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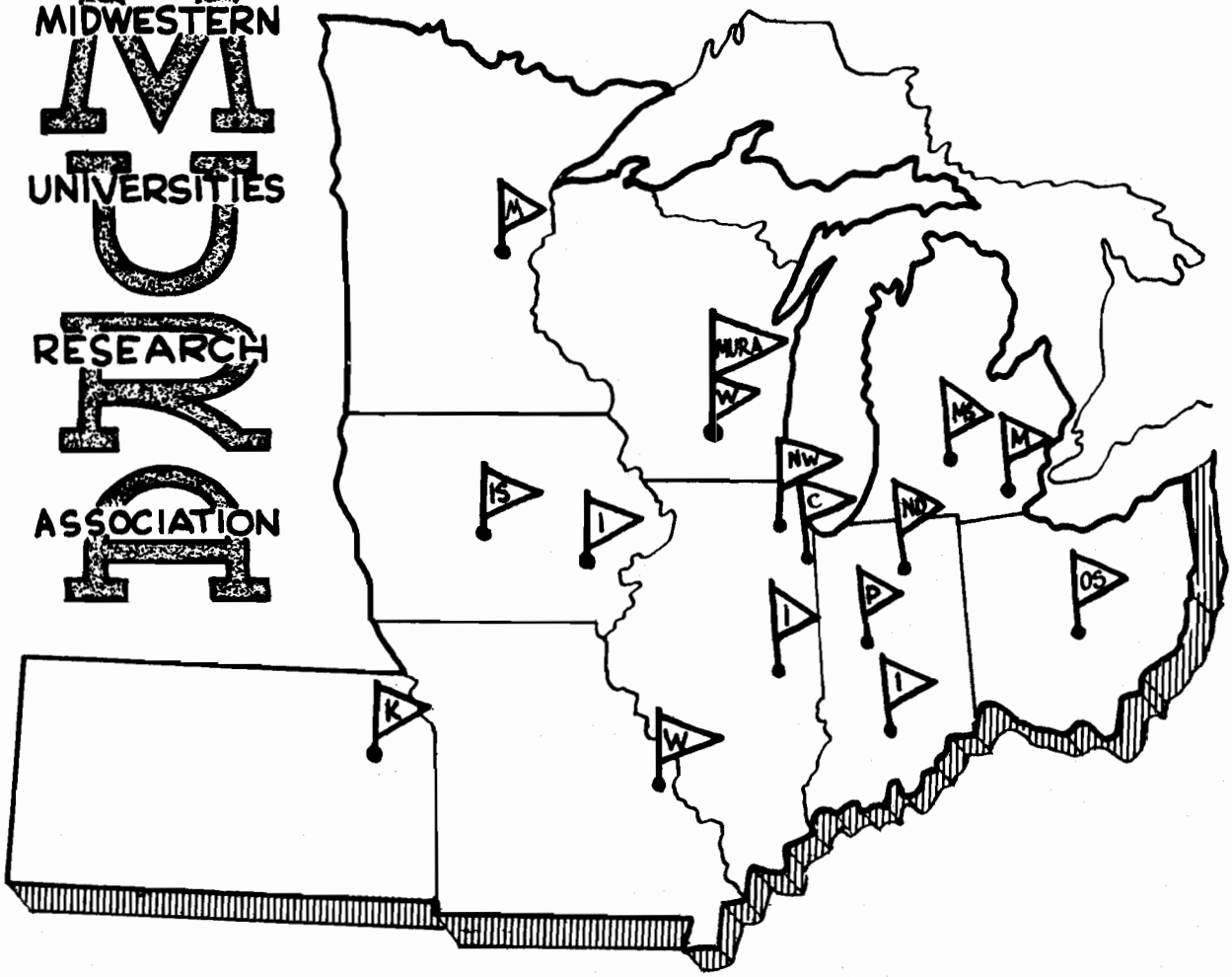

EFFECT OF Δk ERRORS IN ILLINOIS MODEL

L. Jackson Laslett

REPORT NUMBER 257

MURA-257
Internal

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION*

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EFFECT OF Δk ERRORS IN ILLINOIS MODEL

L. Jackson Laslett**

April 19, 1957

1. We have examined with the MURA computer the effect of a Δk error in the Illinois model now being assembled in Madison. The investigation employed the FORMESH-GRUMBLEBUMP programs, since the MUMBLEBUMP program employs the same constants throughout and hence would be unsuitable for the present purpose.

