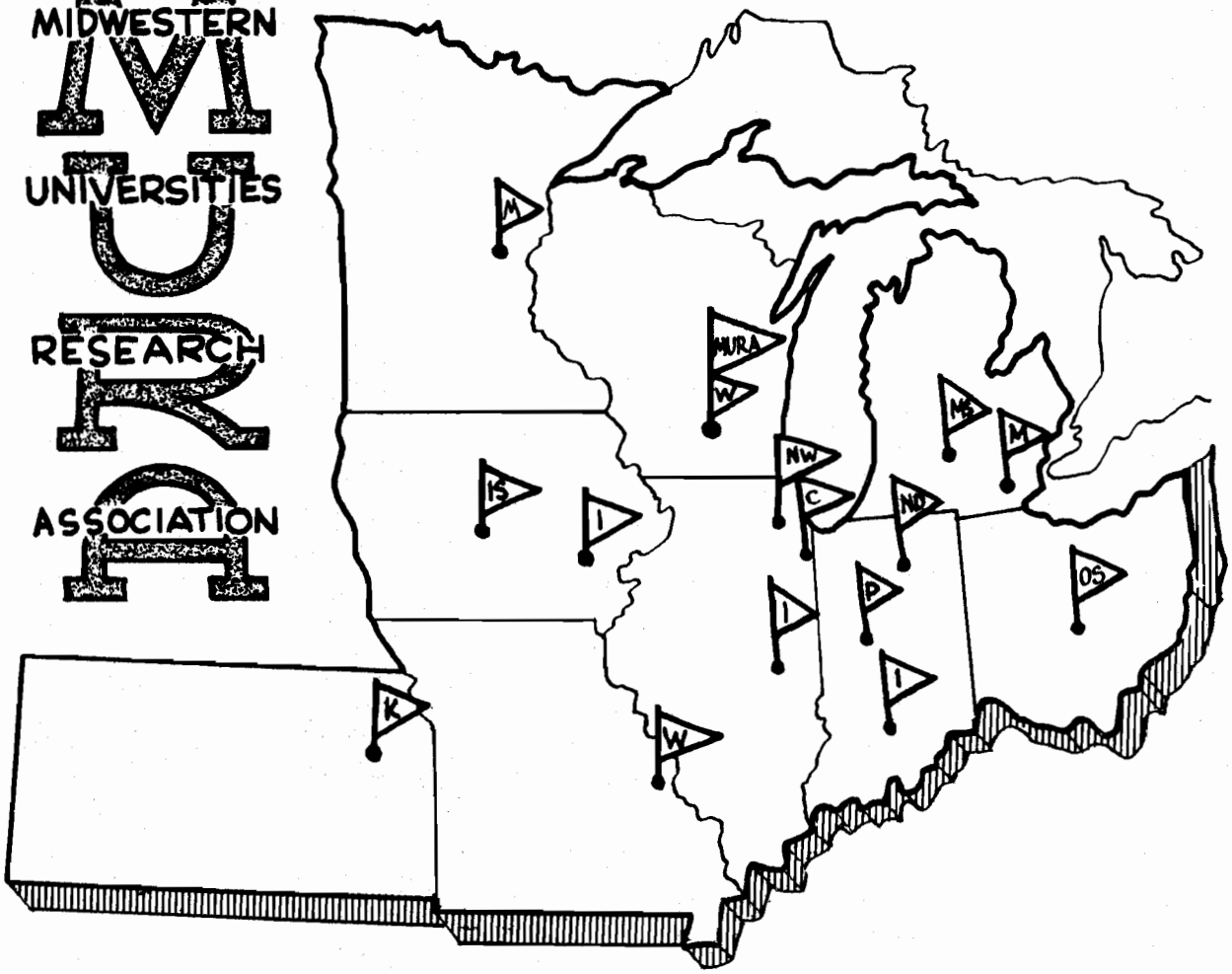
Fermilab Library
0 1160 0038634 6

250

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION



THE 2 $\nu_y - \nu_x = 0$ DIFFERENCE RESONANCE

REPORT **NUMBER** 250

MURA-250
Internal

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION*

2203 University Avenue, Madison, Wisconsin

THE $2\nu_y - \nu_x = 0$ DIFFERENCE RESONANCE

G. Parzen +

April 1, 1957

ABSTRACT: Some results for the $2\nu_y - \nu_x = 0$ coupling resonance are presented in a rough form. No derivations are given. Threshold values of μ and η for growth are given. Results are also given for the turnover point when growth occurs and for the true stability limits.

