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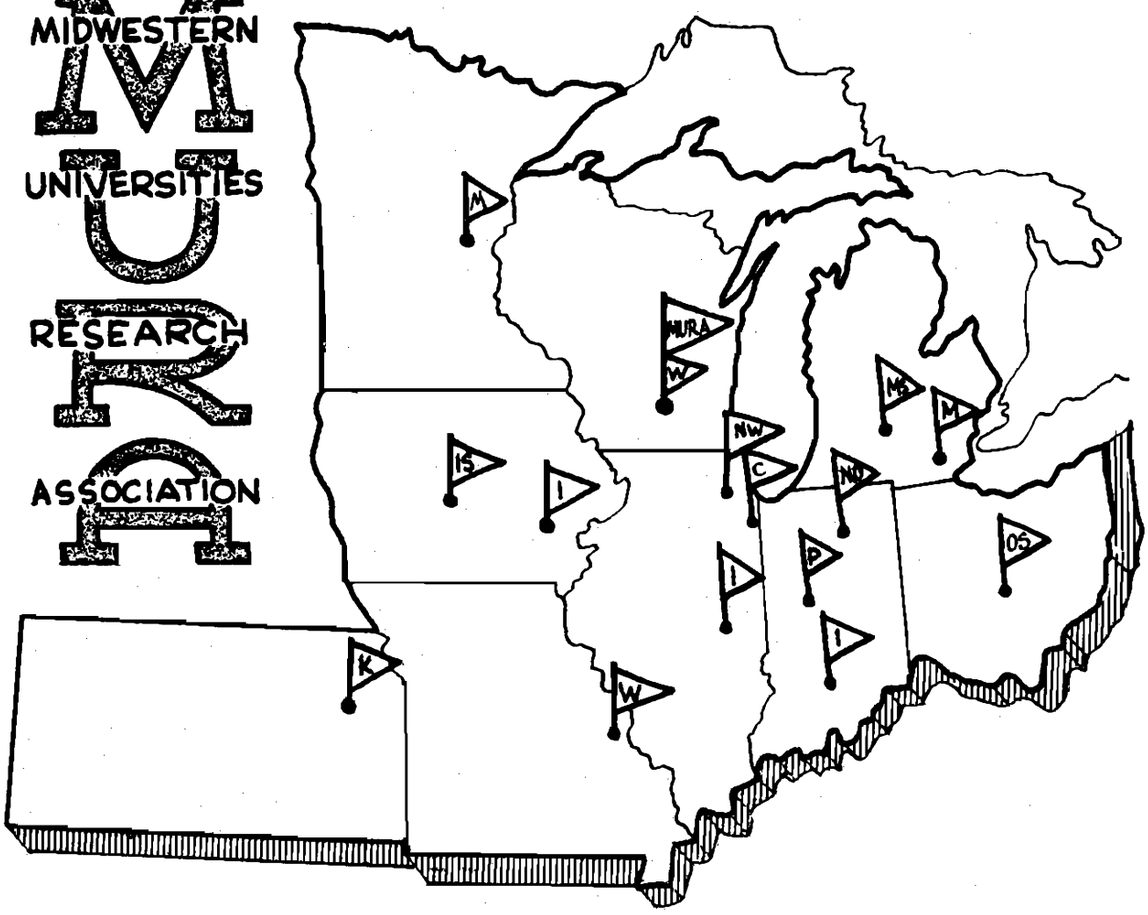

RADIAL SECTOR ACCELERATORS WITH SMALLER CIRCUMFERENCE FACTOR

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REPORT **NUMBER** 256

APRIL, 1957

MURA-256
Internal

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ABSTRACT: By considering geometries intermediate between the usual radial sector design and accelerators designed to operate on a "patch" of the stability diagram (Mark III) it is shown that radial sector accelerators with circumference factors of all values down to two are possible, although the width of the z stability area is reduced with smaller circumference factors. Other possible ramifications of the introduction of sectors containing more than one lens pair are discussed.

