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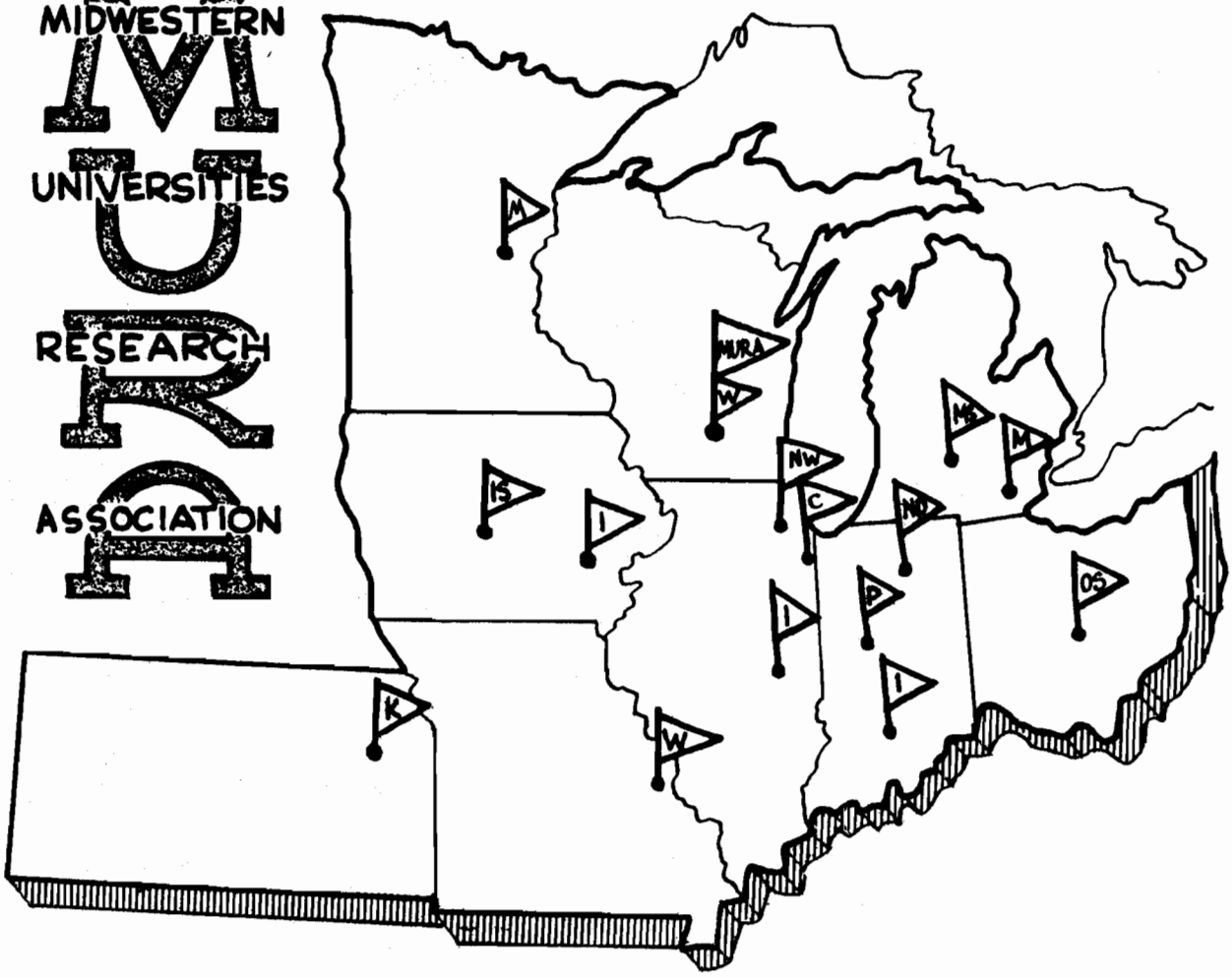

SUPPLEMENTAL NOTE CONCERNING

THE ALGEBRAIC TRANSFORMATIONS OF MURA-246 (Int.)

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MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION *
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SUPPLEMENTAL NOTE CONCERNING

THE ALGEBRAIC TRANSFORMATIONS OF MURA - 246 (Int.)

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ABSTRACT:

It is shown that the algebraic transformations of MURA - 246 (Int.) may be written in a form which more directly shows symmetry between the form of the forward and inverse transformation.

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1. In MURA-246 (Int.) the writer introduced and discussed second-degree algebraic transformations, capable of exhibiting y-growth, which were evidently able to represent certain features of the differential equations previously used to represent particle motion in FFAG accelerators. In Section 6 of that report the inverse transformation was presented and it was noted to be of a form superficially somewhat different from the original forward transformation. It is the purpose of the present note to indicate that this disparity of form is more apparent than real.

2. A simple change of variables given by a linear transformation of the form

$$u_j = (\cos\beta)\,\bar{X}_j + (\sin\beta)\,P_{\bar{X}_j}$$
$$p_{u_j} = -(\sin\beta)\,\bar{X}_j + (\cos\beta)\,P_{\bar{X}_j}$$
$$v_j = (\cos\gamma)\,Y_j + (\sin\gamma)\,P_{Y_j}$$
$$p_{v_j} = -(\sin\gamma)\,Y_j + (\cos\gamma)\,P_{Y_j}$$

when introduced into transformation (4) of MURA-246 (Int.), converts the latter into

$$u_n = (\cos v)\,u_{n-1} + (\sin v)\,p_{u_{n-1}} + \frac{\lambda'}{2}\,\frac{\sin(v+\beta)}{\sin K}\left[(\cos\gamma)\,v_{n-1} - (\sin\gamma)\,p_{v_{n-1}}\right]^2$$