* Supported by Contract AEC No. AT(11-1)-384.

+ On leave from the University of Notre Dame

I. Introduction

This is a short note to present the results which have been found for the $2\nu_y - \nu_x = 0$ difference resonance. No derivations will be given and the results will be given in rough form only. In general this means that certain AG effects have been neglected in the rough form of the result. These AG effects can cause the results to be in error by 30% to 50% in some cases. In most cases, the neglected AG effects can be included without too much trouble.

The equations we have investigated have the form,

$$\left\{\frac{d^2}{d\theta^2} + E_0 - g_1(\theta)\right\} u = B(\theta) u^2 + C(\theta) y^2, \tag{1. 1a}$$

$$\left\{\frac{d^2}{d\theta^2} + E_0' - g_2(\theta)\right\} y = 2C(\theta) u y. \tag{1. 1b}$$

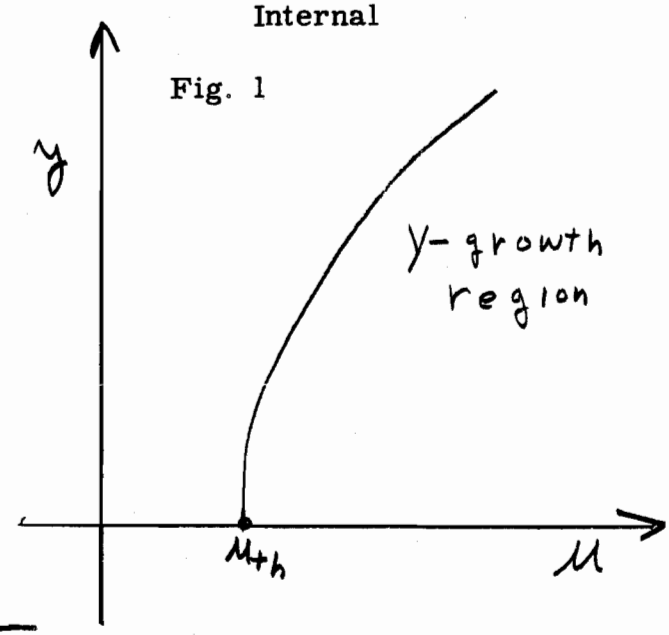
The notation used in this note is more fully defined in the report MURA-217.

II. Y-Growth

The difference resonance $2\nu_y - \nu_x = 0$ does not seem to cause instability which was already pointed out by Hagedorn and by Moser. The resonance does, however, cause y-growth and also x-growth.

Thus the difference resonance does not by itself give rise to any stability limits, but it does give rise to threshold values of u and y at which growth sets in. The threshold values of u and y for y-growth are shown in Fig. 1.

Fig. 1



The curve shown has the significance that if one starts the motion at some point (μ, y) to the right of the curve then y-growth will occur. If the initial μ' and y' are zero, then the curve in Fig. 1 is given by the equation

$$y = 2.8 \sqrt{\mu(\mu - \mu_{th})}, \tag{2.1}$$

where

$$\mu_{th} = \frac{\nu_y}{\bar{C}} \left| 2\nu_y - \nu_x \right|, \tag{2.2}$$

and

$$\bar{C} = \frac{N}{2\pi} \int_0^{2\pi/N} d\theta \; \mu^*(\theta)\, C(\theta)\, y^2(\theta)\, e^{i(\nu_x - 2\nu_y)\theta}, \tag{2.3}$$

and $\mu(\theta)$ and $y(\theta)$ are the Flocquet solutions which give the linear motion. $\mu(\theta)$ and $y(\theta)$ are so normalized that they have the form $e^{i\nu\theta} f(\theta)$ where $f(\theta)$ is periodic and $2\pi/N \int_0^{2\pi/N} |f(\theta)|^2 d\theta = 1$.

The quantity μ_{th} is the threshold value of μ when the motion is started with $y = 0$. The eq. (2.2) for μ_{th} is essentially the same result obtained previously by Laslett and Sessler.

A question that is of interest is how far will y-grow if one starts the motion with a value of μ beyond the threshold. An empirical answer has been obtained to this question. If one starts at the initial value of $\mu = \mu_0$ with $\mu' = y' = 0$ then y will grow to the value

$$y = 2.8 \sqrt{\mu_0(\mu_0 - \mu_{th})}, \tag{2.4}$$

Although the result eq. (2.4) was only obtained empirically, it agrees very well with computer results and it is possible it is the correct answer.

