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FEBRUARY 6, 1956 TO MARCH 31, 1957

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INTRODUCTION

This general report to the Atomic Energy Commission includes the technical reports and publications issued since June, 1956, at the time when Atomic Energy Commission support was used for the technical effort. Many of the reports and publications cover work performed before that date and received support also from the National Science Foundation and the Office of Naval Research. The work of the project is very well described in the MURA reports and publications which are given here. These MURA reports have been used as a means of consolidating and recording developments taking place in the work. Nevertheless, additional over-all summaries are being included in this report to the Atomic Energy Commission.

With the collection of the technical group at Madison as a step in implementing the Atomic Energy Commission's wish to create the most advanced accelerator which we could conceive, the theoreticians and experimenters from various universities could finally work more closely and conveniently with one another. Most of the experimental equipment which had been built in Purdue, Michigan, Ames, Iowa, and Illinois was brought to Madison. In addition, the digital computer effort was transferred from Illinois to Madison. The growing consumption of digital computer time on the University of Illinois Illiac made it clear that MURA could use a faster computer of its own. Consequently, an IBM 704 computer was installed in quarters prepared for it in the summer of 1956. The computer went into operation in the middle of November, 1956.

A large visiting staff was present in the summer while quarters were being prepared in a former automobile agency garage at 2203 University Avenue. MURA moved into the building on August 18, 1957.
The scientific staff is composed of the following members:

Professor F. T. Cole, Theory, on leave from the University of Iowa

Mr. Edward A. Day, Engineer

Dr. Jacob Enoch, Theory

Dr. Lloyd Fosdick, Digital Computer

Dr. Marvin Freiser, Theory

Professor Robert Haxby, Experimental, on leave from Purdue University

Professor Lawrence Jones, Experimental, on leave from the University of Michigan

Professor Donald W. Kerst, Technical Director, on leave from the University of Illinois

Dr. Richard F. King, Digital Computer

Professor P. G. Kruger, Director, on leave from the University of Illinois

Professor L. J. Laslett, Theory and Computer, on leave from Iowa State College

Dr. Frederick Mills, Experimental

Mr. Tihiro Ohkawa, on leave from Tokyo University

Professor George Parzen, Theory, on leave from the University of Notre Dame

Mr. Charles Pruett, Experimental

Mr. Ednor Rowe, Experimental

Professor James Snyder, Head of Computer, on leave from the University of Illinois

Mr. Melvin Storm, Digital Computer

Professor Keith Symon, In Charge of Theory, on leave from the University of Wisconsin

Professor Kent Terwilliger, on leave from the University of Michigan

Professor Jean Van Bladel, RF Engineering, on leave from the University of Wisconsin

Dr. William Wallenmeyer, Experimental
<table>
<thead>
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<th>Administrative and Supporting Staff</th>
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</tr>
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<td>(Average Approximately 15 Hours Per Week Each)</td>
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PART II

PUBLICATIONS AND TECHNICAL REPORTS ISSUED SINCE JUNE, 1956, AND THEIR ABSTRACTS

PROPOSED METHOD FOR DETERMINING MARK V TRAJECTORIES BY AID OF GRID STORAGE. L. J. Laslett. MURA-99, 2/21/56.

It has been proposed that the trajectories of particles in an FFAG accelerator of the spirally-ridged (Mark V) type may conveniently be obtained with an electronic digital computer by storage of values from which the fields may be derived. Based on the scaling properties of the assumed structure, the present report suggests suitable coordinates to be used in representing the field, develops the differential equation describing the magnetostatic problem in these coordinates, indicates the means of obtaining the field-components from the solution of this differential equation and discusses the use of these results in dynamical equations for the trajectories. Proposed limits for the parameters are indicated, certain cases in which $l/w$ vanishes being admissible.

TABLE PERTAINING TO SOLUTIONS OF A HILL EQUATION. G. Belford, L. Jackson Laslett, J. N. Snyder. MURA Notes, 4/3/56.

The digital computer of the University of Illinois has been employed to determine the characteristic exponent and a quantity related to the phase function for solutions of a Hill equation of the form.

$$\frac{d^2 y}{dt^2} + \left[A + B \cos 2t + C \cos 4t + D \cos 6t\right] y = 0$$

for a series of values of the coefficients with $-0.5 \leq A \leq 0.5$, $-0.5 \leq B \leq 2.0$, $-0.5 \leq C \leq 0.5$, $-0.05 \leq D \leq 0.1$

METHODS OF RADIO FREQUENCY ACCELERATION IN FIXED FIELD ACCELERATORS WITH APPLICATIONS TO HIGH CURRENT AND INTERSECTING BEAM ACCELERATORS. K. R. Symon and A. M. Sessler. MURA-106, 4/16/56.

A number of schemes for accelerating particles in fixed field accelerators are described, some of which show promise of giving high beam currents. Included are schemes using conventional synchrotron acceleration of high repetition rate, "bucket lifts", phase displacement acceleration, beam stacking and multiple oscillators. A classification of schemes as scheduled or stochastic is proposed. An analytic theory of radio frequency acceleration is presented which permits an estimate of the performance of any of these schemes. The theory is applied to the study of the acceleration process with constant and with changing radio frequency, to acceleration near the transition energy and to the process of beam stacking and beam capture. The analytic results are compared with results of digital computation.

It is proposed that a beam of particles be stored in a ring magnet and used for intersecting beam experiments. Methods of ejection from the accelerator and injection into the storage ring are discussed.


Designs for a colliding beam accelerator are proposed, using radial sector FFAG machines. In one design, the machines are of positive and negative momentum compaction, respectively, and are concentric. The second type uses two eccentric radial sector machines with high and low energy orbits crossing. Both designs have large circumference.


Spherical coordinates are especially suited for the expression on the magnetic field of the Mark V, FFAG accelerator. The magnetic induction is assumed given in the median plane in the form of a series of circular harmonics. Expansions are then given for the magnetic induction, and its vector and scalar potentials on both sides of this plane. An exact solution is given by making use of associated Legendre functions. Certain approximations and series expansions are introduced. A particularly simple approximation is worked out in detail. If used for the equations of motion it is always required that any approximation satisfy the Liouville theorem exactly. The Lagrangian and Hamiltonian equations are derived.


Expansions for the characteristic exponents and the Floquet solutions for the linear and homogeneous second order differential equation with periodic coefficients are derived. The convergence of these series is established.

A SHORT SURVEY OF DIGITAL COMPUTER RESULTS FOR RADIAL MOTION IN FFAG MARK V SPIRAL RIDGE ACCELERATOR. Nils Vogt-Nilsen. MURA-123, 6/20/56.

General remarks are made pertaining to the graphical presentation and interpretation of numerical computations on non-linear one-dimensional motion. A survey of numerical results is then given for the radial motion on the median plane in a set of large scale FFAG Mark V spiral ridge accelerators.
A SCALED RADIAL SECTOR FFAG FOR INTERSECTING BEAMS.
Tiihiro Ohkawa. MURA-124, 7/9/56.