2. The relevant work involves the following parts.

(i) Orientation and stability-limits for design parameters ($k = 0.7$) - - LJL MURA-213 (Internal);

(ii) FORMESH-TEMPERMESH orientation and stability-limits for $k=0.8$ throughout;

(iii) FORMESH-FOROCYL, employing FOROCYL 61 ($k = 0.7$) but with GRUMBLEBUMP always present to effect a change of k to 0.8 (comparison with (ii) constitutes a check of the bump coefficients);

(iv) FORMESH-FOROCYL, with GRUMBLEBUMP introduced one sector per revolution.

It appears from the results of this work that the main effect of the increase of k in one sector is a partial decrease of the radial phase-space available, similar to the larger decrease found when k is increased in <u>all</u> sectors and σ_{x_0} becomes

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* Supported by Contract AEC #AT(11-1)-384

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more nearly $\frac{2\pi}{4}$, but with no notable effects attributable to the decrease of periodicity in the structure.

3. The selection of GRUMBLEBUMP coefficients was based on the following requirements. We denote the original k-value of the mesh by k_o and the value which we intend to represent by k_1. The magnetic fields are described by the reduced field $\vec{h}$, such that the field employed in the computations is $\vec{H} = (1 + x)^{k_o} \vec{h}$. We wish to achieve a field with components representative* of a median-plane field

$$-H_z\Big|_o = (1+x)^{K_1}\left\{1+\Sigma\left[\mu_{o,m}\cos m\left(\frac{\ln(1+x)}{w}-N\theta\right)+\nu_{o,m}\sin m\left(\frac{\ln(1+x)}{w}-N\theta\right)\right]\right\}:$$

$$\begin{aligned}
H_x = & -K_1 y + K_1(1-K_1)xy \\
& -\mu_{o,m} K_1(\cos mN\theta)y - \mu_{o,m}\frac{m}{w}(\sin mN\theta)y \\
& +\mu_{o,m}\left[\frac{m^2}{w^2}-K_1(K_1-1)\right](\cos mN\theta)xy - \mu_{o,m}[2K_1-1]\frac{m}{w}(\sin mN\theta)xy \\
& -\nu_{o,m}\frac{m}{w}(\cos mN\theta)y + \nu_{o,m}K_1(\sin mN\theta)y \\
& -\nu_{o,m}(2K_1-1)\frac{m}{w}(\cos mN\theta)xy + \nu_{o,m}\left[K_1(K_1-1)-\frac{m^2}{w^2}\right](\sin mN\theta)xy \\
& +O(x^2 y),
\end{aligned}$$

$$\begin{aligned}
H_y = & -1 - K_1 x - \frac{K_1(K_1-1)}{2}x^2 + \frac{K_1^2}{2}y^2 - \mu_{o,m}\cos mN\theta \\
& -\mu_{o,m}K_1(\cos mN\theta)x - \mu_{o,m}\frac{m}{w}(\sin mN\theta)x \\
& +\frac{\mu_{o,m}}{2}\left[\frac{m^2}{w^2}-K_1(K_1-1)\right](\cos mN\theta)x^2
\end{aligned}$$

cont.

*cf. LJL (MAC)-4

$$-\frac{\mu_{0,m}}{2}[2K_1-1]\frac{m}{w}(\sin mN\theta)x^2 + \frac{\mu_{0,m}}{2}\left[K_1^2 - m^2N^2 - \frac{m^2}{w^2}\right](\cos mN\theta)y^2$$

$$+\mu_{0,m}K_1\frac{m}{w}(\sin mN\theta)y^2 + \nu_{0,m}\sin mN\theta$$

$$-\nu_{0,m}\frac{m}{w}(\cos mN\theta)x + \nu_{0,m}K_1(\sin mN\theta)x$$

$$-\frac{\nu_{0,m}}{2}(2K_1-1)\frac{m}{w}(\cos mN\theta)x^2 + \frac{\nu_{0,m}}{2}\left[K_1(K_1-1)-\frac{m^2}{w^2}\right](\sin mN\theta)x^2$$

$$+\nu_{0,m}K_1\frac{m}{w}(\cos mN\theta)y^2 - \frac{\nu_{0,m}}{2}\left[K_1^2 - m^2N^2 - \frac{m^2}{w^2}\right](\sin mN\theta)y^2$$

$$+ O(x\cdot y^2),$$

$$H_\theta = \mu_{0,m}\, mN(\sin mN\theta)y$$

$$-\mu_{0,m}\frac{m^2N}{w}(\cos mN\theta)xy + \mu_{0,m}(K_1-1)\,mN(\sin mN\theta)xy$$

$$+\nu_{0,m}\, mN(\cos mN\theta)y$$

$$+\nu_{0,m}(K_1-1)\,mN(\cos mN\theta)xy + \nu_{0,m}\frac{m^2N}{w}(\sin mN\theta)xy + \ldots,$$

Summation over the index m being understood.