$$p_{u_n} = -(\sin v)\,u_{n-1} + (\cos v)\,p_{u_{n-1}} + \frac{\lambda'}{2}\,\frac{\cos(v+\beta)}{\sin K}\left[(\cos\gamma)\,v_{n-1} - (\sin\gamma)\,p_{v_{n-1}}\right]^2$$

$$v_n = (\cos K)\,v_{n-1} + (\sin K)\,p_{n-1} + \lambda'\,\frac{\sin(K+\gamma)}{\sin K}\left[(\cos\beta)\,u_{n-1} - (\sin\beta)\,p_{u_{n-1}}\right]\left[(\cos\gamma)\,v_{n-1} - (\sin\gamma)\,p_{v_{n-1}}\right]$$

$$p_{v_n} = -(\sin K)\,v_{n-1} + (\cos K)\,p_{v_{n-1}} + \lambda'\,\frac{\cos(K+\gamma)}{\sin K}\left[(\cos\beta)\,u_{n-1} - (\sin\beta)\,p_{u_{n-1}}\right]\left[(\cos\gamma)\,v_{n-1} - (\sin\gamma)\,p_{v_{n-1}}\right]$$

and the inverse transformation becomes

$$u_{n-1} = (\cos v)u_n - (\sin v)p_{u_n} - \frac{\lambda'}{2}\frac{\sin\beta}{\sin K}\left[\cos(K+\gamma)v_n - \sin(K+\gamma)p_{v_n}\right]^2$$

$$p_{u_{n-1}} = (\sin v)u_n + (\cos v)p_{u_n} - \frac{\lambda'}{2}\frac{\cos\beta}{\sin K}\left[\cos(K+\gamma)v_n - \sin(K+\gamma)p_{v_n}\right]^2$$

$$v_{n-1} = (\cos K)v_n - (\sin K)p_{v_n} - \lambda'\frac{\sin\gamma}{\sin K}\left[\cos(v+\beta)u_n - \sin(v+\beta)p_{u_n}\right]\left[\cos(K+\gamma)v_n - \sin(K+\gamma)p_{v_n}\right]$$

$$p_{v_n} = (\sin K)v_n + (\cos K)p_{v_n} - \lambda'\frac{\cos\gamma}{\sin K}\left[\cos(v+\beta)u_n - \sin(v+\beta)p_{u_n}\right]\left[\cos(K+\gamma)v_n - \sin(K+\gamma)p_{v_n}\right]$$

3. We accordingly may emphasis the symmetry between direct and inverse transformations of the type under consideration by choosing $\beta = -v/2$ and $\gamma = -K/2$.

One than obtains

$$u_n = (\cos v)u_{n-1} + (\sin v)p_{u_{n-1}} + (\mu/2)(\sin v/2)\left[(\cos K/2)v_{n-1} + (\sin K/2)p_{v_{n-1}}\right]^2$$

$$p_{u_n} = -(\sin v)u_{n-1} + (\cos v)p_{u_{n-1}} + (\mu/2)(\cos v/2)\left[(\cos K/2)v_{n-1} + (\sin K/2)p_{v_{n-1}}\right]^2$$

$$v_n = (\cos K)v_{n-1} + (\sin K)p_{v_{n-1}} + \mu(\sin K/2)\left[(\cos v/2)u_{n-1} + (\sin v/2)p_{u_{n-1}}\right]\left[(\cos K/2)v_{n-1} + (\sin K/2)p_{v_{n-1}}\right]$$

$$p_{v_n} = -(\sin K)v_{n-1} + (\cos K)p_{v_{n-1}} + \mu(\cos K/2)\left[(\cos v/2)u_{n-1} + (\sin v/2)p_{u_{n-1}}\right]\left[(\cos K/2)v_{n-1} + (\sin K/2)p_{v_{n-1}}\right]$$

and

$$u_{n-1} = (\cos v)u_n - (\sin v)p_{u_n} + (\mu/2)(\sin v/2)\left[(\cos K/2)v_n - (\sin K/2)p_{v_n}\right]^2$$

$$p_{u_{n-1}} = (\sin v)u_n + (\cos v)p_{u_n} - (\mu/2)(\cos v/2)\left[(\cos K/2)v_n - (\sin K/2)p_{v_n}\right]^2$$

$$v_{n-1} = (\cos K)\, v_n - (\sin K)\, p_{v_n} + \mu (\sin K/2)\left[(\cos \nu/2)\, u_n - (\sin \nu/2)\, p_{u_n}\right]\left[(\cos K/2)\, v_n - (\sin K/2)\, p_{v_n}\right]$$

$$p_{v_{n-1}} = (\sin K)\, v_n + (\cos K)\, p_{v_n} - \mu (\cos K/2)\left[(\cos \nu/2)\, u_n - (\sin \nu/2)\, p_{u_n}\right]\left[(\cos K/2)\, v_n - (\sin K/2)\, p_{v_n}\right]$$

$$\text{with } \mu \equiv \lambda'/(\sin K)$$

It is evident that, in this form, the inverse transformation differs from the direct only by a reversal of sign of ν, K, and μ

4. It is perhaps worth noting that, according to the foregoing transformation,

$$(\cos \nu/2)\, u_{n-1} + (\sin \nu/2)\, p_{u_{n-1}} = (\cos \nu/2)\, u_n - (\sin \nu/2)\, p_{u_n}.$$

and

$$(\cos K/2)\, v_{n-1} + (\sin K/2)\, p_{v_{n-1}} = (\cos K/2)\, v_n - (\sin K/2)\, p_{v_n}.$$