A design is proposed for a colliding beams accelerating system consisting of a radial sector FFAG accelerator with two beams circulating in opposite directions. Only one magnet is involved and there is no beam transfer problem, but the system has the large circumference of radial sector machines. There are many target areas available, since the beams cross in every straight section. Details of orbit geometry are discussed.


The change in volume of a region of phase space when dissipative forces are present is calculated. Application is made to the problem of decreasing the phase space occupied by particles in a high energy particle storage ring by the use of foils. It is concluded that foil thickness required is so large that scattering in the foil would cancel the compression in phase space.


Computations have been made of the median plane fields which results magnetostatically from various two-dimensional pole configurations resembling those which would be employed in a spiral sector FFAG accelerator. The flutter (the root mean square deviation of the field from its average in units of its average), gap size and circumference factor are tabulated.

THE OHKAWA INTERSECTING BEAMS MACHINE. L. W. Jones. MURA-134, 8/16/56.

Ohkawa has suggested a geometry for intersecting beams in which particles of the same charge travel in opposite directions in a single accelerator. This is achieved with fields which reverse periodically with azimuth around a circle and which increase radially. Since the amounts of iron, copper, power, R. F., etc., required in this case are characteristic of one machine only instead of two, and since stacked beams would intersect 2N (where N is the number of sectors) times around the machine instead of at only one point, it was thought worthwhile to look into some possible design parameters to determine to what extent such a machine would be feasible.

RESULTS OF MISCELLANEOUS TWO-DIMENSIONAL POTENTIAL COMPUTATIONS, INVOLVING POLE-FACE CURRENTS, OF POSSIBLE INTEREST IN FFAG MAGNET DESIGN. L. J. Laslett. MURA-135, 8/17/56.

For orientation purposes a few two-dimensional potential computations have been made with the ILLIAC to study fields of possible interest in FFAG structure. The results which are reported thus supplement those summarized in MURA-119. One case concerns a structure similar to the reversed-field FFAG model which is in operation at the University of Michigan.

An adiabatic accelerator is one in which all accelerating and guide fields vary slowly in the time of revolution of particles. It is shown that if betatron oscillations may be neglected, the momentum gained by a particle in such an accelerator depends essentially only on the difference in magnetic flux enclosed by the initial and final orbits.

A SIMPLE METHOD FOR RF ACCELERATION IN A FIXED FIELD MACHINE. D. B. Lichtenberg. MURA-137, 8/6/56.

Gas scattering, space charge, resonance and oscillator limitation effects on methods of R.F. acceleration in FFAG accelerators are discussed. Two set of parameters for "intermediate stacking" systems are developed which accelerate particles rapidly through the low energy region where gas scattering is important and do not have resonances between R.F. and betatron oscillation frequencies, thus overcoming the difficulties of earlier schemes.

BEAM LOADING ON AN R.F. CAVITY AND AN ACCELERATION SCHEME BY VELOCITY MODULATION. T. Ohkawa. MURA-140, 8/18/56.

Several effects of beam loading on an R.F. cavity are discussed. An acceleration scheme with a non-linear negative resistance and a passive cavity is developed. A second scheme using velocity modulation effects similar to those in klystrons is also developed with two cavities close together. The first method appears to be useful when particle energies are large compared to R.F. voltages and the second when these quantities are comparable.

NON-LINEAR RESONANCES IN ALTERNATING GRADIENT ACCELERATORS. G. Parzen. MURA-200, 10/12/56.

A method is presented for treating the non-linear resonances which occur in the radial oscillations of the strong focusing alternating gradient type of high energy accelerator. The method predicts the location of the resonances, calculates the stability limits on the amplitude of the radial oscillations and also gives the stability limit orbit. The results of the theory are compared with those of a numerical calculation.


Experiments and calculations are described which test the effects of misalignments in the Michigan radial sector FFAG betatron. In the experiments, one magnet is moved and the equilibrium orbit deviation measured. The calculations are done in the "hard-edge" approximation, where the effects of magnet edges are approximated by impulsive lenses. Agreement between theory and experiment is considered good.
ELECTRICAL CHARACTERISTICS OF A MODEL MECHANICALLY TUNED RADIO FREQUENCY CAVITY FOR A MULTI-BEV SYNCHROTRON

A radio-frequency cavity is described where the frequency is varied by changing the shape of the cavity mechanically. A frequency range of 7:1 and a gap voltage of 10,000 volts are obtained with less than 500 watts r.f. power going into the cavity. A Q of several thousand is found. Electrical properties of the cavity with an oscillator tube in operation are discussed in detail.


Numerical calculations were carried out for the energies available in the center of mass system of two protons of energy, 20 Bev each, colliding at various angles in the laboratory system, and the velocities of the center of mass for such collisions. Formulas for angular distributions of mesons produced in the laboratory system have been derived by assuming certain angular distributions for mono-energetic mesons in the center of mass system, corresponding to two cases of high energy proton-proton collisions (i) oblique collisions of two proton beams of same incident energy and (ii) head-on collisions of two proton beams of different incident energies. Numerical calculations for these meson angular distributions were confined to oblique collisions of two proton beams of 20 Bev each at an angle 120° and head-on collisions of two proton beams, one with 20 Bev while the other has incident energy 10 Bev in the laboratory system.

EFFECTS OF RESONANCES IN THE RADIAL SECTOR FFAG MODEL.

A survey of a portion of the $\sqrt{\chi} - \sqrt{\beta}$ plane has been made in the Michigan radial sector FFAG model to study the effects of resonances on the accelerated beam. No beam is observed on integral, half-integral, and linear sum resonances, as well as on the essential non-linear resonances $3\beta = 2\pi$ and $2\beta + \beta = 2\pi$. Some other quadratic resonances attenuate the beam, but no effect is observed due to cubic or higher order resonances. $\beta = \beta$, $\sqrt{\chi} = \sqrt{\beta}$, and other difference resonances have no noticeable effect on the beam.

SURPRESSION OF BETATRON OSCILLATION EXCITATION (R. F. KNOCKOUT) BY THE R. F. ACCELERATING SYSTEM OF A FIXED FIELD ACCELERATOR.
D. W. Kerst. MURA-215, 2/7/57.

Two recent suggestions for avoiding radio frequency knockout of a beam due to R. F. excitation of betatron oscillations are discussed. The first is the use of two cavities tipped to give electric fields at angles $\phi = \pm \phi$ with the equilibrium orbit and spaced so that $\phi$ radians of betatron oscillation can occur between them with $\tan \phi = \frac{1 + \cos \delta}{\sin \delta}$. The R. F. phase must travel from
one cavity to the other with the speed of the stacked particles. The second suggestion is that it is possible to turn off the cavity voltage briefly while it is passing the R. F. K. O. frequency without losing an appreciable number of particles and probably without enlarging the dimensions of the stacked beams seriously.

