APPROVED BY
J N Snyder

IDENTIFICATION

Fixed Point Logarithm, Base 2, MU LOG2

E. M. Zographos - December 17, 1956

Midwestern Universities Research Association, Madison, Wisconsin

PURPOSE

Given a fixed point fraction x, $2^{-35} \leq x \leq 1 - 2^{-35}$, find fixed point $\log_2 x$.

METHOD

Procedure: $\text{Log}_2\, x$ is computed by replacing x by $2^{-m} \cdot w$, where $1/2 \leq w \leq 1 - 2^{-35}$ and m is a positive integer. Then

$$\log_2 x = \log_2 2^{-m} \cdot w = -m + \log_2 w,$$

$\log_2 w$ is fractional and -m is the integer part of $\log_2 x$. The quantity w is then used to calculate the fractional part of $\log_2 x$ from the series

$$\log_2 w = -1 + \sum_{i=1}^{35} a_i\, 2^{-i},$$

where a_i is either zero or one. The quantities a_i are determined from the iteration process:

$$P_o = w$$
$$P_{i+1} = P_i^2 \text{ if } P_i^2 \geq 1/2$$
$$P_{i+1} = 2P_i^2 \text{ if } P_i^2 < 1/2$$

$$a_{i+1} = 1 \text{ if } P_i^2 \geq 1/2$$
$$a_{i+1} = 0 \text{ if } P_i^2 < 1/2.$$

Range: $2^{-35} \leq x \leq 1 - 2^{-35}$, an error return results for $x \leq 0$.

Accuracy: The error will lie in the interval $\pm 2^{-34}$ but will be biased into the interval 0 to -2^{-35}.

(Cf. ILLIAC Subroutine Library - Program 130 - S3)

USAGE

Calling Sequence:

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX LOG2, 4	α	BTSX A 4
$\alpha + 1$	error return	$\alpha + 1$	error return
$\alpha + 2$	normal return	$\alpha + 2$	normal return

Upon entry x must be in the AC. The normal return is made with the integer part of $\log_2 x$ in the AC and the fractional part in the MQ. Whenever $x \leq 0$, the error return is made with x still in the AC.

Space Required:

	UA SAP	MURASS
46 words program at:	LOG2	A
5 words temporary at:	COMMON	T

CODING INFORMATION

Timing: The minimum time is 15.9 ms. and the maximum time is 19.2 ms.

```
      REM LOG2 MURA FIXED POINT LOGARITHM, BASE 2                    MU   LOG2
LOG2  SXD COMMON,1        SAVE IR1                                 LOG2 0001
      SXD COMMON+1,2      SAVE IR2                                 LOG2 0002
      TZE 1,4             RETURN IF X=0                            LOG2 0003
      TMI 1,4             RETURN IF X NEGATIVE                     LOG2 0004
      STO COMMON+2        STORE X                                  LOG2 0005
      PXD 35,0            CLEAR AC                                 LOG2 0006
      STO COMMON+3        SET SUM =0                               LOG2 0007
      PAX 0,1             SET IR1=0                                LOG2 0008
      LXA LOG2+5,2        SET IR2=35                               LOG2 0009
      CLA COMMON+2                                                 LOG2 0010
      ALS 1               TEST FOR X GREATER THAN                  LOG2 0011
      PBT 1               OR=1/2                                   LOG2 0012
      TXI LOG2+10,1,1     CONTINUE UNTIL X GREATER,=1/2            LOG2 0013
      ALS 2                                                        LOG2 0014
      TZE LOG2+43         TEST FOR X=1/2                           LOG2 0015
      ARS 3                                                        LOG2 0016
      ADD LOG2+45         +1/2                                     LOG2 0017
      STO COMMON+2        STORE W                                  LOG2 0018
      PXD 0,1             NUMBER OF SHIFTS=M, INTEGER PART         LOG2 0019
      ARS 18                                                       LOG2 0020
      STO COMMON+4                                                 LOG2 0021
      LDQ COMMON+2        P SUB I                                  LOG2 0022
      MPR COMMON+2        (P SUB I) EXP 2                          LOG2 0023
      STO COMMON+2                                                 LOG2 0024
      SUB LOG2+45         TEST FOR (P SUB I) EXP 2                 LOG2 0025
      TMI LOG2+38         GREATER OR=1/2                           LOG2 0026
      CLA LOG2+45         A SUB I=1                                LOG2 0027
      ARS 35,2            POSITION (A SUB I) (2 EXP-I)             LOG2 0028
      ORS COMMON+3        ADD TO SUM SO FAR                        LOG2 0029
      TIX LOG2+21,2,1     LOOP RE-ENTRY, OUT AFTER 35 PASSES       LOG2 0030
      CLS COMMON+3        -(SUM OF FRACTIONAL PART)                LOG2 0031
      COM                 FORM - 1 + SUM                           LOG2 0032
      ADD LOG2+44                                                  LOG2 0033
      LRS 35              FRACTIONAL PART TO MQ                    LOG2 0034
      CLS COMMON+4        INTEGER PART TO AC                       LOG2 0035
      LXD COMMON,1        RESTORE IR1                              LOG2 0036
      LXD COMMON+1,2      RESTORE IR2                              LOG2 0037
      TRA 2,4             OUT                                      LOG2 0038
      CLA COMMON+2        A SUB I=0                                LOG2 0039
      ALS 1               FORM 2(P SUB I) EXP2                     LOG2 0040
      STO COMMON+2                                                 LOG2 0041
      TIX LOG2+21,2,1                                              LOG2 0042
      TRA LOG2+30                                                  LOG2 0043
      TXI LOG2+16,1,1     HERE IF W=1/2                            LOG2 0044
      OCT 400000000001    CONSTANTS                                LOG2 0045
      OCT 200000000000                                             LOG2 0046
```