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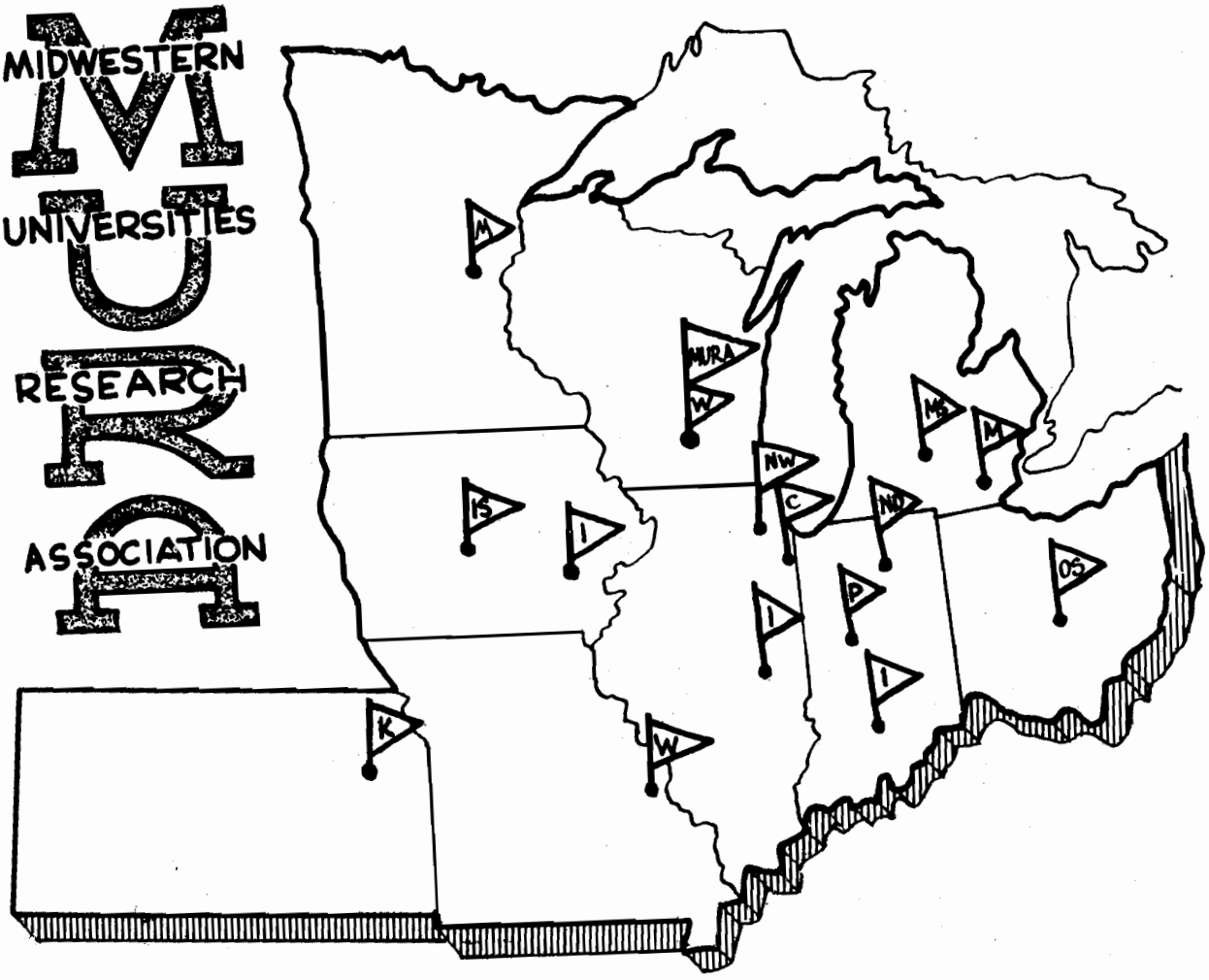

REPORT FORERUNNER
(Problem 57)
March, 1957

NUMBER 240
Internal
(IBM Program)

FORERUNNER
(Problem 57)

March 12, 1957

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This is an IBM 704 program, similar to RIDGE RUNNER on ILLIAC, to solve numerically the equations of motion describing median-plane betatron oscillations of particles in a fixed field alternating gradient accelerator. The system of equations is as follows:

$$x' = (1+x)\frac{p}{\sqrt{1-p^2}},$$

$$p' = \sqrt{1-p^2} - (1+x)\left\{\lambda + \sum_{i=1}^{3} f_i \sin\left[m_i\left(\frac{1}{w}\log(1+x) - N\theta\right) + \alpha_i\right]\right\}$$

where $x' = \frac{dx}{d\theta}$, $p' = \frac{dp}{d\theta}$, and $m_1 \equiv 1$.

