y/\pi)$, but it is felt that the change of this quantity with the parameter k is sufficiently well established for the present purpose. σ_x lay close to 0.37

* $\xi \equiv \frac{1}{2\pi}\left[\frac{\ln(1+x)}{w} - N\theta\right]$.

and σ_y near 0.45π.

In the study of y-growth arising from the resonance in question the computations were of sufficient duration to permit a satisfactory determination of the lapse-rate (involving a growth of y-amplitude through at least one and one half decades from the initial value $y_0 = 10^{-6}$), but the small value often found for the lapse-rate frequently precluded reaching y-amplitudes at which "turn-over" might occur.

The results of the computations are summarized in Table I, in which the absolute value of $3(\sigma_x/\pi) + 2(\sigma_y/\pi)$ is not to be regarded as accurate through all the digits indicated. The values of x_0, the initial radial -amplitude u_0, and the turn-over values of y all pertain to $N\theta = 0$, mod. 2π. The results of the computations with regard to lapse-rate are also portrayed in Fig. 1.

III. DISCUSSION

Examination of the computational results afforded no indication that the threshold for y-growth did not become zero for a certain critical value of tune. The results moreover suggested that the width of the resonance [measured, for example, in units of $3(\sigma_x/\pi) + 2(\sigma_y/\pi)$] varied approximately as the cube of the radial amplitude. The resonance was not found to be symmetrical, however, y-growth being more marked below than above the critical resonant betatron frequency. A consistent presentation of the computational results appears to be that illustrated in Figs. 2 and 3.

In common with the results found for other coupling resonances, the square of the lapse-rate in the unstable zone was found to be roughly propor-

tional to the product of the distances from the operating point to the two stability limits for the given radial amplitude. Empirically the factor of proportionality in this relationship was estimated as $\cong 0.49$, for μ expressed in decades per sector and distances on the resonance diagram measured in units of $3(\sigma_x/\pi) + 2(\sigma_y/\pi)$. From the simplified estimate of this factor, as given by McLachlan for an equation for y of the Mathieu or Hill form, one expects the proportionality factor $\cong \left[(\pi/\lambda)\log e\right]^2 \doteq 0.465$. In addition, again from the computational results, one finds the width of the resonance to vary roughly as the cube of the radial amplitude -- with the width measured in the same units as before and with the radial amplitude (in units of the mean radius) taken at $N\theta = 0$, the proportionality constant for this assumed relationship appears to be in the range 3.5 to 4.0×10^7 for the structure under consideration.

A number of the runs were continued far enough to reveal "turn-over" of the axial oscillation amplitude at values of y (taken at $N\theta \equiv 0$, mod. 2π) in the range 2 to 11×10^{-4}, but the data concerning this feature are not available for all the runs and the information must be regarded as less complete than that pertaining to the lapse-rate. The presence of "turn-over", similar in character to that found for simple difference resonances, might suggest that y-growth for the sum resonance of interest here is less harmful than in the case of lower-order sum resonances. The possibility remains, however, that such growth may make effective instabilities basically due to other resonances.

TABLE I

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CHARACTERISTICS OF COUPLED MOTION NEAR $3(\sigma_x/\pi)+2(\sigma_y/\pi)=2$

$y_0 = 10^{-6}$ -- Coördinates refer to $N\theta = 0$, mod. 2π.