COUPLED NON-LINEAR RESONANCES IN ALTERNATING GRADIENT ACCELERATORS. G. Parzen. MURA-217, 2/20/57.

The method presented in a previous paper for treating the non-linear resonances in alternating gradient type of high energy accelerators has been extended to treat the two dimensional coupled resonances. The sum resonance $2 \nu_f + \nu_x = N$ is treated in detail and the same methods can be applied to the higher order sum resonances. The difference resonances are not treated. The results of the theory are compared with those of a numerical calculation.


A radial sector FFAG accelerator has been constructed and successfully operated. In this 8 sector accelerator electrons are betatron accelerated from 25 to 400 Kev using both continuous and pulsed injection. The number of radial betatron oscillations per revolution may be varied from 2.3 to 3 and the number of vertical oscillations per revolution from 1 to 3. Calculations of these oscillation frequencies using various approximations are described and discussed. These frequencies have been measured statically with the unaccelerated beam and dynamically using an rf perturbing voltage on the accelerated beam. Results of these calculations and experiments are in satisfactory agreement. Effects of misalignments have been measured and are in close agreement with calculations presented. A survey has been made over a large area of the betatron oscillation stability region. The effects of the many resonances observed are in good qualitative agreement with theory.


The computational program necessary to study the stability of particle trajectories in A.G. Synchrotrons may be very much reduced, if one has available phase plane transformations. The transformations are used on high-speed computers, and so must be exactly area preserving, as well as of simple functional form, thus precluding the use of most simple perturbation theories. A method will be given, using the results of G. D. Birkhoff, to construct an approximate transformation in the neighborhood of an equilibrium orbit, which satisfies these requirements. A perturbation solution in powers of the amplitude of oscillation is first obtained, and then transformed to normal form, in such a way that one can formally obtain an area preserving transformation of arbitrary accuracy. In terms of the newly introduced independent variables, the transformation is particularly simple, and one may by inspection obtain the betatron oscillation wavelengths as a function of amplitude. The method does not correctly yield the stability limit of the oscillations. Application will be made to a one-dimensional A.G. Synchrotron with cubic non-linearities.
MEDIAN PLANE MOTION IN THE SPIRALLY RIDGED FFAG ACCELERATOR.

We have investigated the motion in the median plane of the spirally ridged FFAG accelerator with a magnetic field
\[ H_z = -H_0 \left( \frac{r}{r_0} \right)^k \left[ 1 + f \sin \left( w^{-1} \ln \left( \frac{r}{r_0} \right) - N \theta \right) \right] \]
using the Electronic Digital Computer of the Graduate College of the University of Illinois (Iliiac) to integrate numerically the exact equations of motion. Parameters were chosen in the ranges characteristic of the small FFAG accelerator under construction at Illinois, but the orbit phenomena appear to be independent of the machine size. The closed equilibrium orbit is very closely a biased sine wave, in good agreement with the analytical theory of Laslett. The size of the region in the phase plane of stable oscillations about this closed orbit depends critically on the phase change per sector \((\phi)\) of the oscillation, which is a function of amplitude. The motion becomes unstable in most cases when \(\phi\) approaches \(2/3\pi\), though in some cases instability occurs at some other rational fraction of \(\pi\). Interpretation of this instability will be discussed.


In order to investigate the properties of particles subject to radio-frequency acceleration in fixed field accelerators a rather flexible program has been developed for the Electronic Digital Computer of the University of Illinois (Iliiac). The coupling of the phase motion with the betatron oscillations is ignored, but the effect of an arbitrary number of oscillators with arbitrary voltage and frequency programs is included. The particle frequency as a function of energy may also be chosen. It takes the ILLIAC approximately 7 sec. to calculate the passage of a particle through 100 oscillators. The program is being used to study the size and shapes of stable zones in synchrotron phase space as a function of particle energy. Acceleration through the transition energy, and the feasibility of various proposed radio-frequency acceleration schemes will be discussed. Comparisons will be made between these results and the theory of particles in fixed field accelerators.


We have examined analytically the character of small amplitude betatron oscillations about the equilibrium orbit of the spirally ridged accelerator in greater detail than reported previously. The frequencies of betatron oscillations are predicted from the coefficients in the equations of motion through the aid of tables computed for linear equations of the Hill type. Approximate dynamical equations, not limited to small amplitude motion, were also integrated with the Electronic Digital Computer of the Graduate College of the University of Illinois (ILLIAC) for a variety of machine parameters characteristic of small or model-sized structures.
The last-mentioned work not only affords a check of the predicted features of small amplitude oscillation, but also permits a survey to be made of the axial limits of stable oscillation. The results of such a survey for the axial motion will be presented, supplementing work reported previously for the radial motion.


Fixed field accelerators and particularly FFAG accelerators permit a wide variety of acceleration schemes. If accelerator gaps are driven at radio frequency which may or may not be synchronized, then in the neighborhood of each energy for which the frequency of revolution of the particles is equal to the radio frequency or any of its sub-harmonics, there is a region of particle energies and phases ("bucket") within which particles execute stable phase oscillations around the synchronous energies. If the radio frequency is modulated the "buckets" move up or down the energy scale. Under suitable conditions, particles in any of these buckets can be accelerated by this process. Phase displacement acceleration is also possible in which the frequency modulation is reversed from the normal direction so that empty buckets move down the energy scale and particles outside the bucket are accelerated upward. One can classify acceleration schemes as scheduled or unscheduled according to whether or not the various radio frequencies follow a synchronized program so that you accelerate every particle according to a definite schedule. Thus a wide variety of schemes are available, each with its particular advantages and disadvantages.


The density of particles that can be attained in an accelerator beam is limited by Liouville's theorem if the forces operating on the particles are Hamiltonian. If dissipative forces are present, the rate of change in a volume in phase space can be related to an integral over phase space of the partial derivatives of the dissipative forces with respect to the momenta of the phase points. This result has been applied to the case of foils of matter placed in the path of a beam of particles. The compression of a volume in phase space is found to be comparable to the degradation of particle momenta in the foils, independent of the shape of the foils.


A major problem of very high energy alternating gradient and FFAG accelerators is their sensitivity to small misalignments of their component magnets. The effect of such misalignments has been tested in a small FFAG betatron by moving individual magnets radially and vertically and measuring the resultant position of the equilibrium orbit. All the magnets of positive or of negative curvature are moved in a test with the beam probe position held fixed, which is equivalent to measuring the position of the equilibrium orbit at different azimuths with
a fixed misalignment. We have calculated the position of the equilibrium orbit in the presence of these misalignments in the linear approximation. The effects of magnet edges were replaced by impulsive lenses. Table I shows the theoretical maximum deviation of the equilibrium orbit per unit magnet displacement as determined in straight sections. The experimental points fall on the theoretical curves to within the accuracy of the measurements.