The components of the reduced field $\vec{h}$ which we desire to be effective are then selected to be consistent with the foregoing terms in $\vec{H}$ and to contain otherwise-arbitrary higher-order terms to insure $\nabla\cdot\vec{H} = 0$ exactly:

$$
\begin{aligned}
h_x = \; & -K_1 y + K_1\left[1-(K_1-K_0)\right]xy + 2K_1\left[1-\tfrac{1}{2}(K_1-K_0)\right]x^2 y \\
& -\mu_{0,m} K_1 (\cos mN\theta) y \qquad -\mu_{0,m}\frac{m}{w}(\sin mN\theta) y \\
& -\mu_{0,m}\left[K_1(K_1-K_0-1)-\frac{m^2}{w^2}\right](\cos mN\theta)xy \\
& +\mu_{0,m}(K_0-2K_1+1)\frac{m}{w}(\sin mN\theta)\, xy \\
& -\mu_{0,m}\left[\left(K_1-\frac{m^2N^2}{K_0+1}\right)(K_1-K_0-2)-\frac{m^2}{w^2}\right](\cos mN\theta)\, x^2 y \\
& +\mu_{0,m}\left[K_0-2K_1+2+\frac{m^2N^2}{K_0+1}\right]\frac{m}{w}(\sin mN\theta)\, x^2 y \\
& -\nu_{0,m}\frac{m}{w}(\cos mN\theta) y + \nu_{0,m} K_1 (\sin mN\theta) y \\
& +\nu_{0,m}(K_0-2K_1+1)\frac{m}{w}(\cos mN\theta) xy + \nu_{0,m}\left[K_1(K_1-K_0-1)-\frac{m^2}{w^2}\right](\sin mN\theta) xy \\
& +\nu_{0,m}\left[K_0-2K_1+2+\frac{m^2N^2}{K_0+1}\right]\frac{m}{w}(\cos mN\theta)\, x^2 y \\
& +\nu_{0,m}\left[\left(K_1-\frac{m^2N^2}{K_0+1}\right)(K_1-K_0-2)-\frac{m^2}{w^2}\right](\sin mN\theta)\, x^2 y,
\end{aligned}
$$

$$
\begin{aligned}
h_y = \; & -1-(K_1-K_0)x - \frac{(K_1-K_0)[K_1-K_0-1]}{2}x^2 + \frac{K_1^2}{2}y^2 \\
& -K_1(3+K_0)\left[1-\tfrac{1}{2}(K_1-K_0)\right]xy^2 \quad -\mu_{0,m}\cos mN\theta \\
& -\mu_{0,m}(K_1-K_0)(\cos mN\theta)x - \mu_{0,m}\frac{m}{w}(\sin mN\theta)x \\
& +\frac{\mu_{0,m}}{2}\left\{(K_1-K_0)\left[1-(K_1-K_0)\right]+\frac{m^2}{w^2}\right\}(\cos mN\theta)\, x^2
\end{aligned}
$$

cont.

$$-\frac{\mu_{0,m}}{2}(K_1-K_0-1)\frac{m}{w}(\sin mN\theta)x^2+\frac{\mu_{0,m}}{2}\left[K_1^2-m^2N^2-\frac{m^2}{w^2}\right](\cos mN\theta)y^2$$

$$+\mu_{0,m}K_1\frac{m}{w}(\sin mN\theta)y^2+\mu_{0,m}\frac{K_0+3}{2}\left[\left(K_1-\frac{m^2N^2}{K_0+1}\right)(K_1-K_0-2)-\frac{m^2}{w^2}\right](\cos mN\theta)xy^2$$

$$-\mu_{0,m}\frac{K_0+3}{2}\left[K_0-2K_1+2+\frac{m^2N^2}{K_0+1}\right]\frac{m}{w}(\sin mN\theta)xy^2$$

$$+V_{0,m}\sin mN\theta-V_{0,m}\frac{m}{w}(\cos mN\theta)x+V_{0,m}(K_1-K_0)(\sin mN\theta)x$$

$$-\frac{V_{0,m}}{2}(K_1-K_0-1)\frac{m}{2}(\cos mN\theta)x^2$$

$$-\frac{V_{0,m}}{2}\left\{(K_1-K_0)[1-(K_1-K_0)]+\frac{m^2}{w^2}\right\}(\sin mN\theta)x^2+V_{0,m}K_1\frac{m}{w}(\cos mN\theta)y^2$$