PARAMETERS (N=40) k	$1/\omega$	x_0 / u_0	-.0018 / -.0012	-.0017 / -.0011	-.0016 / -.0010	-.0015 / -.0009	-.0014 / -.0008	-.0013 / -.0007	-.0012 / -.0006	-.0011 / -.0005	$\frac{3\sigma_x+2\sigma_y}{\pi}$ (Nominal)
46.3	361.25	RUN Sects. Dec./Sect. y-T.O.	267 350 0	268 500 0							2.007
46.0	360.00	RUN Sects. Dec./Sect. y-T.O.	240 200 .0134	269 350 .00897 2.9×10^{-4}	270 550 .00548 1.9×10^{-4}	271 700 .001976	260 240 0				2.000
45.85	359.375	RUN Sects. Dec./Sect. y-T.O.								297 920 .00179	1.996
45.775	359.0625	RUN Sects. Dec./Sect. y-T.O.								298 630 .00147	1.994
45.7	358.75	RUN Sects. Dec./Sect. y-T.O.	272 200 .01754 5.2×10^{-4}	273 250 .01415 4.9×10^{-4}	274 300 .011 5.5×10^{-4}	275 400 .00833 4.1×10^{-4}	276 550 .0064 3.4×10^{-4}	277 601 .00457 3.4×10^{-4}		299 383 0	1.992
45.625	358.4375	RUN Sects. Dec./Sect. y-T.O.								300 830 0	1.990
45.4	357.50	RUN Sects. Dec./Sect. y-T.O.	261 240 .0205 6.3×10^{-4}	278 250 .01595 6.5×10^{-4}	279 218 .0125	280 350 .00956 5×10^{-4}	262 240 .0053	281 550 0			1.984
45.1	356.25	RUN Sects. Dec./Sect. y-T.O.	283 200 .0203 $7\text{or}8\times10^{-4}$	284 250 .0165 $7\text{or}8\times10^{-4}$	285 300 .0119 7.5×10^{-4}	286 400 .0056	287 550 0				1.977
44.8	355.00	RUN Sects. Dec./Sect. y-T.O.	171 200 .0189 11×10^{-4}	289 350 .01364 8×10^{-4}	290 257 .0059	291 162 0	263 240 0				1.969
44.5	353.75	RUN Sects. Dec./Sect. y-T.O.	292 200 .016 No T.O.	293 450 .00617							1.962
44.08	352.00	RUN Sects. Dec./Sect. y-T.O.	0								1.951

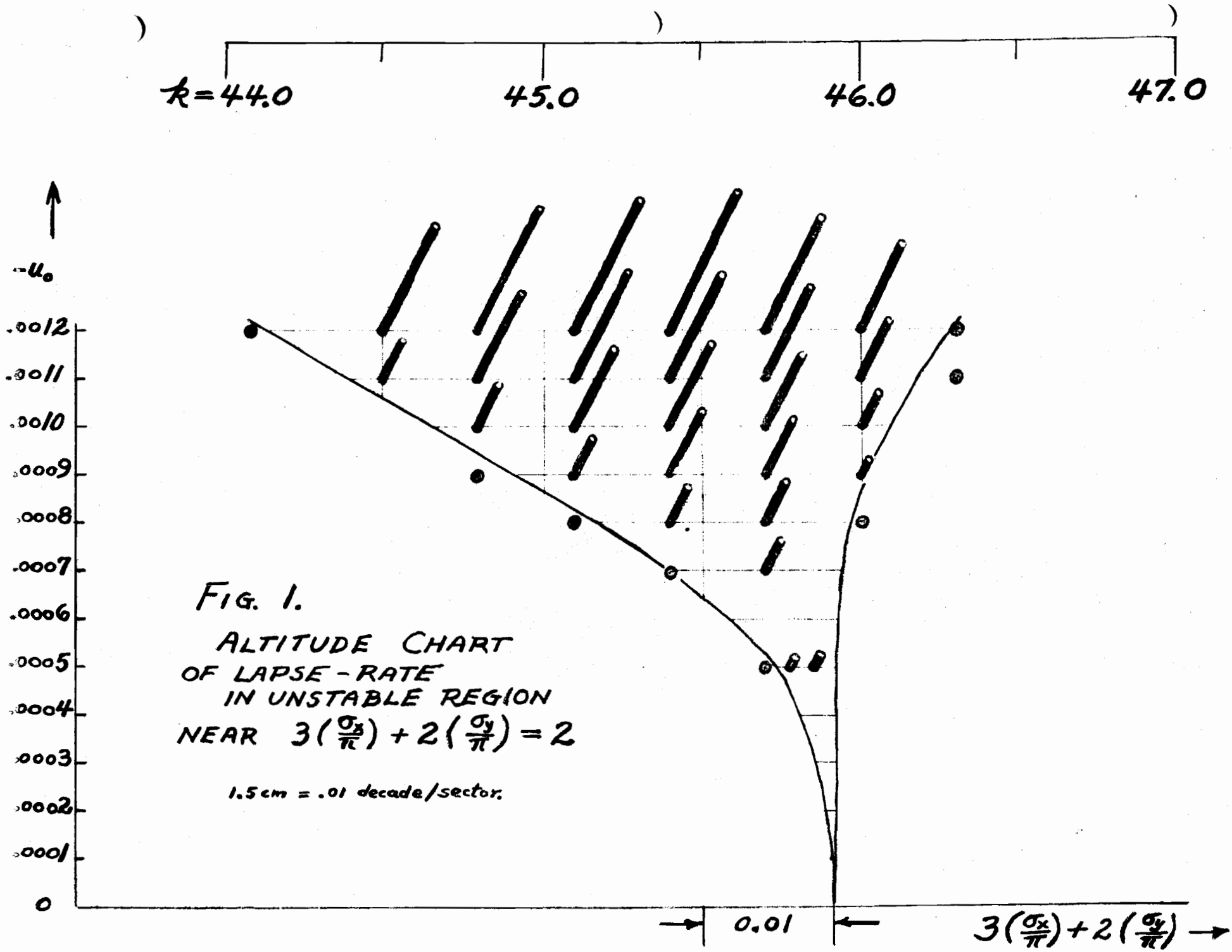

FIG. 1.