<table>
<thead>
<tr>
<th>Amplitude per unit magnet displacement</th>
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<tr>
<td>Positive magnet moved radially</td>
</tr>
<tr>
<td>Negative magnet moved radially</td>
</tr>
<tr>
<td>Positive magnet moved vertically</td>
</tr>
<tr>
<td>Negative magnet moved vertically</td>
</tr>
</tbody>
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BETATRON OSCILLATION RESONANCES IN A SMALL FFAG BETATRON.

The frequencies of betatron oscillation in the small FFAG betatron may be varied independently. The intensity of accelerated beam has been measured over the range between 2.3 and 3 in \( \nu_r \) and 1 and 3 in \( \nu_z \), the frequencies being determined within 1% accuracy by the rf perturbation technique. The beam is destroyed or attenuated only in regions where the frequencies satisfy a relation of the form \( p\nu_r + q\nu_z = m \), with \( p, q, \) and \( m \) positive integers. In particular, no beam is obtained when \( p + q = 1 \) or 2, as expected in a linear machine with imperfections. The beam is usually affected when \( p + q = 3 \) because of quadratic forces. In this case it is always destroyed when \( q \) is even and \( m = 8 \), corresponding to nonlinear instabilities in this 8-sector machine, always attenuated when \( q \) is even and \( m \neq 8 \), due to imperfections, and rarely attenuated when \( q \) is odd because of symmetry about the median plane. There is no effect on the beam where \( p + q > 4 \), or when either \( p \) or \( q \) is negative, consistent with the theory of nonlinear equations.

NONLINEAR RESONANCES IN ALTERNATING GRADIENT ACCELERATORS.

A method has been developed for treating the nonlinear resonances which occur in the betatron oscillations of the strong focusing alternating gradient type of high-energy accelerator. The method predicts the location of the resonances, calculated the stability limits on the amplitude of the oscillations, and also gives the stability limit orbits. The theory has been compared with numerical calculations for the \( \nu_r = 1/3 \) N resonance and for the \( 2\nu_x + \nu_y = N \) coupled resonance.


The several suggestions for achieving very high center-of-mass energies with head-on collisions between particles of intersecting beams require two
tangent accelerators or one accelerator with storage rings. It is possible, using a modification of the radial sector fixed field alternating gradient accelerator, to have particles of the same mass and charge circulating simultaneously in opposite directions in a single accelerator. The geometry consists of a ring of identical magnets, with the field in successive magnets alternating in sign but of the same magnitude. With the field in each magnet increasing radially as $r^k$, the equilibrium orbits are scalloped so that they lie in stronger fields in magnets of one sign and hence progress around the accelerator. Some advantages may be realized by spiraling the magnets. Calculations will be summarized which show that, for realistic parameters, the circumference factor (ratio of the radius of the accelerator to the minimum particle radius of curvature) may be five or six. While the large circumference of such a machine is a severe disadvantage, the possibility of many intersecting beam target areas, variability of center-of-mass energy, and the relative design and structural simplicity of this configuration make it an interesting possibility.


In radial sector FFAG accelerators the variation of the median plane magnetic field in the azimuthal direction (the direction of particle motion) is large compared to its azimuthal average. Because of this large variation, the equilibrium (closed) orbit, which is that orbit with the period of the structure, undergoes large oscillations about a circle. The analytical methods used in the discussion of spiral sector FFAG accelerators do not give accurate expressions for the equilibrium orbit because the variation of the field is so large compared to its average. Approximate analytical expressions have been found for this equilibrium orbit by a variation-iteration technique, which includes "mixing" effects of various Fourier components of each other. The free (betatron) oscillations about this orbit are strongly affected by the oscillations of the equilibrium orbit because of the inherent nonlinearities of FFAG. The equations of motion of these betatron oscillations have been derived. Solutions will be discussed and comparisons with digital computer results given for the nonlinear motion.


The attainment of very high intensity beams and the practicality of circulating intersecting beams in FFAG accelerators depend heavily on the successful application of the Symon-Sessler radio-frequency acceleration theory. The FFAG electron accelerator described earlier is being used with a combination of betatron and rf acceleration to study various phases of this theory. A betatron pulse accelerates the beam to an intermediate energy, where it is allowed to coast. Then one or a number of identical frequency modulated rf accelerating pulses can be applied to the circulating electrons. The starting frequency of these rf pulses can be adjusted so as to sweep through the coasting beam or to pick it up. A second betatron pulse then accelerates all remaining electrons to the target. The first betatron pulse is used to limit the range of frequency modulation required. Polarization of ferroelectric condensers then produces about a 15% frequency change for the rf acceleration.
RADIO-FREQUENCY EXPERIMENTS WITH A FFAG ACCELERATOR. II. RESULTS.

A particular prediction of the Symon-Sessler general rf theory for FFAG accelerators states that an rf pulse, starting at a frequency corresponding to an energy below that of a coasting beam of particles and frequency modulating through it does not capture, but lowers the energy of the coasting beam by a predictable amount. This phase displacement has been tested experimentally with the radial sector FFAG electron accelerator. Measurements of the average energy displacement of the circulating beam by an rf pass through it have been made by two different methods which agree with each other and with the theoretically predicted values to within experimental error. In a second experiment beam of particles coasting at one energy has been transferred (accelerated) to a higher energy by a sequence of identical rf pulses. The resulting energy distribution of the electrons in the vacuum chamber as inferred from the time distribution of beam output from the second betatron pulse indicates a good transfer efficiency. Details of these experiments and numerical results will be presented.


When a large circulating beam is to be stored at high energy in an accelerator while additional particles are being accelerated up to the stacking energy, the frequency modulated accelerating voltage can excite resonant betatron oscillations in the stack at certain frequencies. Methods of suppressing this knockout have been considered by members of the MURA staff. Some of the methods are: (1) Separate cavities at different orbit radii. (2) Fast passage through knockout resonance. (3) Multiplephased cavities to eliminate certain knockouts and to simulate traveling accelerating waves. (4) Controlled angle of the electric field of a phased pair of cavities spaced a half of a betatron wavelength or less along the orbit at the stacking radius. (5) Jumping over the narrow knockout frequency with a cavity holding a wide bucket. (6) Programed operations to cancel the excited oscillations. Knockout resulting from both nontangential electric fields and from discrete energy jumps is important.


An important advantage of fixed field accelerators is the possibility of stacking intense beams of circulating particles by acceleration to the stacking energy of successive groups of injected particles. Theoretical analyses and computational results will be presented which permit the calculation of the magnitude and energy distribution of stacked currents which result from any given injection and radio-frequency acceleration process. Stacking processes which make use of betatron accelerations have also been studied.