$$-\frac{V_{0,m}}{2}\left[K_1^2-m^2N^2-\frac{m^2}{w^2}\right](\sin mN\theta)y^2$$

$$-V_{0,m}\frac{K_0+3}{2}\left[K_0-2K_1+2+\frac{m^2N^2}{K_0+1}\right]\frac{m}{w}(\cos mN\theta)xy^2$$

$$-V_{0,m}\frac{K_0+3}{2}\left[\left(K_1-\frac{m^2N^2}{K_0+1}\right)(K_1-K_0-2)-\frac{m^2}{w^2}\right](\sin mN\theta)xy^2,$$

$$h_\theta=\mu_{0,m}mN(\sin mN\theta)y-\mu_{0,m}\frac{m^2N}{w}(\cos mN\theta)xy$$

$$+\mu_{0,m}(K_1-K_0-1)mN(\sin mN\theta)xy+V_{0,m}mN(\cos mN\theta)y$$

$$+V_{0,m}(K_1-K_0-1)mN(\cos mN\theta)xy+V_{0,m}\frac{m^2N}{w}(\sin mN\theta)xy.$$

The foregoing expressions should be found to satisfy the curl condition (θ component)

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0 \quad \text{or} \quad (1+x)\left[\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right] + K_0 h_y = 0$$

through terms of first order and the divergence condition

$$(1+x)\left[\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y}\right] + H_x + \frac{\partial H_\theta}{\partial \theta} = 0$$

$$\text{or} \quad (1+x)\left[\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y}\right] + (K_0+1)h_x + \frac{\partial h_\theta}{\partial \theta} = 0$$

exactly.

To remove with the GRUMBLEBUMP program the r^{K_0} field and replace it with a similar r^{K_1} field we therefore **add** the following reduced-field components:

$$\begin{aligned}
\Delta h_x = {} & -(K_1-K_0)y + (K_1-K_0)(1-K_1)xy + (K_1-K_0)(2-K_1)x^2y \\
& -\mu_{0,m}(K_1-K_0)(\cos mN\theta)y + \mu_{0,m}(K_1-K_0)(1-K_1)(\cos mN\theta)xy \\
& -2\mu_{0,m}(K_1-K_0)\frac{m}{w}(\sin mN\theta)xy + \mu_{0,m}(K_1-K_0)\left(2-K_1+\frac{m^2N^2}{K_0+1}\right)(\cos mN\theta)x^2y \\
& -2\mu_{0,m}(K_1-K_0)\frac{m}{w}(\sin mN\theta)x^2y + \nu_{0,m}(K_1-K_0)(\sin mN\theta)y \\
& -2\nu_{0,m}(K_1-K_0)\frac{m}{w}(\cos mN\theta)xy - \nu_{0,m}(K_1-K_0)(1-K_1)(\sin mN\theta)xy \\
& -2\nu_{0,m}(K_1-K_0)\frac{m}{w}(\cos mN\theta)x^2y - \nu_{0,m}(K_1-K_0)\left(2-K_1+\frac{m^2N^2}{K_0+1}\right)(\sin mN\theta)x^2y ,
\end{aligned}$$

$$\begin{aligned}
\Delta h_y = {} & -(K_1-K_0)x + \frac{(K_1-K_0)[1-(K_1-K_0)]}{2}x^2 + \frac{(K_1-K_0)(K_1+K_0)}{2}y^2 \\
& -(K_1-K_0)\frac{3+K_0}{2}(2-K_1)xy^2 - \mu_{0,m}(K_1-K_0)(\cos mN\theta)x \\
& +\frac{\mu_{0,m}}{2}(K_1-K_0)[1-(K_1-K_0)](\cos mN\theta)x^2 - \mu_{0,m}(K_1-K_0)\frac{m}{w}(\sin mN\theta)x^2
\end{aligned}$$

cont.

$$+ \frac{\mu_{0,m}}{2}(K_1-K_0)(K_1+K_0)(\cos mN\theta)y^2 + \mu_{0,m}(K_1-K_0)\frac{m}{w}(\sin mN\theta)y^2$$

$$-\mu_{0,m}(K_1-K_0)\frac{K_0+3}{2}\left[2-K_1+\frac{m^2N^2}{K_0+1}\right](\cos mN\theta)xy^2$$

$$+\mu_{0,m}(K_1-K_0)(K_0+3)\frac{m}{w}(\sin mN\theta)xy^2 + \nu_{0,m}(K_1-K_0)(\sin mN\theta)x$$