* Supported by Contract AEC #AT(11-1)-384

** On leave from the University of Michigan

I. INTRODUCTION

Ohkawa has suggested that the circumference factor of the colliding beams radial sector accelerator[1] might be reduced by introducing "superperiodicities"; that is, by choosing a magnet structure which repeats less frequently than the magnet pairs. This idea bears some resemblance to the Mark III concept[2] in which the phase change, σ, of one mode of oscillation was made greater than π for one period of the structure. The original Mark III is not symmetric with respect to particles going both directions, and in addition it has very undesirable properties for one mode of oscillation. (Large amplitudes for a given angle of crossing the equilibrium orbit and very closely spaced resonances) In the present note a class of geometries is investigated which are intermediate between Mark Ib and Mark III. While this example does not have the symmetry required for the colliding beams machine, it is simple and instructive. The methods of calculation used are simple matrix multiplication assuming no straight sections nor edge effects, and considering fields along an equilibrium orbit as plus or minus a single value.

II. METHOD OF CALCULATION

Consider a machine made up of sectors where each sector consists of four magnets; D, C, B, A. In magnets D and B the particles are bent toward the machine center and (with positive k) are radially focusing. Magnets C and A have fields in the opposite direction (radially defocusing). Characterize each magnet by the parameter ψ where $\psi = \sqrt{n}\,\frac{s}{\rho}$, $n = \left| \frac{\rho}{B} \frac{dB}{dx} \right|$,

[1] MURA-TO-6

[2] MURA-LWJ-6

and s is the length of the magnet. Only cases where

$$\rho_A = \rho_B = \rho_C = \rho_D \;,\; m_A = m_B = m_C = m_D \text{ and } s_D = s_B$$

will be considered for simplicity. Therefore $\psi_D \equiv \psi_B$. Neglecting all edges and straight sections and considering $n \gg 1$, $\cos\sigma$ is found on multiplying the appropriate transfer matrices;

$$\cos\sigma_x = \cos 2\psi_B \cosh\psi_A \cosh\psi_C + \sinh\psi_A \sinh\psi_C$$

$$\cos\sigma_z = \cosh 2\psi_B \cos\psi_A \cos\psi_C - \sin\psi_A \sin\psi_C .$$

Three cases were considered; 1) $\psi_A = \psi_C$, 2) $\psi_A = 1/2\,\psi_C$, and 3) $\psi_A = 0$

The stability boundaries for these three cases are presented in figures 1, 2, and 3.

III. RESULTS

Case 1) $\psi_A = \psi_C$ This is the conventional Mark Ib except that σ is defined for a period of the structure containing two identical magnet pairs, so that the stability boundary corresponds to $\sigma = 2\pi$.

Case 3) $\psi_A = 0$ The second shaded region corresponds to the conventional Mark III with a sector defined between the centers of focusing magnets. For the center of this region $\psi_B = \frac{3}{2}\psi_C$ and $s_B = \frac{3}{2} s_C$.

Case 2) $\psi_A = 1/2\,\psi_C$ This is seen to be in every respect intermediate between cases 1) and 3). The $\sigma_x = \pi$ line of case 1) has broadened into a band of x instability, although this band is considerably narrower than the corresponding region of case 3). The circumference factor of these machines is given by

$$C = \frac{2s_B + s_A + s_C}{2s_B - s_A - s_C} = \frac{2\psi_B + \psi_A + \psi_C}{2\psi_B - \psi_A - \psi_C}$$

For case 1) a point in the corner of the stability region has $C \cong 5$, for case 3) a point in the center of the second stability region has $C = 2$. For the point $\psi_B = 2.0$, $\psi_C = 1.5$ of case 2), $C = 3.57$.

The width of the region of z-stability in units of ψ_c for three cases for given values of ψ_B is:

