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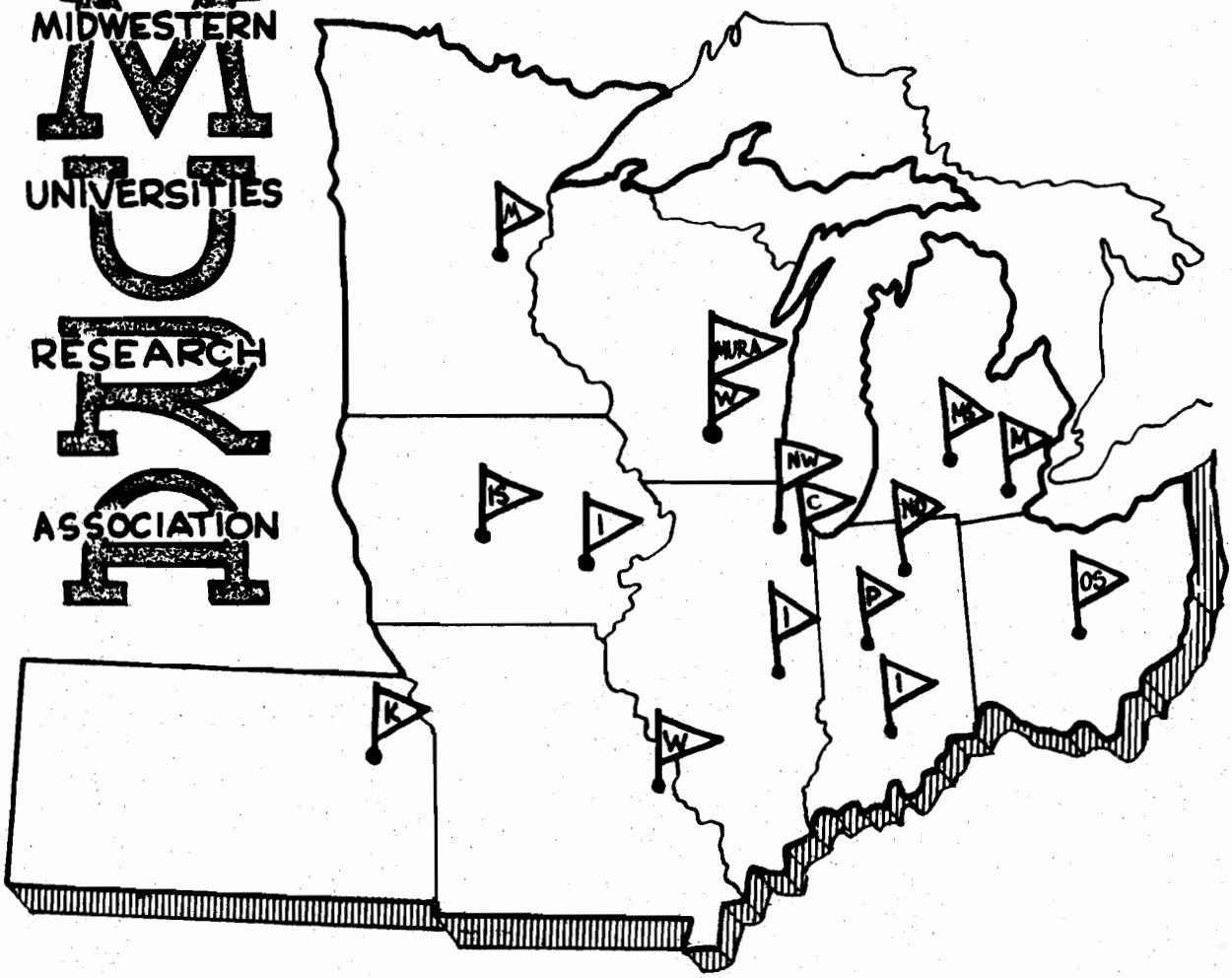

STABILITY LIMIT IN SPIRAL SECTOR STRUCTURES NEAR $\sigma_x = \frac{2\pi}{4}$

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STABILITY LIMIT IN SPIRAL SECTOR STRUCTURES NEAR $\sigma_x = \frac{2\pi}{4}$

L. J. Laslett* and A. M. Sessler**

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Renewed interest in the "handy formulas", stimulated by the possibility of applying them to the study of imperfection resonances, may make the following simple derivation of a stability limit near $\sigma_x = \frac{2\pi}{4}$ of general interest.

We start with the equation describing motion about the forced orbit in a spiral sector accelerator,

$$u'' + \left[k+1 - \frac{f^2}{2\omega^2 N^2} + f/\omega \cos N\theta\right]u = \tfrac{1}{2} f/\omega^2 \sin N\theta\, u^2 + \frac{1}{6}\frac{f^3}{\omega^3}\cos N\theta\, u^3$$

where only the dominant terms have been retained.[1]

In the neighborhood of $\sigma_x = \frac{2\pi}{4}$ we may keep only the resonant terms, or:

$$u'' + \nu_x^2 u = \frac{1}{6} f/\omega^3 \cos N\theta\, u^3$$

where ν_x^2 is the x-tune, and approximately:

$$\nu_x^2 = k+1$$

A trial function is now employed and solution is obtained by the method of harmonic balance.

Let $x = A\sin\left(\frac{N\theta}{4} + \epsilon\right)$, then:

$$\left(\nu_x^2 - \frac{N^2}{16}\right) A \sin\left(\frac{N\theta}{4} + \epsilon\right) = \frac{1}{6}\frac{fA^3}{\omega^3}\cos N\theta \sin^3\left(\frac{N\theta}{4} + \epsilon\right)$$

$$= \frac{1}{6}\frac{fA^3}{\omega^3}\left\{\frac{3}{8}\sin\left(\frac{5N\theta}{4} + \epsilon\right) + \frac{3}{8}\sin\left(-\frac{3N\theta}{4} + \epsilon\right) - \frac{1}{8}\sin\left(\frac{7N\theta}{4} + 3\epsilon\right) - \frac{1}{8}\sin\left(-\frac{N\theta}{4} - 3\epsilon\right)\right\}$$

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[1] F. T. Cole MURA Report #95

by harmonic balance:

$$\left(\nu_x^2 - \frac{N^2}{16}\right) A \sin\left(\frac{N\theta}{4} + \epsilon\right) = + \frac{fA^3}{48\omega^3} \sin\left(\frac{N\theta}{4} + 3\epsilon\right)$$

Thus if

$$\left(\nu_x^2 - \frac{N^2}{16}\right) > 0 \qquad \epsilon = 0$$

$$\left(\nu_x^2 - \frac{N^2}{16}\right) < 0 \qquad \epsilon = \frac{\pi}{2}$$

and in either case.

$$\frac{fA^2}{48\omega^3} = \left|\nu_x^2 - \frac{N^2}{16}\right| \quad \text{or} \quad A = \sqrt{\frac{48\omega^3}{f}} \left|\nu_x^2 - \frac{N^2}{16}\right|^{\frac{1}{2}}$$

It might be remarked that this method is the very same as that used previously to obtain the stability limit near $\sigma_x = \frac{2\pi}{3}$.[2]

This result is, of course, the same as that obtained by Parzen by his more sophisticated techniques.[3]

[2] L. J. Laslett and A. M. Sessler, MURA Notes, 6/1/56.

[3] G. Parzen, Non-Linear Resonances in Alternating Gradient Accelerators, MURA-200