$$-\nu_{0,m}(K_1-K_0)\frac{m}{w}(\cos mN\theta)x^2 - \frac{\nu_{0,m}}{2}(K_1-K_0)[1-(K_1-K_0)](\sin mN\theta)x^2$$

$$+\nu_{0,m}(K_1-K_0)\frac{m}{w}(\cos mN\theta)y^2 - \frac{\nu_{0,m}}{2}(K_1-K_0)(K_1+K_0)(\sin mN\theta)y^2$$

$$+\nu_{0,m}(K_1-K_0)(K_0+3)\frac{m}{w}(\cos mN\theta)xy^2$$

$$+\nu_{0,m}(K_1-K_0)\frac{K_0+3}{2}\left[2-K_1+\frac{m^2N^2}{K_0+1}\right](\sin mN\theta)xy^2,$$

$$\Delta h_0 = \mu_{0,m}(K_1-K_0)mN(\sin mN\theta)xy + \nu_{0,m}(K_1-K_0)mN(\cos mN\theta)xy.$$

4. In the present application, we represent the change in k for the non-AG portion of the field, for the fundamental AG component, and for the second harmonic:

$K_1 = 0.8$ $\quad K_0 = 0.7$ $\quad \frac{1}{w} = 6.25$ $\quad N = 6$

$m_1 = 1$ $\quad m_2 = 2$

$\mu_{01} = -.13119843$ $\quad \mu_{02} = -.08750375$

$\nu_{01} = +1.0687883$ $\quad \nu_{02} = -.02577238$.

We find approximately

$$
\begin{aligned}
\Delta h_x = & -.1\,y + .02\,xy + .12\,x^2 y \\
& +.013120\,(\cos N\theta)\,y + .106879\,(\sin N\theta)\,y \\
& -1.338609\,(\cos N\theta)\,xy + .142622\,(\sin N\theta)\,xy \\
& -1.629561\,(\cos N\theta)\,x^2 y - 2.227573\,(\sin N\theta)\,x^2 y \\
& +.008750\,(\cos 2N\theta)\,y - .002577\,(\sin 2N\theta)\,y \\
& +.062681\,(\cos 2N\theta)\,xy + .219275\,(\sin 2N\theta)\,xy \\
& -.687278\,(\cos 2N\theta)\,x^2 y + .440159\,(\sin 2N\theta)\,x^2 y \,,
\end{aligned}
$$

$$
\begin{aligned}
\Delta h_y = & -.1\,x + .045\,x^2 + .075\,y^2 - .222\,xy^2 \\
& +.013120\,(\cos N\theta)\,x + .106879\,(\sin N\theta)\,x \\
& -.673897\,(\cos N\theta)\,x^2 + .033904\,(\sin N\theta)\,x^2 \\
& +.658153\,(\cos N\theta)\,y^2 - .162158\,(\sin N\theta)\,y^2 \\
& +3.014688\,(\cos N\theta)\,xy^2 + 4.121010\,(\sin N\theta)\,xy^2 \\
& +.008750\,(\cos 2N\theta)\,x - .002577\,(\sin 2N\theta)\,x \\
& +.028278\,(\cos 2N\theta)\,x^2 + .110539\,(\sin 2N\theta)\,x^2 \\
& -.038778\,(\cos 2N\theta)\,y^2 - .107447\,(\sin 2N\theta)\,y^2 \\
& +1.271464\,(\cos 2N\theta)\,xy^2 - .814295\,(\sin 2N\theta)\,xy^2 \,,
\end{aligned}
$$

$$
\begin{aligned}
\Delta h_\theta = & .641273\,(\cos N\theta)\,xy - .078719\,(\sin N\theta)\,xy \\
& -.030927\,(\cos 2N\theta)\,xy - .105004_5\,(\sin 2N\theta)\,xy \,.
\end{aligned}
$$

The foregoing coefficients, divided by 64, are entered on the GRUMBLEBUMP agenda, with