Case 1) $\Delta \psi_c \cong 0.5$ at $\psi_B \cong 1.8$

Case 2) $\Delta \psi_c \cong 0.075$ at $\psi_B \cong 2.0$

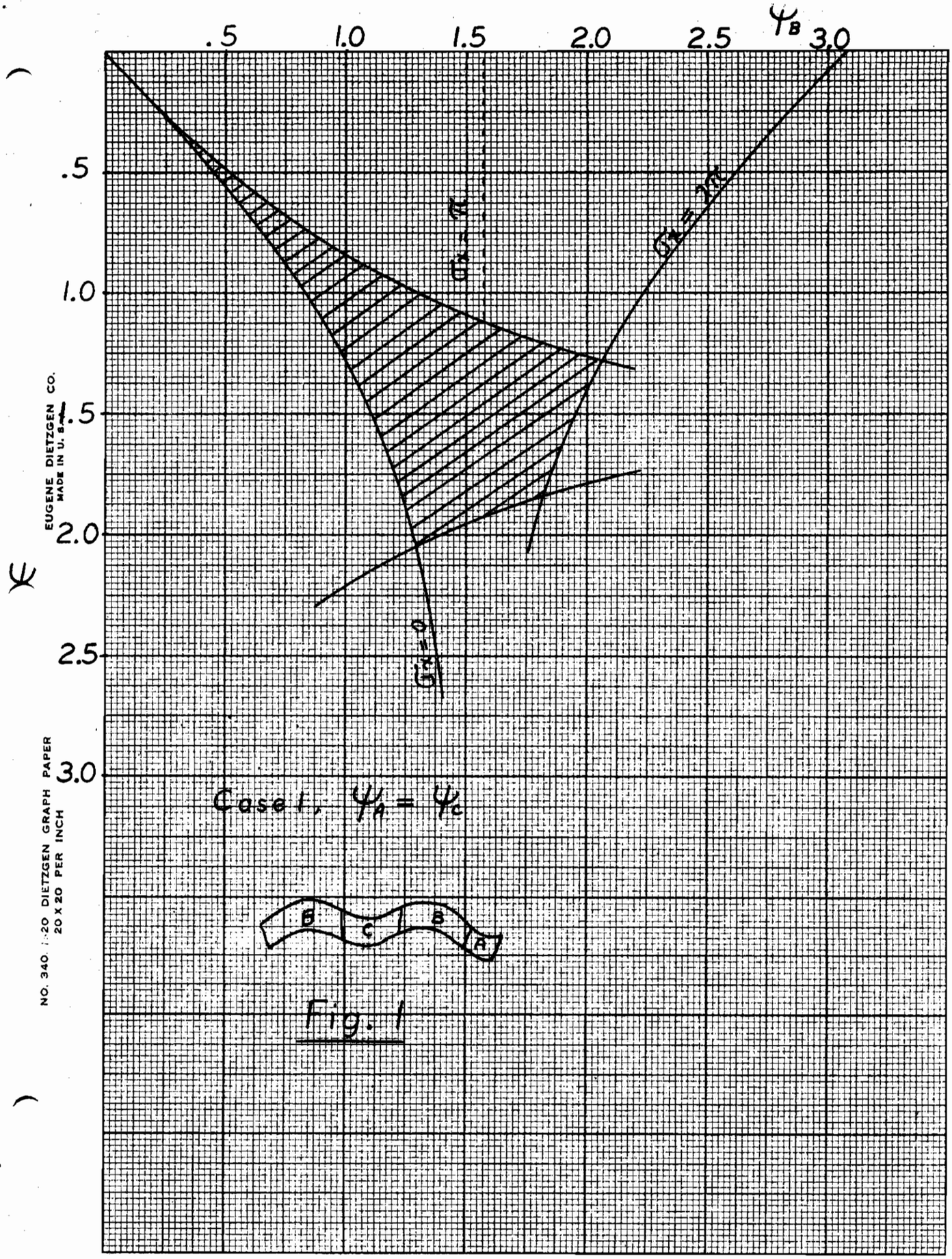
Case 3) $\Delta \psi_c \cong 0.036$ at $\psi_B \cong 2.35$

The amplitudes of z oscillation for given angles of crossing the equilibrium orbit have not been studied.

IV. DISCUSSION

Characterizing machines by the parameter ψ (n, s, p) is convenient for calculations using transfer matrices, however, the more conventional parameters k and N are more difficult to infer. Here k = n C, so that along any diagonal through the origin of figs. 1-3 (constant C lines) k is proportional to ψ^2. The circumference factor increases as the angle between the ψ_B axis and a diagonal increases (e.g. approaching infinity in case 1) when $\psi_B = \psi_c$). The number of sectors, N, depends on the particular point on the $\psi_B \cdot \psi_c$ plane as well as the chosen value of n.

