231

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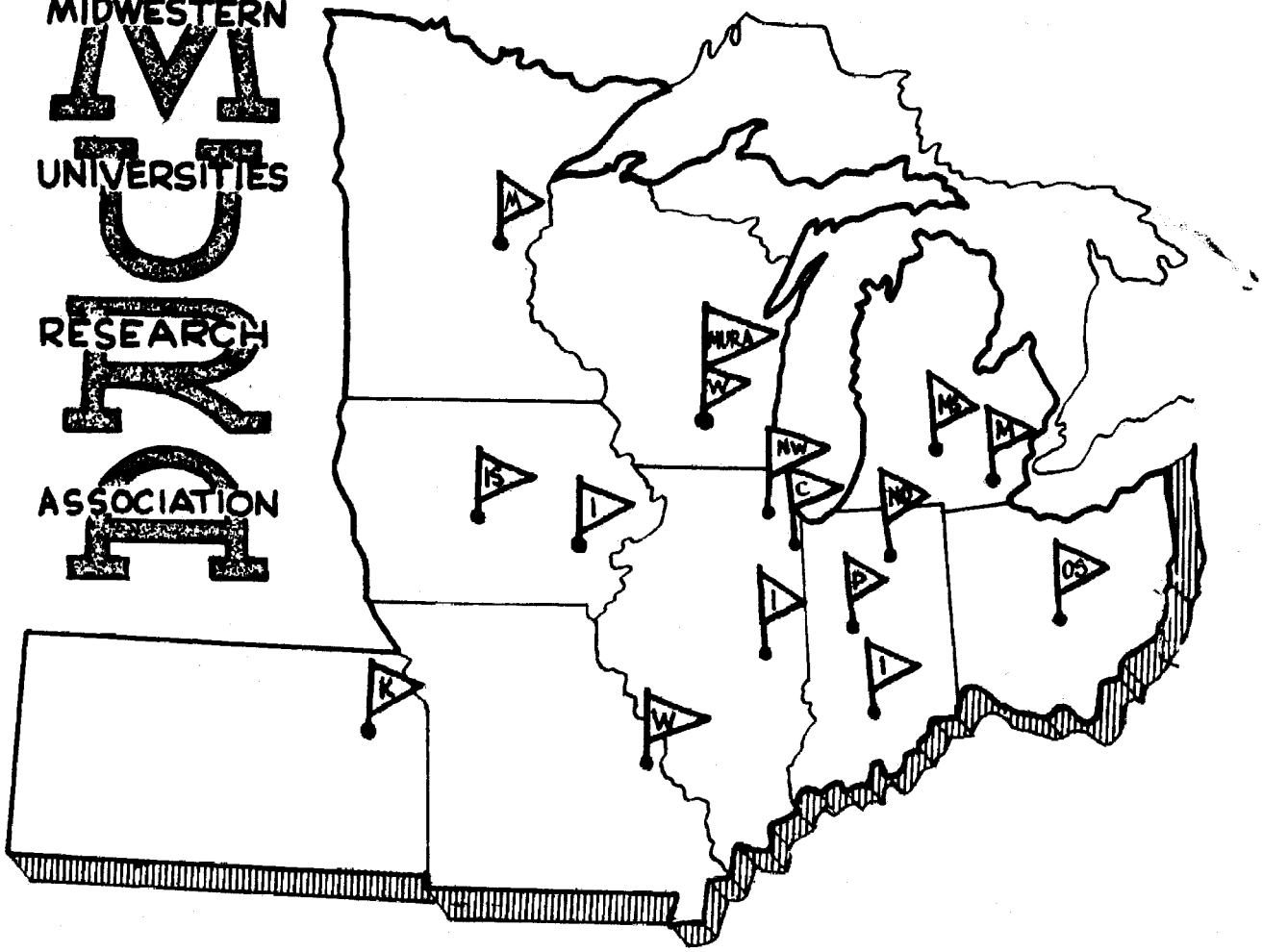

REPORT FIXED POINT SEARCH, ROTATION NUMBER, AND INVARIANT COEFFICIENT PROGRAMMES

NUMBER 231

Internal
(IBM Program)

FORFIX POINT (PROGRAMME 55)
FORFIX POINT WITH FUMBLEBUMPS (PROGRAMME 66)
FORFIX POINT WITH GRUMBLEBUMPS (PROGRAMME 67)

February-March, 1957

FIXED POINT SEARCH, ROTATION NUMBER, AND INVARIANT COEFFICIENT PROGRAMMES

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FORFIX POINT WITH GRUMBLEBUMPS (PROGRAMME 67)

J. N. Snyder

The desirability of having this family of programs can be traced back to a memorandum L. Jackson Laslett to J. N. Snyder 17 December 1956. Although different in internal machine execution, the three programs have a common data input format, a common goal, and a common output format. The latter two introduce the two types of bumps which have been adequately described elsewhere (FORMESH FUMBLEBUMPS, Programme 60 and FORMESH GRUMBLEBUMPS, Programme 62). Hence one description (below) will suffice for the three programmes in hand. It should be recalled that the basic problem treated is that of FORMESH, Programme 26.

The programme is divided into Part I and Part II. The former finds a fixed point, the latter finds the rotation number about that fixed point and obtains the quantities necessary to easily compute the coefficients in the invariant function. Either part may be omitted; Part I by moving SS3 up, Part II by moving SS6 up.

