APPROVED BY
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IDENTIFICATION

Fixed Point Square Root, MU SQR3

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PURPOSE

To extract the square root of a non-negative fixed point fraction, F.

RESTRICTIONS

The binary point is assumed to be located between the sign and first bit of the accumulator.

METHOD

Accuracy: Maximum error = $+ 2^{-35}$

Range: $+ 0 \leq F \leq 1 - 2^{-35}$, thus allowing the equality at the upper limit which is not permitted in MU SQR2.

Procedure:

$x = \sqrt{F}$ is obtained by the iterative procedure

$$x_{n+1} = x_n + 1/2 \left[F/x_n - x_n\right]$$

with $x_o = 1 - 2^{-35}$

Convergence is assumed to have occurred when

$$1/2 \left[F/x_n - x_n\right] = x_{n+1} - x_n = 0$$

(Cf. ILLIAC Subroutine Library - Program 116 - R1)

USAGE

Calling Sequence: Two modes of entry are available. In the first mode entry is made with the most significant part of F in the accumulator and the least significant part in the MQ-register, with the calling sequence:

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX SQR3, 4	α	BTSX A 4
$\alpha + 1$	error return	$\alpha + 1$	error return
$\alpha + 2$	normal return	$\alpha + 2$	normal return

where exit to the error return is made when $F \leq -0$ with F unchanged in the accumulator and MQ-register.

In the second mode of entry the most significant part of F is in COMMON and the least significant part of F is in COMMON + 1 (the state of the accumulator and the MQ-register is irrelevant) and the following calling sequence is used:

UASAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX SQR3 + 3, 4	α	BTSX 3A 4
$\alpha + 1$	error return	$\alpha + 1$	error return
$\alpha + 2$	normal return	$\alpha + 2$	normal return

where it is the burden of the programmer to assure that $F \geq +0$ since no error test is made in this mode of entry. Failure to do this will result in nonsense or a division halt.

The routine is left with $\sqrt{F}$ in both the accumulator and COMMON + 2.

Space Required:	UA SAP	MURASS
21 words program at:	SQR3	A
3 words temporary at:	COMMON	T

CODING INFORMATION

Timing: 0.5 n milliseconds, where n is the smallest integer such that

$$\left(\frac{1 - \sqrt{F}}{1 + \sqrt{F}}\right)^{2^{n-1}} < 2^{-35}$$

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        REM SQR3 MURA FIXED POINT SQUARE ROOT ROUTINE                  MU  SQR3
SQR3    TMI 1,4             IF NEGATIVE, ERROR RETURN                 SQR3  0001
        STO COMMON          MSP OF F STORED                           SQR3  0002
        STQ COMMON+1        LSP OF F STORED                           SQR3  0003
        CLA SQR3+20         PREPARE INITIAL X=X SUB ZERO              SQR3  0004
        STO COMMON+2                                                  SQR3  0005
        SUB COMMON          (1-2EXP-35)-(F)                           SQR3  0006
        TZE SQR3+18         IF ZERO, LEAVE                            SQR3  0007
        CLA COMMON          F IN AC-MQ                                SQR3  0008
        LDQ COMMON+1                                                  SQR3  0009
        DVH COMMON+2        F/XSUBN IN MQ                             SQR3  0010
        PXD 0,0             CLEAR AC                                  SQR3  0011
        LLS 35              F/XSUBN IN AC                             SQR3  0012
        SUB COMMON+2                                                  SQR3  0013
        ARS 1               1/2(F/XSUBN-XSUBN) IN AC                  SQR3  0014
        TZE SQR3+18         IF 0, DONE                                SQR3  0015
        ADD COMMON+2        IF NOT 0, FORM XSUBN+1                    SQR3  0016
        STO COMMON+2        XSUB(N+1) RESTORED                        SQR3  0017
        TRA SQR3+6          REPEAT ITERATION                          SQR3  0018
        CLA COMMON+2        ANSWER IN AC                              SQR3  0019
        TRA 2,4             NORMAL RETURN                             SQR3  0020
        OCT 377777777777    CONSTANT, 1-2EXP-35                       SQR3  0021
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