It has been proposed by Veksler that there is a possibility of accelerating particles by transferring momenta from photons to particles in "coherent Thomson scattering". When the density of particles is sufficiently high, the scattered wave will interfere with the incident wave and finally a complete reflection occurs. It has been observed that a plasma in a microwave system reflects or attenuates the wave strongly. With a cloud of charged particles inside the guide, the cut-off frequency of the guide is higher than that of the guide alone. When a wave having frequency between the two cut-off frequencies encounters the cloud in the guide, it will be reflected completely. Therefore the momentum carried by the wave must be transferred to the particles provided that the phase and group velocities are higher than the velocity of the particles. This method of acceleration appears feasible with the realistic densities of particles and microwave frequency and power. Since the necessary density of particles is considerably lowered by using a frequency near cutoff for the wave, the acceleration per particle is large compared to Veksler's original scheme.


It is shown that the amount of phase space in FFAG accelerators is sufficient to allow the accumulation of successively accelerated groups of particles to form an intense beam of high energy particles circulating for many minutes in a fixed field accelerator. Such intense circulating beams when directed against one another would give usable yield of nuclear disintegrations with the great advantage that the center of mass would be at rest in the laboratory and that the full energy of the accelerated particles would be available for nuclear disintegrations.


It is possible, by using alternating-gradient focusing, to design circular accelerators with magnetic guide fields which are constant in time, and which can accommodate stable orbits at all energies from injection to output energy. Such accelerators are in some respects simpler to construct and operate, and moreover, they show promise of greater output currents than conventional synchrotrons and synchrocyclotrons. Two important types of magnetic field patterns are described, the radial-sector and spiral-sector patterns, the former being easier to understand and simpler to construct, the latter resulting in a much smaller accelerator for a given energy. A theory of orbits in fixed-field alternating-gradient accelerators has been worked out in linear approximation, which
yields approximate general relationships between machine parameters, as well as more accurate formulas which can be used for design purposes. There are promising applications of these principles to the design of fixed-field synchrotrons, betatrons, and high-energy cyclotrons.


The equations describing the distribution of magnetic field in the orbital plane and the differential equations of motion and spiral sector accelerators are given. The stability diagram for the motion is shown and some of the non-linear effects examined by the MURA group are described. Various types of magnet structures are described such as the separated spiral structure with an example of a 15 billion volt accelerator having a large flutter, a small flutter \((f = 0.25)\) spirally ridged structure, and floating equipotential structures are shown. Some of the coil winding problems are discussed.


It is pointed out that in fixed field alternating gradient accelerators it is possible to have beams which have been built up in intensity by acceleration of successive groups of particles passing in opposite directions through a common section of the structure so the interacting beams cause nuclear disintegration with the center of the mass of reacting particles at rest in the laboratory. The advantage of such a system is that it is equivalent to an accelerator producing a single beam striking a stationery target having an energy \(2E^2\), where \(E\) is the energy of one of the double beams in units of \(mc^2\). It is shown that at 15 billion volts with approximately 100 amperes circulating in the stacked beams, beam life of the order of 1000 seconds is feasible and \(10^7\) interactions per second under certain target configurations would be expected. The difficulties of background from residual gases are discussed.


The radial sector type accelerator designed and constructed at Purdue University, the University of Illinois and the University of Michigan by the MURA staff from other universities and finally assembled and tested at the University of Michigan is described. The construction and winding of the magnets and methods of testing the frequency of orbital and radial oscillations are explained. One such test is the static test using a fine beam of electrons and another method is the system of radio frequency excitation of betatron oscillations to determine their frequency. The behavior of the accelerator with pulses or continuously injected beams is described.


An extensive description of analytical and digital computer results for the
radio frequency acceleration of particles in fixed field accelerators are given. Conventional frequency modulated accelerators, the bucket lift, phase displacement mechanisms of accelerators, beam stacking, multiple oscillators, schedule and random schemes are described. Phase stable regions determined analytically by the computer are shown and the method of calculating the amount of phase flux accelerated is given.


A detailed treatment of the general theory for fixed field alternating gradient accelerators is given. It is applicable to radial and spiral sector designs with results of digital computer computations on non-linear effects.


This is a survey of several types of FFAG particle accelerators, the radial sector type, spiral sector type. Cyclotrons and betatrons are discussed.


Analytical questions such as the equations of motion, the linear theory and resulting stability limits and the non-linear characteristics and stability limits of the motion are given. Methods of handling magnetostatic problems on the digital computer are described. Characteristics of constant frequency alternating gradient cyclotrons and acceleration methods are discussed. Interesting radio frequency possibilities arise in fixed field accelerators because many particles can be simultaneously accelerated with different harmonics and different accelerating cavities. The possibility is brought up by the high intensities of FFAG accelerators for bombarding oppositely directed beams for the achievement of high energy.


The acceleration of electrons in small radial sector models is up to 400 kilovolts as described. Operation under pulses and continuous injections is possible and the sensitivity of operation near resonances is examined.

Weekly staff meeting minutes and bi-monthly general meeting minutes provide current information on the work. Minutes #1 - 29.
PART III
SUMMARY OF EXPERIMENTAL WORK

The theoretical work of the project centering around the digital computer could be instrumented by renting the IBM 704 and changing over from the already going Illiac program to a 704 program. This change-over was organized and carried out by Professor Snyder. The experimental program, on the other hand, required bringing in shop equipment, engineers, technicians, and experimental equipment and instruments from various places.

The radial sector accelerator which had been designed and constructed at Purdue, Illinois and Michigan and which had been assembled and tested at Michigan, was moved to Madison after the completion of the study described in the MURA report on effects of resonance (MURA-212). It was thought that this model could be made use of further in studies of RF acceleration in a fixed field accelerator.

Toward the end of November a six sector spiral FFAG ring magnet with its coils was moved to Madison from Illinois. At the end of the summer work, the parameters for this structure had been settled, and preliminary manufacturing and field tests by Peterson and Hauseman had been carried out at Illinois. In the next three months all magnets with two sets of k coils and k tuning coils and one set of flutter coils were constructed and shipped to Madison. Pulsing auxiliary circuits for models and for RF experiments were also brought from Illinois.

Magnetic field detecting equipment was brought from Purdue for the purpose of establishing the orbital plane in the new radial sector model and for the purpose of measuring the small 30 to 100 gauss field. In addition, various magnet supply equipment and test magnets made at Purdue for the Michigan model and for other tests were brought.
A mechanically modulated radiofrequency system was shipped from Ames, Iowa. The experiments carried out with this equipment and with additional equipment at the MURA laboratory are well described in MURA reports and publications.