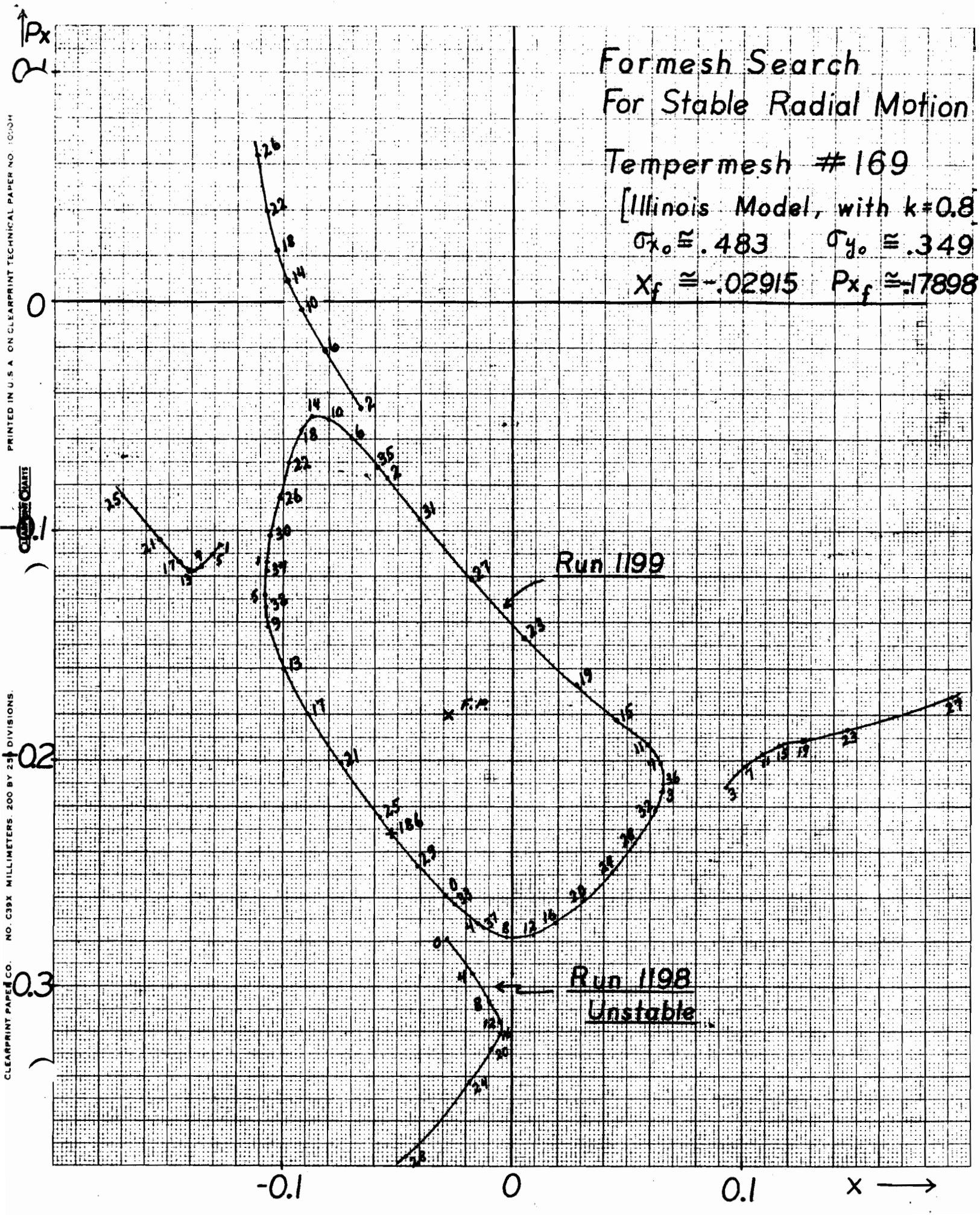
$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_8 = \lambda_9 = 1 \,.$$

When it is desired that the bump represent the perturbation of one sector only, the bump is introduced and removed at values of θ at which the field is rather low, employing

$N_B = 192$, $N_{IN} = 72$, and $N_{OUT} = 104$.

5. Since the results of computations pertaining to the unperturbed model have already been reported in LJL-MURA-213 (Internal), we present here only the recent results graphically. The accompanying table summarizes the relevant data.

)	k = 0.7	k=0.7 for 5 sect. k=0.8 for 1 sect.	k = 0.8)	
	Formesh-Forocyl Forocyl 61	Forocyl 61 with Grumblebump one sect/rev.	Formesh-Tempermesh 169	Forocyl 61 with Grumblebumps always present
$\sigma x_0/\pi$	.466	.470	.483	.480
$\sigma y_0/\pi$	.374	.370	.349	.358
x_f	-.02915	-.0297	-.0296	-.0292
p_{x_f}	-.17898	-.1784	-.1786	-.1790
AREA (x, p_x units)				
Last Stable Curve		.043	.020	.020
Est. Limiting "	.061	.052	.028	.028
y_{0max}				
Unstable	.038 after 8 sect	.038 after 124 sect	.040 after 7 sects	.040 after 7 sects
Unstable	.036 " 56 "	.036 " 32 "	.038 " 40 "	.038 " 15 "
Stable	.034 thru 100 sect	.034 thru 192 sect	.036 thru 192 sect	.036 thru 192 sect


