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APPROVED BY

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IDENTIFICATION

Fixed Point Exponential, Base 2, MU EXP2

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PURPOSE

Given the fixed point fraction x, $-(1 - 2^{-35}) \leq x \leq +0$, to compute 2^x as a fixed point fraction.

METHOD

2^x is computed from the series

$$2^x \cong \sum_{i=0}^{12} \frac{(x \log_e 2)^i}{i!}$$

with the leading term in the series taken to be $1 - 2^{-35}$ rather than 1.

Accuracy: Since the above series is alternating the truncation error must be less than $\frac{(\log_e 2)^{13}}{13!} < 2^{-35}$ and is therefore negligible in comparison with the errors from other sources. The other sources of error are roundoff from the 12 multiplications, each contributing a maximum error of 2^{-36}, and the initial inaccuracies in the 13 constants. 12 of the constants contribute a maximum error of 2^{-36} each and one, namely the leading term is in error by 2^{-35}. Hence the total maximum error is $13 \cdot 2^{-35} < 2^{-31}$. In most cases the error is considerably less than this limit since under the assumption that roundoff errors are randomly and uniformly distributed the RMS error from these sources is about $\sqrt{2} \cdot 2^{-35}$. Furthermore if $x \leq 1/2$ it can be easily shown that the error from these sources never exceeds 2^{-33}.

Range: $-(1 - 2^{-35}) \leq x \leq +0$, an error return results for $x \geq 2^{-35}$.

USAGE

Calling Sequence: Entry is made with x in the accumulator.

UA SAP		MURASS	
Loc.	Instruction	Loc.	Instruction
α	TSX EXP2	α	BTSX A 4
$\alpha + 1$	error return	$\alpha + 1$	error return
$\alpha + 2$	normal return	$\alpha + 2$	normal return

The normal return is made with 2^x in the accumulator. The error return is made whenever $x \geqslant 2^{-35}$, with x still in the accumulator. If entry is made with $x = \pm 0$ the routine gives $1 - 2^{-35}$ for the exponential.

Space Required:

	UA SAP	MURASS
26 words program at:	EXP2	A
1 word temporary at:	COMMON	T

CODING INFORMATION

Timing: 4.2 ms.

The number of terms taken in the exponential series, cited above under METHOD, may be reduced to some number $N < 12$ by inserting N for the integer end constant in location EXP2 + 12, which is now 12.

For each term removed from the series there will be a saving of 0.34 ms. The truncation error will be less than the last term neglected.

```
        REM EXP2 MURA EXPONENTIAL, BASE 2                                MU   EXP2
EXP2    TZE EXP2+2          TEST FOR X EQUAL TO ZERO                   EXP2  0001
        TPL 1,4                                                        EXP2  0002
        SXD EXP2+12,1       SAVE IR1                                   EXP2  0003
        LXA EXP2+12,1       SET IR1=12                                 EXP2  0004
        STO COMMON                                                     EXP2  0005
        CLA EXP2+25,1                                                  EXP2  0006
        LRS 35              LOOP TO SUM EXPONENTIAL SERIES             EXP2  0007
        MPR COMMON                                                     EXP2  0008
        ADD EXP2+26,1                                                  EXP2  0009
        TIX EXP2+6,1,1      LOOP TRANSFER, OUT AFTER 12 PASSES         EXP2  0010
        LXD EXP2+12,1       RESTORE IR1                                EXP2  0011
        TRA 2,4                                                        EXP2  0012
        HTR 12                                                         EXP2  0013
        OCT 000000000001    (LN2) TO THE 12TH/12 FACTORIAL             EXP2  0014
        OCT 000000000017    (LN2) TO THE 11TH/11 FACTORIAL             EXP2  0015
        OCT 000000000363    (LN2) TO THE 10TH/10 FACTORIAL             EXP2  0016
        OCT 000000006651    (LN2) TO THE 9TH/9 FACTORIAL               EXP2  0017
        OCT 000000130540    (LN2) TO THE 8TH/8 FACTORIAL               EXP2  0018
        OCT 000001777460    (LN2) TO THE 7TH/7 FACTORIAL               EXP2  0019
        OCT 000024141105    (LN2) TO THE 6TH/6 FACTORIAL               EXP2  0020
        OCT 000256607775    (LN2) TO THE 5TH/5 FACTORIAL               EXP2  0021
        OCT 002354525560    (LN2) TO THE 4TH/4 FACTORIAL               EXP2  0022
        OCT 016153021534    (LN2) TO THE 3RD/3 FACTORIAL               EXP2  0023
        OCT 075376757770    (LN2) TO THE 2ND/2 FACTORIAL               EXP2  0024
        OCT 261344137677    LN2                                        EXP2  0025
        OCT 377777777777    1-2 TO THE MINUS 35                        EXP2  0026
```