After installation of the radial sector accelerator, the radiofrequency experiments demonstrating the stacking of a beam of particles by accumulation of successive groups of particles was accomplished. In addition, the process of phase displacement acceleration was observed and studied in the same experiments. If a large circulating beam is to be created for the purpose of causing two oppositely directed intense beams to interact, then we can build up that beam by successive injections into the accelerator and successive acceleration of each new group to the final energy of the circulating beam or stack. The fundamental limitations on this process are the amount of phase space brought up to the final energy with each successive group of particles and the space charge and space current effects. As more phase space represented by the area of the phase stable region in the accelerator is brought to the stack, it must displace, by Liouville's Theorem, the phase space occupied by the already stacked particles. This action has the effect of shifting the energy of the stack of particles to make room for the new additions to the stack.

The fundamental theory explaining these effects was developed by Symon and Sessler, and it appears in MURA-106, The Bulletin of the American Physical Society, April 26, 1956, p. 178, in Volume I of the CERN Symposium, 1956, and in the Bulletin of the American Physical Society, Volume 2, p. 188, April 25, 1957, where the experimental verification of the theory is explained. (by Symon, Jones, and Terwilliger at the Washington meeting of the American Physical Society).
The passage of a so-called bucket or RF phase stable region past a group of asynchronous particles producing phase displacement not only can decelerate particles but it can accelerate particles if the frequency modulation is toward the lower energy.

There is a generally small fraction of the particles which are nearly synchronous and which travel along to some extent in the direction of frequency modulation, just outside but very close to the phase stable region. This produces a filamentation of phase space. The theory giving this stirring or mixing effect in phase space was described by K. R. Symon in the minutes of the general MURA meeting of January 4 and 5, 1957, and the effect has been observed by Terwilliger, Jones, and Pruett in the RF experiments with the radial sector model. (Bulletin of American Physical Society, Washington Meeting, Volume 2, Number 4, Page 188.)
Figure one shows an example of stacking and phase displacement. The top sweep shows the burst of electrons arriving at the target at a late time. A group of particles has been stacked by betatron action and subsequently accelerated out to the target by a second pulse of betatron flux.

The next sweep shows a big burst of electrons arriving earlier because one radio frequency operation has lifted particles from the first stack to form a new stack at higher energy which required less flux of the second flux pulse to accelerate a new stack out to the target. A residue of the first stack not captured comes out at the usual time.

The third sweep shows two effects. Four radio frequency operations have been used to lift the first stack to the position of the second stack and consequently the late pulse of particles due to the first stack is almost completely captured and lifted out. However, the second stack has become broader because the second, third and fourth R. F. operations have phase displaced particles restacked by the first R. F. operation. Consequently the second stack has been broadened in energy by the width of the second, third and fourth R. F. buckets. The fourth trace shows so much additional phase displacement from seven R. F. operations that the particles from the first R. F. operation are almost phase displaced down to the energy of the first stack.

The bottom sweep shows ten R. F. operations which now capture the highly phase displaced particles from the first R. F. operation and lifts them back to the position of the energy of the second stack.

These beam stacking experiments were necessarily done at the 200 to 300 kilovolt energies available in the model, and as a result the electron beam had a life-time of only one millisecond. For the purposes of doing experiments with
large space charges and large space currents it would be necessary to go to the order of 50 Mev to obtain life-times of several seconds at $10^{-6}$ millimeters of mercury pressure. With these life-times it should be possible to stack 100 amperes of circulating current very easily, which would model some but not all space charge and space current effects of a stacked beam of protons suitable for a 15 Bev colliding beam accelerator. The design of such an electron accelerator has been under study since the fall of 1956.

The methods of injection available must be, and seem to be, sufficient to scan very easily the large amount of phase space available in an FFAG accelerator to build up a multi-turn injected current reaching the space charge limits for 100 KV injection. In fact by programing the multiturn scanning of betatron phase space with a 500 milliampere beam we would have several times the charge needed to reach the space charge limit.

The amount of phase space available in the electron accelerator studied was sufficient to allow the stacking of the beam far in excess of 100 amperes. A 10,000 ampere circulating current and probably more could be accomodated in a 1 cm. diameter beam. The possibility of pushing the stacking experiments to these high currents where some of the effects mentioned by the Russian, Budker, might be observed has been constantly in mind as an experiment to be pursued with this model. These high current experiments would provide much of the necessary experience in compensation for the demagnetization effects of the relativistic beam on the
ring magnet and for the observation of tuning and resonance effects within the stacked beam due to space charges, space currents, and their images.

Since the collection of ions in a stacked beam can alter the pinching of the beam, ultra high vacuum techniques have been felt necessary for some of the experiments. Vacuum systems for operation at least to $10^{-8}$ millimeters of mercury have been under consideration.

This high current electron model would have both radio-frequency and betatron acceleration methods provided, since combinations of the two methods provide flexibility in the experiment.

Structures having approximately 12 sectors with a $k$ of about 8 and orbits about 100 cm. radius have been under examination on the digital computer.

Frequency modulation of about 1.4 would be necessary for injection at 100 KV. Since there would be approximately three betatron oscillations per revolution, this range of frequency modulation would give us the opportunity to study means of avoiding integral, half integral, and third integral R.F. knock-out effects upon the stacked beam.

Samples of ferrites have been tested for use in the 20-65 megacycle range for the purpose of frequency modulating the RF cavity. However, the extent of modulation required may well be possible by the use of ferroelectric capacitors which have some advantages over ferrites.

In the summer of 1956 the possibility of using such a high current electron model for experiments with interacting beams (synchroclash) of electrons was examined by members of the MURA staff and by theoretical physicists at the University of Wisconsin. It is very likely that electron-electron scattering and other electron-electron effects will never be studied
except by this method. If each of the two colliding electron beams has an energy less than the energy necessary to produce \( \pi \) mesons on the residual gas, certain background effects will be avoided. Beams of around 120 to 200 million volts were considered, but there is a severe problem of detection resulting from the small cross sections concerned. The interacting beams would be of the order of 1,000 amperes of continuously circulating current. Much compensation for energy loss due to radiation would be called for, and consequently an RF cavity would be needed to hold the stack at the final energy.

One interesting configuration of an accelerator which could perform this experiment is the radial sector machine with all magnets identical, as suggested by Ohkawa. This can contain beams traveling in opposite directions in the same structure, thus causing interaction wherever the beams cross. This configuration is probably the type one would use for an electron-electron experiment, although this radial sector type requires far more iron than a pair of spiral sector ring magnets. The amount of iron necessary is not a crucial factor in this small model size.

The construction problems brought up by a high current model are in some cases more difficult than the construction problems of a full size accelerator, partly because the space for copper is more crowded in a small magnet than it is in a large magnet operating at the same flux. Modifications of the methods used for making coils on the first model must be made. Shaped and formed coils fitting onto shaped poles have been studied. The digital computer gives us the correct shape of the poles which would provide for a scaling flux plot in a magnet in which the structure does not scale. This allows the magnet gap at the high field radius to be small, thus to require a
small number of ampere turns.

