APPROVED BY
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IDENTIFICATION

Fixed Point Arctangent, MU ATN1

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PURPOSE

To find the principal value of arctan x for a fixed point fraction x.

RESTRICTIONS

The binary point is assumed to be located immediately to the left of bit 1.

METHOD

Accuracy: Error bounds = $\pm 6.7 \cdot 10^{-10}$
Maximum |observed error| $\leq 1.3 \cdot 10^{-10}$
Root Mean Square error $\leq 2.4 \cdot 10^{-10}$

Range: $-1 < x < 1$, $-\frac{\pi}{4} < \arctan x < \frac{\pi}{4}$

Procedure: Expand arctan x in a truncated Chebyshev series

$$\sum_{i=0}^{23} a_i T_i(x) \quad , \text{ where } a_i = \begin{cases} (-1)^{(i-1)/2}(2/i)(-1+\sqrt{2})^i, & \text{i odd} \\ 0 \quad , & \text{i even} \end{cases} .$$

Combine coefficients of corresponding powers of x to get

$$\arctan x = \sum_{i=1}^{12} c_{2i-1} x^{2i-1}.$$

The sequence $S_{23} = c_{23}$, $S_{21} = x^2 S_{23} + c_{21}, \ldots, S_{2i-3} = x^2 S_{2i-1} + c_{2i-3}$, $\ldots$, $S_1 = x^2 S_3 + c_1$ is formed and $S_1 x$ is taken as arctan x.

USAGE

Calling Sequence: Entry is made with the fixed point fraction x in the accumulator by the following:

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX ATN1, 4	α	BTSX A 4
$\alpha + 1$	normal return	$\alpha + 1$	normal return

The routine is left with arctan x in radians as a fixed point fraction in the accumulator.

Space Required:

	UA SAP	MURASS
27 words program at:	ATN1	A
2 words temporary at:	COMMON	T

CODING INFORMATION

Timing: 4.5 milliseconds.

```
      REM ATN1 MURA FIXED POINT ARCTANGENT                          MU   ATN1
ATN1  SXD ATN1+14,1       SAVE INDEX 1 IN ATN1+14                  ATN1  0001
      LXA ATN1+14,1       SET INDEX 1 TO 11                        ATN1  0002
      STO COMMON          CONTENTS COMMON EQUALS X                 ATN1  0003
      LDQ COMMON                                                   ATN1  0004
      MPR COMMON                                                   ATN1  0005
      STO COMMON+1        CONTENTS COMMON+1 EQUALS X-SQUARED       ATN1  0006
      LDQ ATN1+15         CONTENTS MQ EQUALS C SUB 23              ATN1  0007
      MPR COMMON+1                                                 ATN1  0008
      ADD ATN1+27,1                                                ATN1  0009
      LRS 35                                                       ATN1  0010
      TIX ATN1+7,1,1      FORM S SUB 2I-3 IN MQ                    ATN1  0011
      MPR COMMON          S SUB 1 EQUALS ARCTAN X IN AC            ATN1  0012
      LXD ATN1+14,1       RESTORE INDEX 1                          ATN1  0013
      TRA 1,4             NORMAL RETURN                            ATN1  0014
      DEC 11              INTEGER 11, INDEX 1 IN DECREMENT         ATN1  0015
      OCT 400113062334    C SUB 23                                 ATN1  0016
      OCT 001047524143    C SUB 21                                 ATN1  0017
      OCT 403555603507    C SUB 19                                 ATN1  0018
      OCT 010131144064    C SUB 17                                 ATN1  0019
      OCT 415277606152    C SUB 15                                 ATN1  0020
      OCT 022175717026    C SUB 13                                 ATN1  0021
      OCT 426677314757    C SUB 11                                 ATN1  0022
      OCT 034275514170    C SUB 9                                  ATN1  0023
      OCT 444440104743    C SUB 7                                  ATN1  0024
      OCT 063146065646    C SUB 5                                  ATN1  0025
      OCT 525252520504    C SUB 3                                  ATN1  0026
      OCT 377777777752    C SUB 1                                  ATN1  0027
```