ALTITUDE CHART OF LAPSE-RATE IN UNSTABLE REGION NEAR $3\left(\frac{\sigma_x}{\pi}\right)+2\left(\frac{\sigma_y}{\pi}\right)=2$

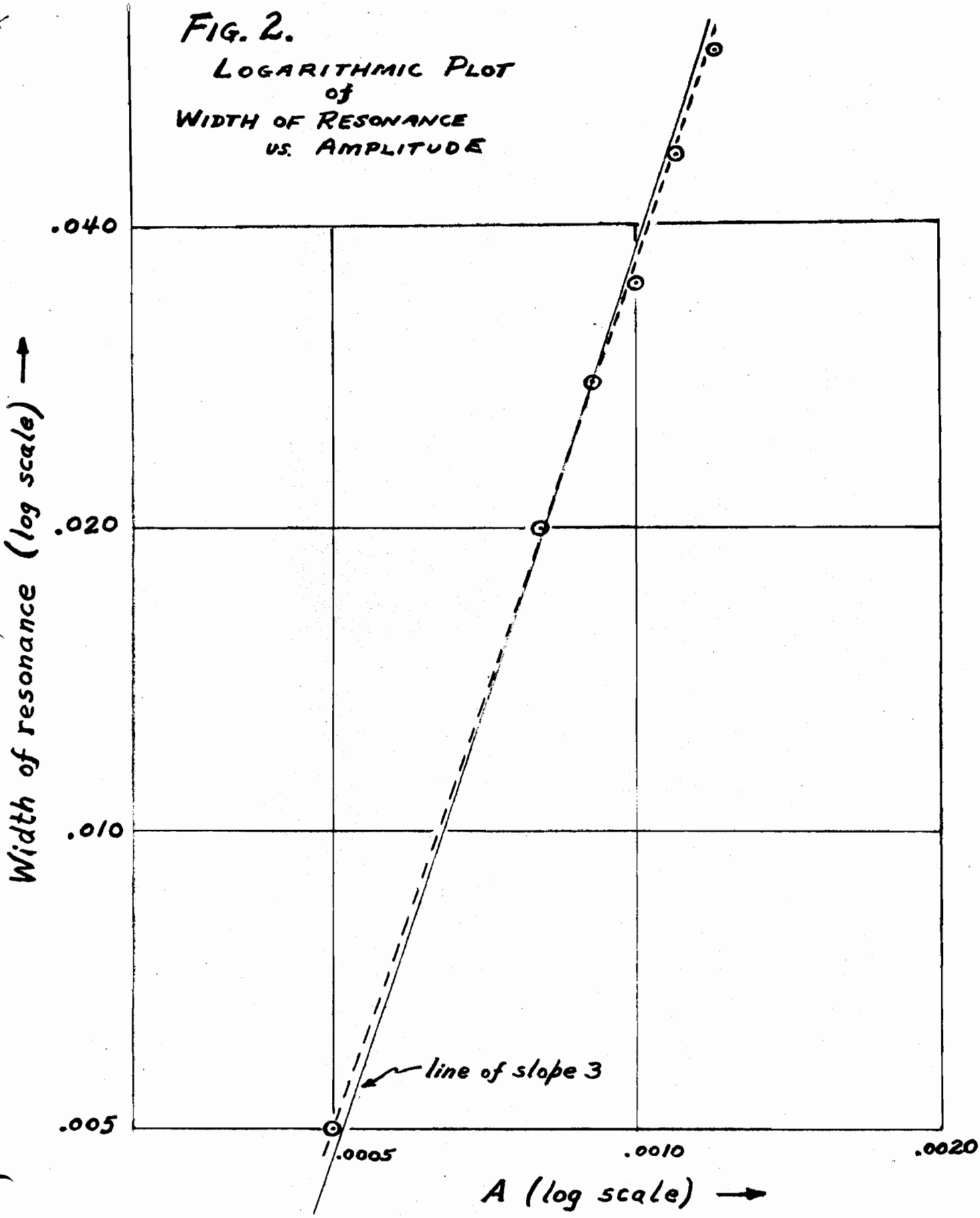
FIG. 2.
LOGARITHMIC PLOT
of
WIDTH OF RESONANCE
vs. AMPLITUDE
.040
.020
.010
.005
Width of resonance (log scale)
line of slope 3
.0005
.0010
.0020
A (log scale)