A very interesting development is the possibility of the so-called eigenpoles, which require no distributed coils over the pole face to produce an \( r^k \) field. Numerous possibilities have been suggested by eigenpoles, some of which require distributed coils but no iron (except a thin shield) and consequently such poles do not require reluctance correction.

Experimental work with the spiral sector model magnet brought from the University of Illinois has been largely in the development of magnetic field measuring techniques and instrumentation and in the examining of the corrections necessary on iron structures. The spiral shape of the magnet requires corrections which are not as simple as those required in a radial sector. For the radial sector case it can be shown that the reluctance correction with scaled iron thickness and an approximately constant permeability can be simply made by winding correction turns on the back leg of the magnet where the forward winding of the main coil passes. This correction is sufficient except at the inside radius of the magnet.

When the spiral sector magnets have been completely examined with instruments being made for orbital plane examination etc., the vacuum tube will be installed with an injector-inflector system brought from the University of Illinois. These auxiliary parts are being readied.

The photographs Figure 2 show the shape of the spiral sector poles. Spiral grooves are machined in the edges of the pole faces so that the edges of the back wound coil can be inserted. This puts the center of the pole at the desired magnetic potential, while the ears or guard edges are at zero magnetic potential. The result is a higher flutter than is possible without
the guard edges. The back wound distributed coil provides a \( k \) of about 0.7, and an additional \( k \) tuning coil is laid down on the pole. The magnetic potential of the guard edges can be altered by flutter coils which are attached to the edges. The adjustment of the current in the flutter tuning coils mainly controls the frequency of axial betatron oscillations, while the adjustment of the current in the \( k \) tuning coil produces mainly a variation in the frequency of the radial betatron oscillations. The careful examination which is being made of magnetic properties of these poles is instructive because the effects are typical and perhaps exaggerated in the case of such a low magnetic field model. Fine details are being examined which were never tested on the radial sector model for which we have very little field information.

The spiral sector poles shown in the photograph were pressed to a conical shape. Some of the dimensional variations in the resulting structure have made it desirable to disassemble and remachine, particularly to produce good magnetic contact at the back leg of the magnet. In addition, the surfaces are being cleaned of the black magnetic oxide of iron which developed in annealing which exhibits random permanent magnetism. This permanent magnetism has disturbed the determination of the internal flux plot within the iron pole piece by the observation of the tangential oersteds on the surfaces of the iron. Figure 3 shows a potential demand plot on the surface of the iron exterior to the magnet. A typical graph of the variation of the \( k \) radius is also shown in Figure 4.

The parameters shown for this model are conservative but the non-linearities are strong and typical of spiral sector accelerators in general. This accelerator has been completely studied on the digital computer and it
remains to be seen how the actual performance compares with the performance of the digital computer. The main difference between this accelerator and a very large one is that a large accelerator would have more imperfection resonances in it which must be avoided.

Instrumentation for injection studies on the spiral sector model takes the form of injector-inflector combination. This structure was examined by the rubber diaphragm method. With the emission of a large current the resulting space charge forces should produce a small source at the exit end of the inflector where a cross over would occur. The resulting source of particles is separated from the orbits by a thin .005" septum which should make the acceptance of particles much more likely than is the case for a filament source within an injector case. Figure 5 shows this general arrangement. Tests are under way bombarding these inflector structures by an injector to see how hot and with how much current we can run them. Modulators for providing various pulse lengths of injectors are being prepared.
PART IV
SUMMARY OF THEORETICAL WORK

Considerable progress has been made in non-linear orbit theory and
the problem of short-time stability is now fairly well in hand. The methods
of Laslett and Sessler\(^1\) for predicting stability boundaries have been extended
utilizing the methods of Parzen\(^2\) and Moser\(^3\) (see abstracts of MURA
reports). It is now possible to predict fairly accurately, short-time stability
boundaries, betatron oscillation frequencies as a function of amplitude, lapse
rates as a function of amplitude in regions where the amplitude grows,
turnover points, and in general, to characterize the behavior of the motion
out to amplitudes at which not more than one resonance dominates the motion.
Analytic studies have been begun on the problem of intermediate and long-
time stability and a few computations utilizing algebraic transformations
have been carried out. Formulas have been worked out for determining
fairly precisely the equilibrium orbits in radial and spiral sector
accelerators with given magnetic fields.\(^4\) The equilibrium orbits are
needed both in the linear and non-linear theory of betatron oscillations.

The theory of the RF stacking process is nearly completed\(^11\). Analytic
and computational methods have been worked out for determining the energy
distribution of stacked particles as a function of time. Tables of energy loss
distribution by phase displacement have been computed. Analytic work is
in progress and computational methods are available for studying the process
of RF knockout of stacked beams. A number of methods have been proposed
and are under study for avoiding RF knockout\(^5\). An analysis of the effect of
space charge on the RF acceleration process is nearly completed.

Proposals have been made for the acceleration of plasmas by means of
radiation pressure in wave guides. Preliminary analyses of this process have
been carried out\(^6\).
Extensive computations of orbits in the radial sector model are under way utilizing the Forocyl Formesh programs on the IBM 704. Results of these computations are compared with experimental results of orbit studies on the model\(^\text{(7)}\). Equilibrium orbit calculations and calculated betatron oscillation frequencies are in good agreement with measured values. Studies of non-linear effects are in the progress.

Magnet configuration has been proposed in which particles may be accelerated simultaneously in opposite directions around a single accelerator\(^\text{(8)}\). This will make it possible to achieve colliding beams in a single accelerator. Analytical and computational studies are under way.

Analytical and computational methods have been developed for designing non-scaling magnetic pole surfaces which will, nevertheless, produce magnetic fields which scale. It will be necessary for reasons of economy to use such non-scaling magnet designs in larger models and in the final accelerator. Methods are being developed for taking into account finite thicknesses of coils and construction errors in the winding of coils. A type of magnetic pole configuration which has been called an "eigenpole" has been discovered which may be useful at least in the construction of models. Eigenpole surfaces are scaling surfaces of zero potential. A pole of infinite permeability whose surface is an eigenpole surface will produce a scaling magnetic field configuration without any backwindings on the pole surface. All of the current windings appear at the outer and inner edges of the magnet. Eigenpoles have the advantage not only of avoiding backwindings on the pole surfaces but also of giving a large gap between poles. Analytic and computational methods are available for computing eigenpole surfaces for a given magnetic field configuration, or conversely for computing the magnetic
field configuration which will result from a given eigenpole shape. Several related types of magnetic pole designs have been suggested, including finite reluctance poles, and "paper poles" in which only backwindings are required without any iron except for shielding purposes.

Exploratory computations are under way to determine optimum magnet shapes and magnet parameters for a high current electron model. Calculations of radiation effects have been made including corrections due to shielding by the metal vacuum tank and coherent radiations from bunches of electrons.

Computations of electromagnetic fields near r.f. accelerating gaps in metal donuts of various shapes have been made(10).

