APPROVED BY
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IDENTIFICATION

Fixed Point Square Root, MU SQR2

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PURPOSE

To extract the square root of a non-negative fixed point fraction, F.

RESTRICTIONS

The binary point is assumed to be located between the sign and first bit of the accumulator.

METHOD

Accuracy: Maximum error = $+ 2^{-35}$

Range: $+ 0 \leq F < 1 - 2^{-35}$

Procedure:

$x = \sqrt{F}$ is obtained by the iterative procedure

$$x_{n+1} = x_n + 1/2 \left[F/x_n - x_n\right]$$

with $x_o = 1 - 2^{-35}$

Convergence is assumed to have occurred when

$$1/2 \left[F/x_n - x_n\right] = x_{n+1} - x_n = 0$$

(Cf. ILLIAC Subroutine Library - Program 116 - R1)

USAGE

Calling Sequence: Two modes of entry are available. In the first mode entry is made with the most significant part of F in the accumulator and the least significant part in the MQ-register, with the calling sequence:

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX SQR2, 4	α	BTSX A 4
$\alpha + 1$	error return	$\alpha + 1$	error return
$\alpha + 2$	normal return	$\alpha + 2$	normal return

where exit to the error return is made when $F \leq -0$ with F unchanged in the accumulator and MQ-register.

In the second mode of entry the most significant part of F is in COMMON and the least significant part of F is in COMMON + 1 (the state of the accumulator and the MQ-register is irrelevant) and the following calling sequence is used:

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX SQR2 + 3, 4	α	BTSX 3A 4
α + 1	error return	α + 1	error return
α + 2	normal return	α + 2	normal return

where it is the burden of the programmer to assure that $F \geq +0$ since no error test is made in this mode of entry. Failure to do this will result in nonsense or a division halt.

The routine is left with $\sqrt{F}$ in both the accumulator and COMMON + 2.

Space Required:	UA SAP	MURASS
18 words program at:	SQR2	A
3 words temporary at:	COMMON	T

CODING INFORMATION

Timing: 0.5 n milliseconds, where n is the smallest integer such that

$$\left(\frac{1 - \sqrt{F}}{1 + \sqrt{F}}\right)^{2^{n-1}} < 2^{-35}$$

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REM SQR2 MURA FIXED POINT SQUARE ROOT ROUTINE                    MU   SQR2
TMI 1,4                    IF NEGATIVE, ERROR RETURN            SQR2  0001
STO COMMON                 MSP OF F STORED                      SQR2  0002
STO COMMON+1               LSP OF F STORED                      SQR2  0003
CLA SQR2+17                PREPARE INITIAL X=XSUBZERO           SQR2  0004
STO COMMON+2                                                    SQR2  0005
CLA COMMON                 F IN AC-MQ                           SQR2  0006
LDQ COMMON+1                                                    SQR2  0007
DVH COMMON+2               F/XSUBN IN MQ                        SQR2  0008
PXD 0,0                    CLEAR AC                             SQR2  0009
LLS 35                     F/XSUBN IN AC                        SQR2  0010
SUB COMMON+2                                                    SQR2  0011
ARS 1                      1/2(F/XSUBN-XSUBN) IN AC             SQR2  0012
TZE SQR2+15                IF 0, DONE                           SQR2  0013
ADD COMMON+2               IF NOT 0, FORM XSUBN+1               SQR2  0014
TRA SQR2+4                 REPEAT ITERATION                     SQR2  0015
CLA COMMON+2               ANSWER IN AC                         SQR2  0016
TRA 2,4                    NORMAL RETURN                        SQR2  0017
OCT 37777777777            CONSTANT, 1-2EXP-35                  SQR2  0018
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