Formesh Search
For Stable Radial Motion
Tempermesh #169
[Illinois Model, with k=0.8]
$\sigma_{x_0} \cong .483$ $\sigma_{y_0} \cong .349$
$X_f \cong -.02915$ $P_{x_f} \cong -.17898$
Run 1199
Run 1198
Unstable
F.P.
Px
x
0
-0.1
0.1
0.2
0.3
PRINTED IN U.S.A. ON CLEARPRINT TECHNICAL PAPER NO. 1000H
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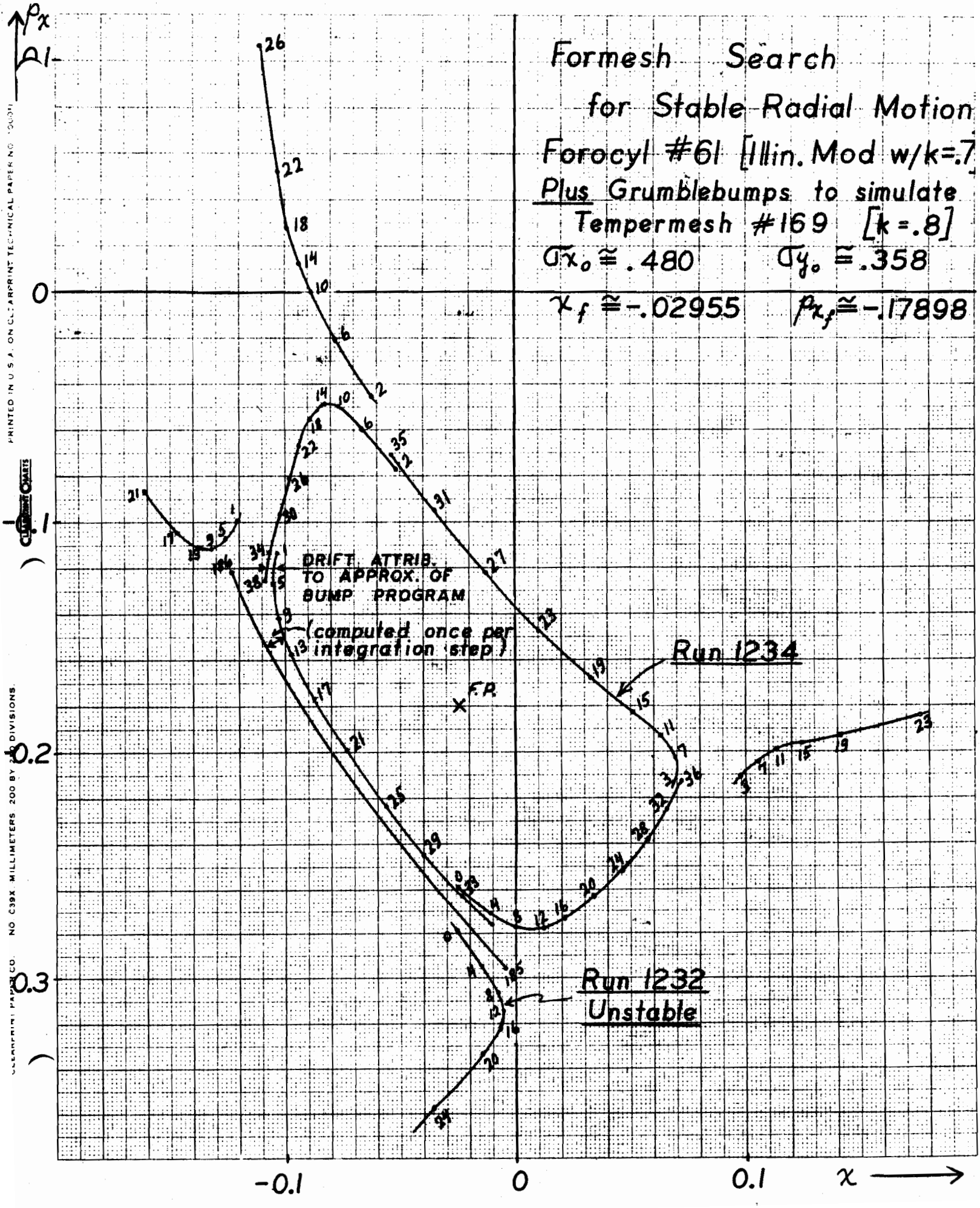
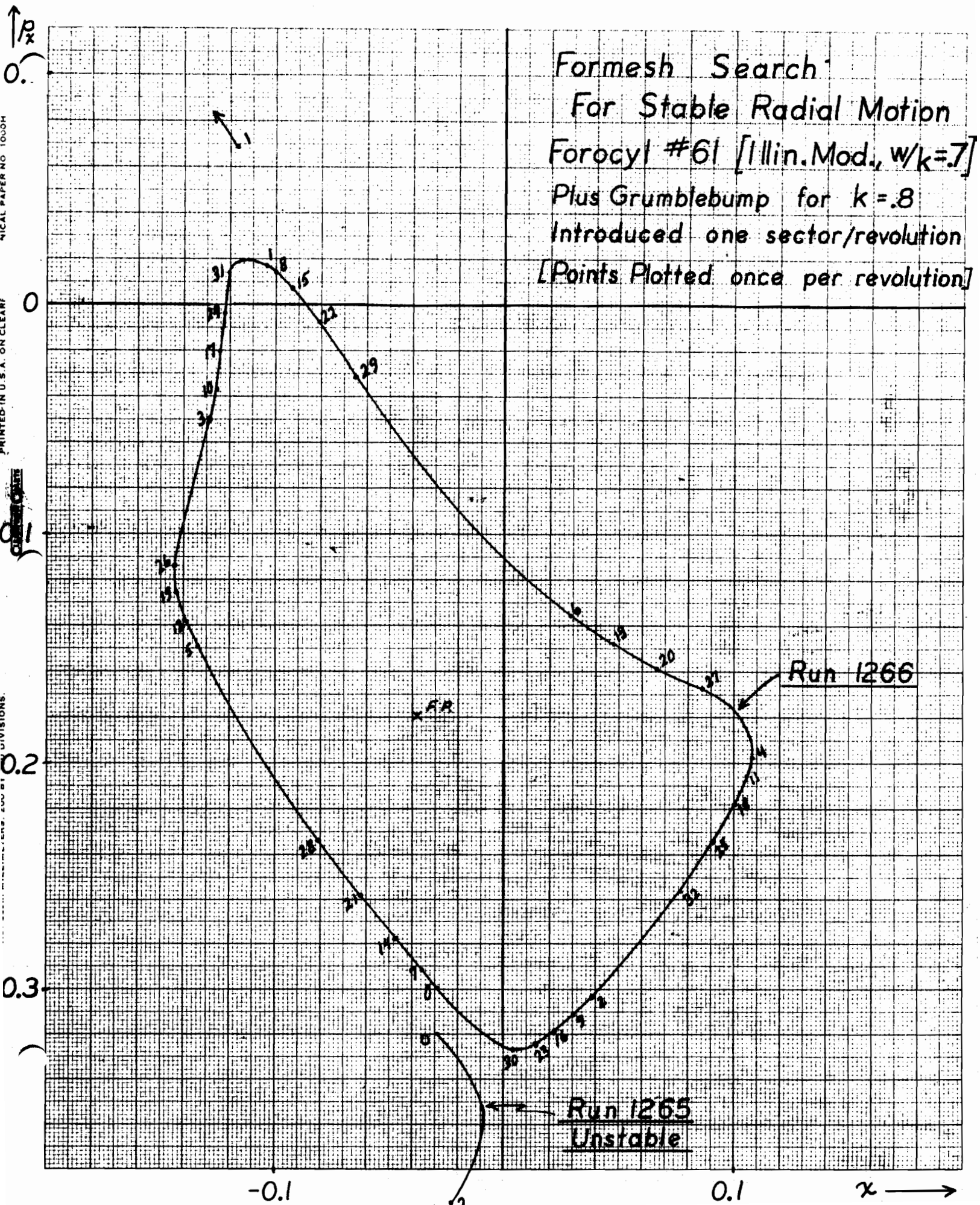

Formesh Search
for Stable Radial Motion
Forocyl #61 [Illin. Mod w/k=.7]
Plus Grumblebumps to simulate
Tempermesh #169 [k=.8]
$\sigma x_0 \cong .480$ $\sigma y_0 \cong .358$
$x_f \cong -.02955$ $p_{x_f} \cong -.17898$
DRIFT ATTRIB. TO APPROX. OF BUMP PROGRAM
(computed once per integration step)
F.P.
Run 1234
Run 1232
Unstable
p_x
x
0.1
0
-0.1
-0.2
-0.3
-0.1
0
0.1


Formesh Search
For Stable Radial Motion
Forocyl #61 [11lin. Mod., w/k=.7]
Plus Grumblebump for k=.8
Introduced one sector/revolution
[Points Plotted once per revolution]
Run 1266
Run 1265
Unstable
F.R.
p_x
x
0.
0
0.1
0.2
0.3
-0.1
0.1