References

1. L. J. Laslett and A. M. Sessler, MURA-120
   A. M. Sessler, MURA-128
   L. J. Laslett, MURA-139, 248.
   #24, March 18, 1957.
PART V
SUMMARY OF DIGITAL COMPUTER WORK

MURA's IBM Electronic Data Processing Machine, Model 704, arrived in Madison on October 19, 1956, as per the delivery schedule set up by IBM in June, 1956. Several weeks were spent installing and checking out the machine; it became available for our use on November 7, 1956. An accurate log is kept of the available time and maintenance time of the computer. The percentage of available time is comparable to or better than that obtained in other 704 installations for which such data is available, starting at 81% for the month of November, 1956, and reaching 91% for the month of March, 1957. The reliability of this machine and the engineering maintenance provided are hence very satisfactory. The utilization of the machine as expressed in the proportion of time used for actual computing greatly exceeds that in other 704 installations which seldom achieve more than 40% of the time spent in computation. MURA's 704 computed approximately 70% of the scheduled time in November, 1956. This has grown to 84% for March, 1957.

Programming effort in the Computer Division began in September, 1956. Since that time 103 separate programs have been written. A large fraction of these are service routines and testing routines which are necessary to form parts of and to test the actual production programs of immediate interest to the accelerator design group. Twenty-five such production programs along with many overwrites which augment and alter their properties have been written.

A detailed listing of these programs follows:
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Descriptions of each production program have been prepared, and have been circulated to interested parties. A brief statement as to the function of each of the production programs follows:

FOROCYL

solves for the magneto-static potential distribution, given the contour of the magnets, the potential distribution on the magnets, and the configuration of the currents flowing in any conductors present. The magnetic field is then obtained at a series of mesh points in the space between the magnetic poles. This magnetic field is then placed upon punched cards so that it can be used in any subsequent computation. The magnetic field is also Fourier analyzed and printed out.

FORMERGE

will linearly combine as many magnetic fields produced by FOROCYL as desired. This process saves time in cases where the magnetic structure is made up of two or more independent magnets.

FORSUPS

regards the entire potential output of a given FOROCYL run as the eigenvector of a boundary value problem. The norm of this eigenvector is found and its scalar product with any similar eigenvector is found.
FORANAL

Fourier analyzes any sequence of magnetic fields.

EQUICYL

finds and prints the coordinates of surfaces of constant magneto-
static potential in a magnetic field produced by FOROCYL.

THE WELL-TEMPERED FIVE

integrates the dynamical equations of motion for betatron oscillations in a magnetic field which is specified by giving the Fourier components of that field on the median plane. The field components are obtained by expanding a series off the median plane. Both radial and axial oscillations are treated.

TEMPERMESH

uses the same series expansion as THE WELL-TEMPERED FIVE to produce and punch a set of field components at a selected set of mesh points. This set has all of the properties of the output of FOROCYL so that it may be used as the input for any subsequent computation.

FORERUNNER

integrates the differential equations for betatron oscillations in the median plane only. The Fourier components of the field on the median plane are given. This program is extremely useful in checking other programs for internal consistency of the entire set of focusing calculations.
FORMESH integrates the equations of motion for radial and axial betatron oscillations in a magnetic field specified on a mesh and produced by FOROCYL. Invariants (quadratic functions of the axial and radial coordinates and their momenta which are constant in linear betatron oscillation theory) can be computed and printed out by any of the programs which treat betatron oscillations.

FORMESH FIELD PRINT OVERWRITE prints out the components of the magnetic field along an orbit integrated by the FORMESH program.

FORFIX POINT finds the fixed points, the rotation number, and the invariant functions for betatron motion in the magnetic field produced by FOROCYL. This may be done for a hypothetically perfect machine or for a machine possessing various kinds of imperfections ("bumps").

A machine being tested using the FORMESH program can be afflicted with several types of field imperfections in order to test their resultant effect on the betatron motion.

FORMESH FUMBLEBUMPS applies an imperfection which is represented by a general non-linear transformation on the radial and axial coordinates and momenta once per machine revolution.
FORMESH MUMBLEBUMPS
represents an imperfection by changing the magnetic field produced by FOROCYL for a portion of each revolution to another magnetic field mesh also produced by FOROCYL.

FORMESH GRUMBLEBUMPS
represents the imperfection by altering the magnetic field of a portion of each revolution by adding to it a harmonic function of the radial and axial coordinates.

FORFIX POINT WITH FUMBLEBUMPS
finds the fixed points, the rotation number, and the invariant functions for betatron motion in a magnetic field produced by FOROCYL which has been augmented by imperfections of the type discussed above under FORMESH FUMBLEBUMPS.

FORFIX POINT WITH GRUMBLEBUMPS
finds the fixed points, the rotation number, and the invariant functions for betatron motion in a magnetic field produced by FOROCYL which has been augmented by imperfections of the type discussed above under FORMESH GRUMBLEBUMPS.

Two programs similar to FORMESH are:

ATEMESH
which uses an 8-point interpolation formula for obtaining the field quantities at a point in the magnetic field mesh rather than the 4-point interpolation formula used by FORMESH.
SIXTEEN MESH

which uses a 16-point interpolation formula to obtain field values from the magnetic field mesh.

Additional programs are:

SCOFFLAW

which integrates the betatron equations of motion in a magnetic field mesh which is read point-wise into the computer. It would thus treat motion in a magnetic field obtained from a series of experimental field measurements.

DUCK CALL and its successor DUCK ANSWER

integrates the equations of motion with driving terms specified by rather general non-linear harmonic functions. It is extremely useful in testing theories of non-linear motion and in making rough searches for types of field structure to which the more specific programs may be applied.

INVARIANT DUCK BUMPS

augments the DUCK ANSWER program by enabling it to compute the standard invariant functions and by providing it with machine imperfections of the type discussed above under FOR-MESH FUMBLEBUMPS.

TTT

treats the RF acceleration problem. A particle is carried around the machine being subjected to an arbitrary number of gaps, the
time dependence of whose frequency and voltage variations can be specified. The particle can also be subjected to energy attenuation due to foils. Betatron oscillations are inserted in a rudimentary way.

**ALGYTEE**

is designed to test various theories of betatron oscillation. Betatron oscillations are represented by a series of non-linear algebraic transformations on the radial and axial coordinates and momenta. The program is fast so that a particle may be carried through a very large number of sectors in a relatively short operating time.

**JOYBUCKETS**

evaluates a certain integral for choosable values of the parameters contained therein. This integral expresses the increment in the energy variable suffered by particles subject to a modulated radial frequency voltage.

**MESSY MESSY**

regards an accelerating machine as being made up of an arbitrarily choosable combination of defocusing portions, focusing portions, and sector edges. The transfer matrices for each such component are multiplied together to obtain the transfer matrix for the entire combination.