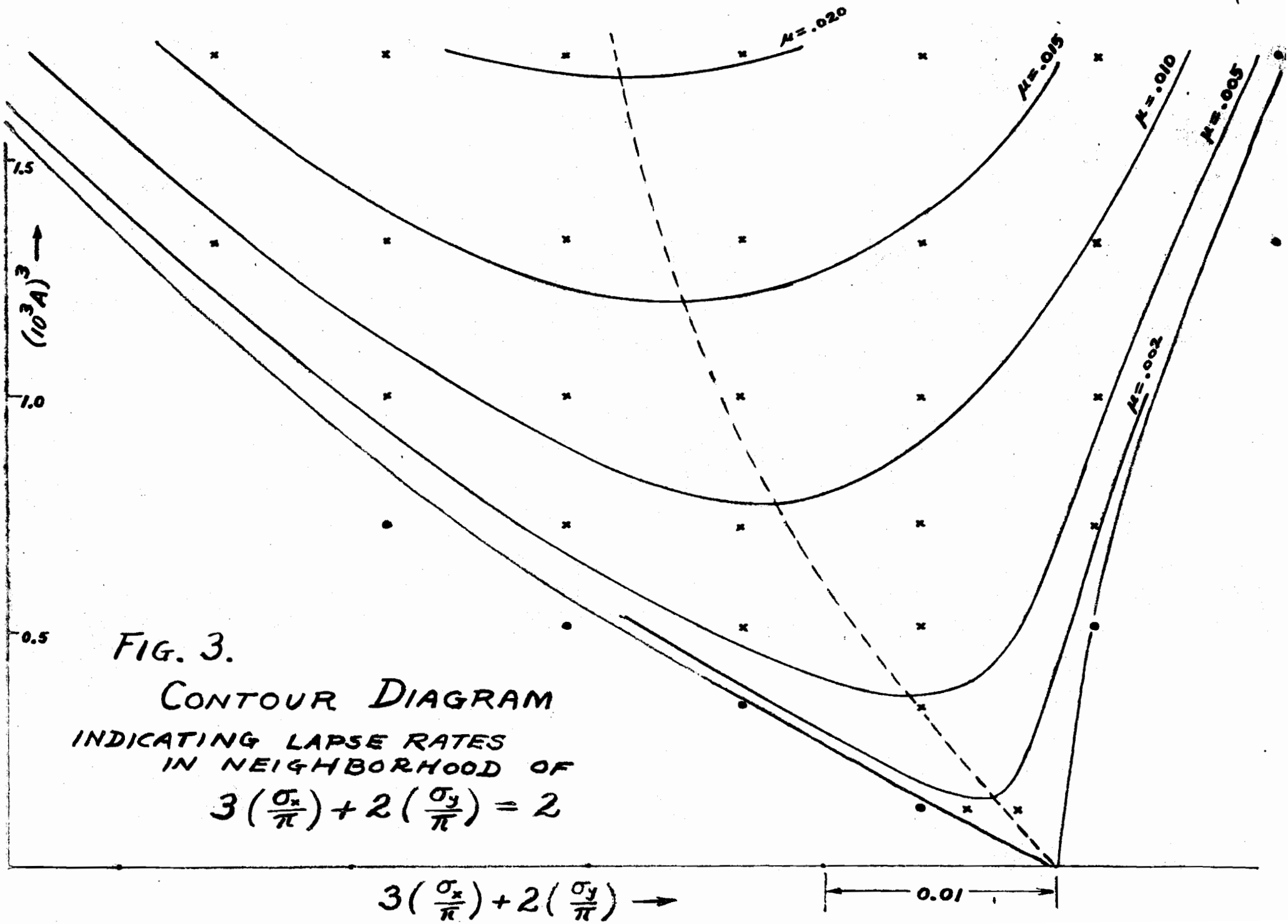

FIG. 3.

CONTOUR DIAGRAM

INDICATING LAPSE RATES IN NEIGHBORHOOD OF

$$3\left(\frac{\sigma_x}{\pi}\right) + 2\left(\frac{\sigma_y}{\pi}\right) = 2$$