The user submits parameters $k' = k+1, \lambda, f_1, f_2, f_3, m_2, m_3, \frac{1}{w}, N, \alpha_1, \alpha_2, \alpha_3$, and others described below, as well as initial conditions x_0, p_0. A run will be stopped when the variables x and p no longer satisfy the conditions

$$|x| < \frac{\beta_1}{\sqrt{(\frac{1}{w})^2 + N^2}}, \quad |p| < \frac{\beta_2 A}{\sqrt{(\frac{1}{w})^2 + N^2}},$$

where A = max (4, N) and β_1, β_2 are parameters supplied by the user. If we define

$$C_1 = \frac{2}{\beta_1}\sqrt{(\tfrac{1}{w})^2 + N^2}, \quad C_2 = \frac{2}{\beta_2 A}\sqrt{(\tfrac{1}{w})^2 + N^2},$$

then these conditions are $|x| < \frac{2}{C_1}, |p| < \frac{2}{C_2}$.

In an effort to obtain more accuracy for this fixed-point program the variables actually carried within the routine are $\bar{x} = \frac{C_1 x}{4}$ and $\bar{p} = \frac{C_2 p}{4}$.

The parameters must satisfy the following restrictions:

$$0 \le k' \le 300$$
$$0 \le \lambda \le 1$$
$$\lambda + \sum_{i=1}^{3} |f_i| < 32$$
$$-8 < m_2 < 8, \quad -8 < m_3 < 8 \quad \} \; m_2, m_3 \text{ integers}$$
$$0 \le \frac{1}{w} \le 5000$$
$$0 \le |N| \le 100$$

$-2\pi < \alpha_i \leq 2\pi, \; i = 1, 2, 3$

$0 \leq \beta_1 \leq 4$

$0 \leq \beta_2 \leq 2$

$\sqrt{\left(\frac{1}{\omega}\right)^2 + N^2} \geq 4$

$|h| = |h' - 1| \leq (.4)\sqrt{\left(\frac{1}{\omega}\right)^2 + N^2}$.

The equations are integrated for N_e Runge-Kutta (RK) steps each of length $h = \frac{2\pi}{N n_{RK}}$, where n_{RK} is an integer which specifies the number of steps per sector. Backward integration (h negative) may be accomplished by assigning a negative integral value to n_{RK}.

Printing of the parameters as listed on the agendum sheet is followed by the initial conditions and then the integrated variables every N_p steps. The four columns of print are: 1) a five-digit count of the number of prints, 2) $\frac{N\theta}{2\pi}$ mod 1, 3) x, 4) p.

The run will end and a characteristic integer will be printed whenever parameters or variables lie outside permissible ranges, according to the following table:

Char. integer printed	Reason for stop
00001	$\lvert x \rvert \geq \frac{2}{C_1}$
00002	$\lvert p \rvert \geq \frac{2}{C_2}$
00003	$\left\lvert 2^m h \left(\frac{C_1}{4}\right) x' \right\rvert \geq 1$
00004	$\lvert p' \rvert \geq 4$, or $\left\lvert 2^m h \left(\frac{C_2}{4}\right) p' \right\rvert \geq 1$
00006	$k' \geq 2^9$
00007	$\lambda + \sum_{i=1}^{3} \lvert f_i \rvert \geq 32$
00008	$\frac{1}{\omega} \geq 2^{11}\pi$
00009	$\lvert m_2 \rvert \geq 8$, or $\lvert m_3 \rvert \geq 8$
00010	$\left(\frac{1}{\omega}\right)^2 + N^2 < 16$