III. u-growth

The threshold values of u and y for u-growth are shown in Fig. 2. The curve shown has the significance that if the motion is started at some point (u, y) to the left of the curve, will grow rapidly. The u-growth is not as marked as the y-growth. If the initial u' and y' are zero, then the curve in Fig. 2. is given by

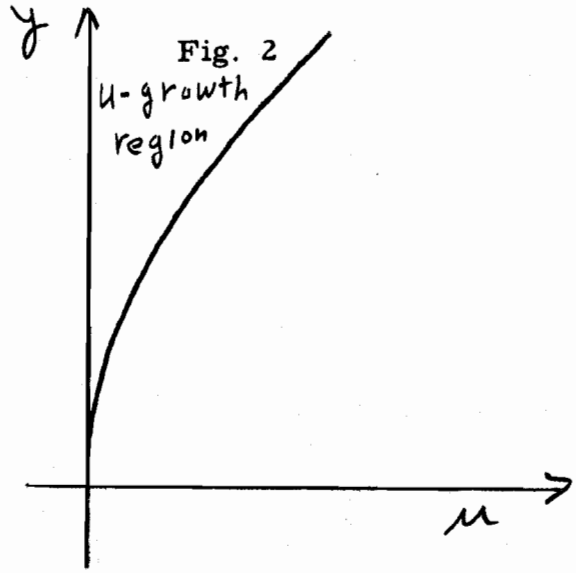


Fig. 2

$$y = 2.8\sqrt{u(u + u_{th})} \tag{3.1}$$

The "growth surface" which corresponds to the stability surface for resonances causing instability has the form shown in Fig. 3. The non-shaded region is the "stable" or no-growth region.

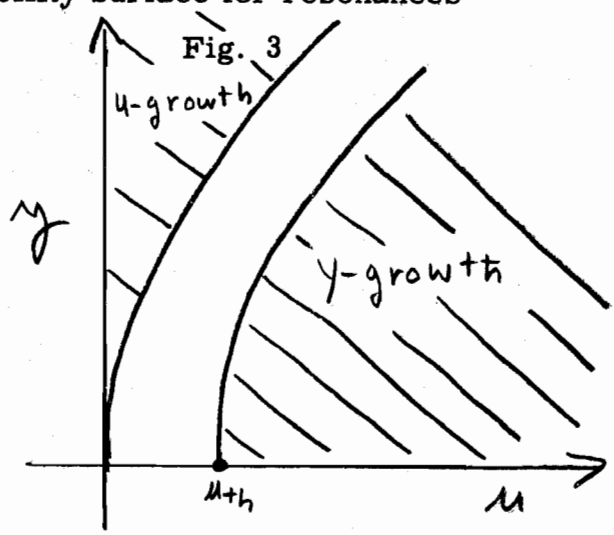


Fig. 3

An empirical result has been obtained for how far the u will grow. If one starts in the u-growth region, with an initial y equal to y_0 and $u' = y' = 0$ then u will grow to

$$u = \sqrt{\left(\frac{y_0}{2.8}\right)^2 + \frac{1}{4}u_{th}^2} - \frac{1}{2}u_{th}, \tag{3.2}$$

which is obtained by solving eq (3.1) for u.

IV. The True Stability Limits

As was mentioned previously, the difference resonance $2\nu_y - \nu_x = 0$ does not give rise to instability. Nevertheless, for sufficiently large amplitudes the motion will become unstable when the linear tune is near the $2\nu_y - \nu_x = 0$ line.

These stability limits are due to other resonances like the $\nu_x = \frac{1}{3} N$ and the $2\nu_y + \nu_x = N$ resonances.

It is difficult to calculate these stability limits since the linear tune is near the $2\nu_y - \nu_x = 0$ line and not near the resonance line causing the instability. The following picture will estimate the stability limits. These estimates may be off by a factor of two.

Suppose the nearest resonance line that can cause instability is the $2\nu_y + \nu_x = N$ line. The stability limits due to this line can be computed. Suppose the line gives rise to a y-limit, y_L.

Then the coupling of the u and y motion and the $2\nu_y - \nu_x = 0$ resonance will cause a u-limit, u_L given by solving the equation

$$y_L = 2.8 \sqrt{u_L (u_L - u_{th})} \quad . \tag{4.1}$$

If the nearest resonance causing instability is the $\nu_x = \frac{1}{3} N$ resonance and if it gives rise to the u-limit, u_L Then the coupling resonance $2\nu_y - \nu_x = 0$ will cause a y-limit, y_L given by

$$y_L = 2.8 \sqrt{u_L (u_L + u_{th})} \quad . \tag{4.2}$$