It is the purpose of this exercise to emphasize that by introducing "superperiodicities" (sectors containing more than one lens pair) one may arrive at new combinations of machine characteristics. Unfortunately, in the cases studied, the width of the z stability region on the ψ plane is reduced by reducing the circumference factor. There is hope that the symmetrical radial sector accelerator of Ohkawa's might be designed with a smaller circumference factor than 5 by the use of such variations.



.5
1.0
1.5
2.0
2.5
3.0
ψ_B
ψ
$\sigma_x = \pi$
$\sigma_x = 2\pi$
$\sigma_x = 0$
Case I, $\psi_A = \psi_C$
B
C
B
A
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Fig. 1

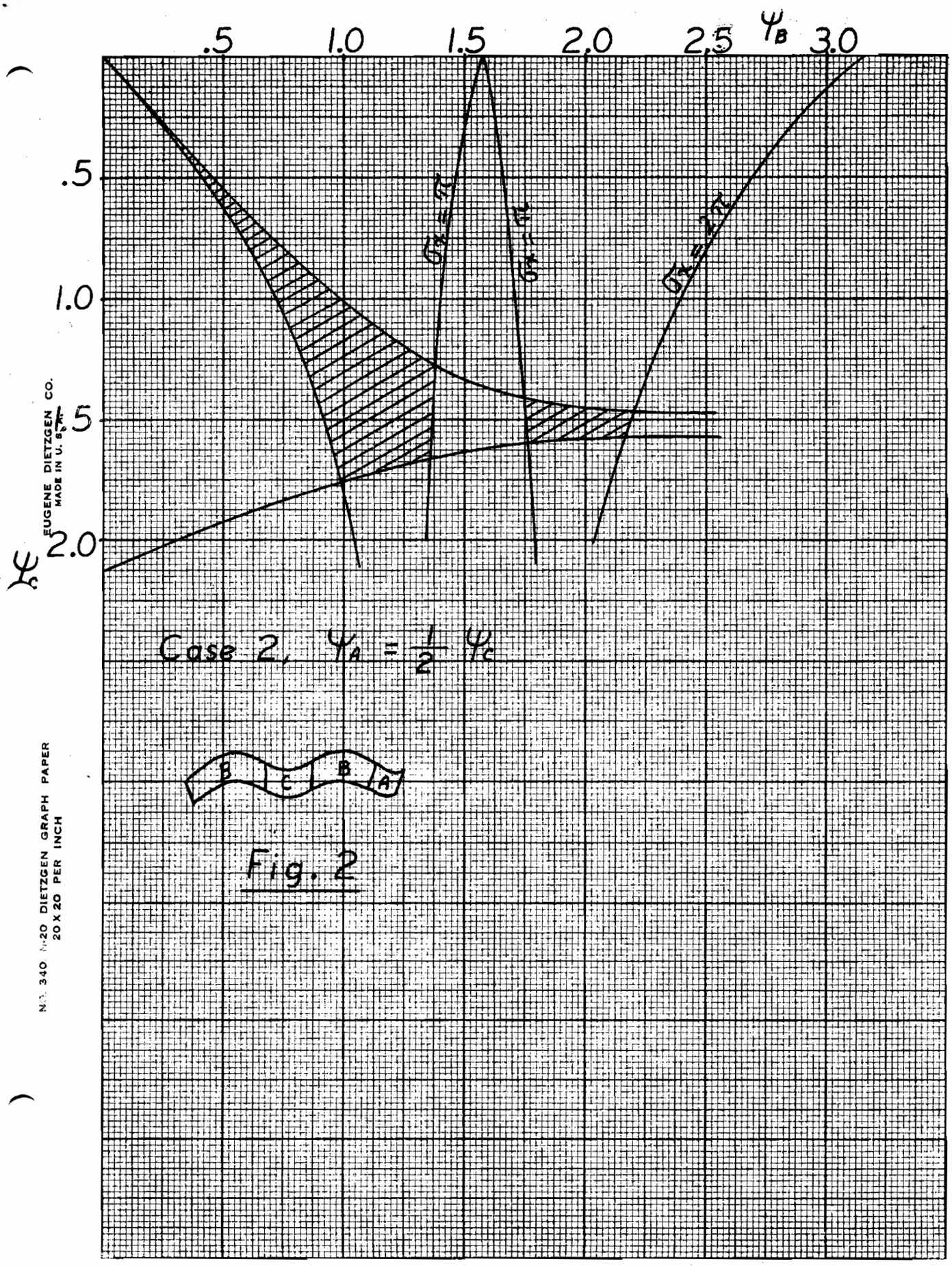

Fig. 2

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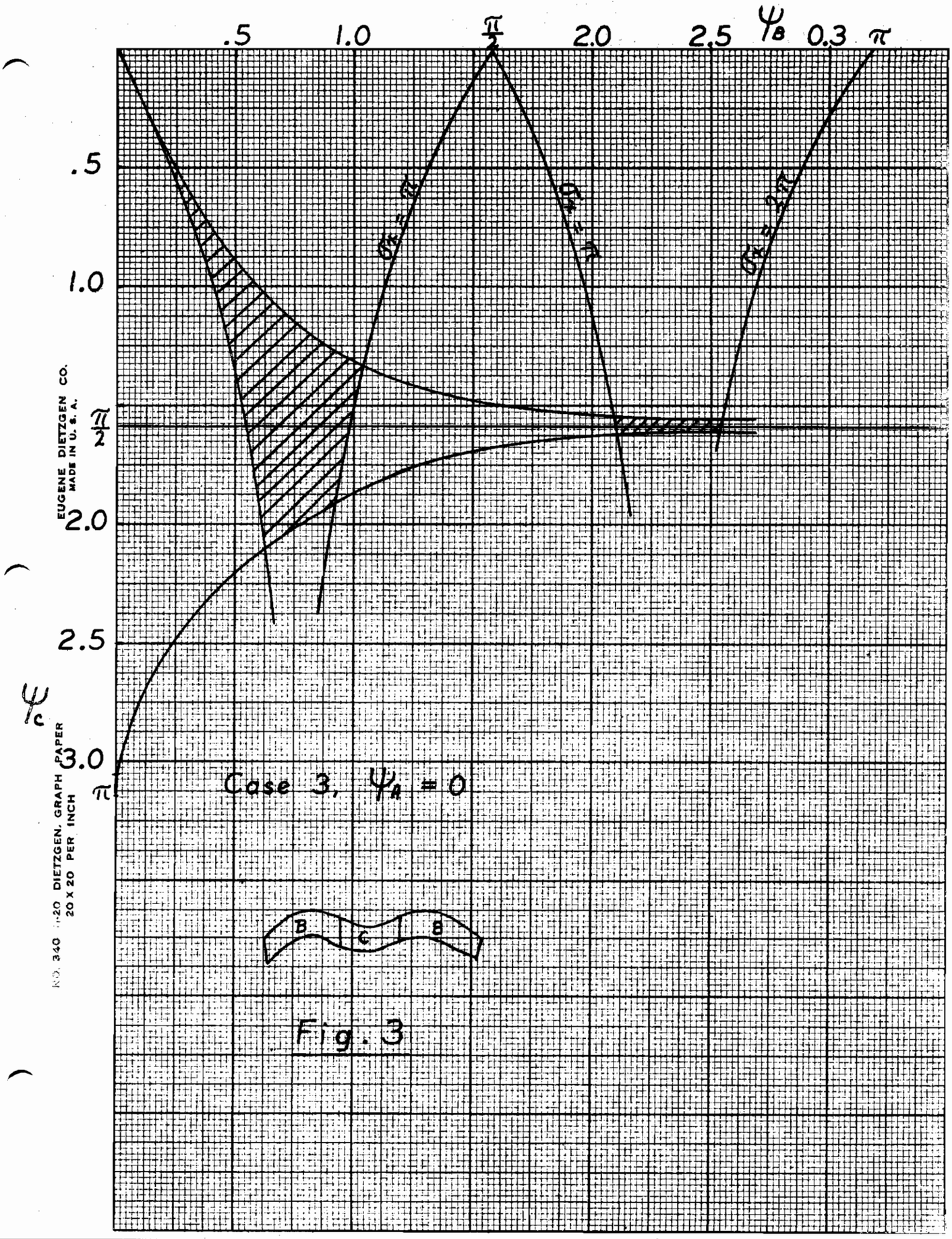
.5
1.0
π/2
2.0
2.5
ψB
0.3
π
.5
1.0
π/2
2.0
2.5
3.0
π
ψc
Case 3, ψA = 0
B
C
B
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Fig. 3