00011 $|N| > 100$

00012 $\beta_1 > 4$

00014 $\beta_2 > 2$

00016 $|k| > (.4)\sqrt{(\frac{1}{w})^2 + N^2}$

Furthermore if $\frac{1}{4} < \frac{1}{C_1} \leq \frac{1}{2}$, then $\frac{1}{C_1}$ is set equal to $\frac{1}{4}$, β_1 is recomputed, and $\beta_1 \cdot 10^{-2}$ is printed along with characteristic integer 00012. Likewise if $\frac{1}{2} < \frac{1}{C_2} \leq 1$, then $\frac{1}{C_2}$ is set equal to $\frac{1}{2}$, β_2 is recomputed, and $\beta_2 \cdot 10^{-2}$ is printed along with characteristic integer 00014. In the latter two cases computation continues.

If N = 0 the user must insert $2^m h$ as a fraction into decimal location 131 and m as an integer into decimal location 1371. Here m is such that $2^{-1} \leq |2^m h| < 1$. It is assumed that $\Theta_0 = 0$; if, however, a nonzero value of Θ_0 is desired it may be inserted in decimal location 155 as $\frac{N\Theta_0}{2\pi}$ mod 1. Both of these cases must be handled on the agendum sheet.

Those parameters of a particular run of a series which are the same as on the preceding run need not be put on the agendum sheet, with the following exceptions: $\frac{N\Theta_0}{2\pi}$ mod 1 in decimal location 155; $\beta_1 \cdot 10^{-2}$, $\beta_2 \cdot 10^{-2}$, λ in decimal locations 115, 116, 117, resp. The parameters β_1, β_2, and λ are preset to 4, 2, and 1, resp., so that if any of these values are desired they need not be put on the agendum.

If the user is present during a series of runs he may kill a particular run by putting sense switch 2 up. The "search" feature is also included: sense switch 6 is put up if it is desired to stop a series of runs after the first such run which is successfully completed, as when the user is looking for stable regions.

FORERUNNER may be compared with WELL-TEMPERED FIVE, where appropriate, by using the following:

$$m_2 = 2,\ m_3 = 3,\ f_i = \nu_{oi}\sqrt{1+\left(\frac{\mu_{oi}}{\nu_{oi}}\right)^2},$$
$$\alpha_i = \sin^{-1}(\mu_{oi}/f_i),\ i = 1, 2, 3.$$

Printing time is about .680 sec. per print and computation time is about .203 sec. per RK step, although the latter is reduced if any of the $f_i = 0$ by the amount of time necessary to compute the corresponding sine function.

FORERUNNER Agendum (Problem 57)

Parameter	Decimal Address	Value	Remarks
ID	100	57	Ten digits: A·10^8 + 57·10^6 + i, where A = human ID, i = run number
N_e	101		Number of RK steps to end of problem
N_p	102		Number of RK steps between prints
n_{RK}	103		Number of RK steps per sector
m_2	104		
m_3	105		
m	1371		Insert only if N = 0. Then m is such that $2^{-1} \le \lvert 2^m h \rvert < 1$
			END INTEGERS
$k' \cdot 10^{-4}$	106		$k' = k + 1$
$t_1 \cdot 10^{-2}$	107		
$t_2 \cdot 10^{-2}$	108		
$t_3 \cdot 10^{-2}$	109		
$\frac{1}{\omega r} \cdot 10^{-4}$	110		
$\alpha_1/(2\pi)$	111		
$\alpha_2/(2\pi)$	112		
$\alpha_3/(2\pi)$	113		
$N \cdot 10^{-4}$	114		
$\beta_1 \cdot 10^{-2}$	115		β_1 = 4 unless otherwise specified
$\beta_2 \cdot 10^{-2}$	116		β_2 = 2 unless otherwise specified
λ	117		λ = 1 unless otherwise specified
x_0	118		
p_0	119		
$2^m h$	131		Insert only if N = 0
$\frac{N\theta_0}{2\pi} \bmod 1$	155		Insert only if $\theta_0 \neq 0$
			END FRACTIONS

Fixed point integers (1-11 digits)

Fixed ground fractions (1-11 digits)

Sense switches normally down. SS6 up: search feature. ☐ Check if desired.