For identification purposes, the programme belongs to Category II, having separate and distinct Run ID and Human ID numbers. If N = Number of sectors $\equiv 0$, it is necessary to supply m and $2^m h$ as in FORMESH (q. v.) During the entire problem (Parts I and II) each of the possibly many runs will be printed every N_P integration steps in the same format as FORMESH (q. v.) As in FORMESH, overflow limits on the variables x, p_x, p_y can be set via x_{max}, $p_{x_{max}} = p_{y_{max}}$. These are now all set to 1/2 on the master cards and should not be <u>increased</u> except in desperate circumstances. The y overflow limit is determined by the b of the mesh (see FORMESH). It can be reduced by executing the well known b_{PHONY} procedure.

In Part I, the fixed point search we define

N_{RK} = integration steps per sector

N_{FP} = integration steps in a possibly larger periodicity

N_{EFP} = integration steps in a run

The first run will be started with initial conditions

$$p_{x_0} = (p_{x_0})_0$$
$$p_{y_0} = (p_{y_0})_0$$
$$x_0 = (x_0)_0$$
$$y_0 = (y_0)_0$$
$$\left(\frac{N\theta}{2\pi}\right)_0 = \left(\frac{N\theta}{2\pi}\right)_{0FP}$$

as entered on the agendum sheet. Every N_{FP} steps p_x, p_y, x, y will be observed and recorded. At the end of the ith run the quantities

$\langle p_x \rangle_i$

$\langle p_y \rangle_i$

$\langle x \rangle_i$

$\langle y \rangle_i$

recomputed, the averaging being over the N_{EFP}/N_{FP} (≤ 32 is necessary) observations mentioned above.

The (i + 1) st run will be started with initial conditions determined by RSR= "Re-Start Rule" as follows:

RSR = +0

$$p_{x_o} = (p_{x_o})_o$$
$$p_{y_o} = (p_{y_o})_o$$
$$x_o = (x_o)_o$$
$$y_o = (y_o)_o$$
$$\left(\frac{N\theta}{2\pi}\right)_o = \left(\frac{N\theta}{2\pi}\right)_{OFP}$$

RSR = -0

$$p_{x_o} = \langle p_x \rangle_i$$
$$p_{y_o} = (p_{y_o})_o$$
$$x_o = \langle x \rangle_i$$
$$y_o = (y_o)_o$$
$$\left(\frac{N\theta}{2\pi}\right)_o = \left(\frac{N\theta}{2\pi}\right)_{OFP}$$

RSR = +1

$$p_{x_o} = (p_{x_o})_o$$
$$p_{y_o} = \langle p_y \rangle_i$$
$$x_o = (x_o)_o$$
$$y_o = \langle y \rangle_i$$
$$\left(\frac{N\theta}{2\pi}\right)_o = \left(\frac{N\theta}{2\pi}\right)_{OFP}$$

RSR = -1

$$p_{x_o} = \langle p_x \rangle_i$$
$$p_{y_o} = \langle p_y \rangle_i$$
$$x_o = \langle x \rangle_i$$
$$y_o = \langle y \rangle_i$$
$$\left(\frac{N\theta}{2\pi}\right)_o = \left(\frac{N\theta}{2\pi}\right)_{OFP}$$

This process will continue until a run is obtained on which for <u>every one</u> of the N_{EFP}/N_{FP} aforementioned observations it is true that

$$|x - x_o| < \epsilon_x$$
$$|y - y_o| < \epsilon_y$$
$$|p_x - p_{x_o}| < \epsilon_{p_x}$$
$$|p_y - p_{y_o}| < \epsilon_{p_y}$$

where x_o, y_o, p_{x_o}, p_{y_o} pertain to the starting values of the run in question. These values are then regarded as being x_f, y_f, p_{x_f}, p_{y_f} the coordinates of the fixed point. These are then printed as a solitary line on a solitary sheet as:

00002 p_{x_f} p_{y_f} x_f y_f

It should be noted that if Part I is to be omitted the user is expected to have filled in p_{x_f}, p_{y_f}, x_f, y_f in place of $(p_{x_o})_o$, $(p_{y_o})_o$, $(x_o)_o$, $(y_o)_o$ on the agendum sheet for use by Part II. (See sample agendum sheet attached.) Further note that several quantities on the agendum sheet are for convenience given values by the master programme. These can be changed singly or severally if desired.

For Part II of the programme we define:

$$p_\rho = p_x - p_{x_f}$$

$$p_\psi = p_y - p_{y_f}$$

$$\rho = x - x_f$$

$$\psi = y - y_f$$

N_{RK} = number of integration steps per sector (the <u>same</u> number as in Part I)

N_{RN} = number of integration steps per larger periodicity

Part II first makes a run with initial conditions

$$p_{x_o} = p_{x_f} + (p_{\rho\,o})_x$$

$$p_{y_o} = p_{y_f} + (p_{\psi_o})_x$$

$$x_o = x_f + (\rho_o)_x$$

$$y_o = y_f + (\psi_o)_x$$

$$\left(\frac{N\theta}{2\pi}\right)_o = \left(\frac{N\theta}{2\pi}\right)_{ox}$$

The user must also enter in which quadrant $(Q_o)_x$ the initial $(\rho_o)_x$, $(p_{\rho_o})_x$ vector lies. Quadrants are labelled 1, 2, 3, 4 counterclockwise as is conventional. Every N_{RN} integration steps ρ and p_ρ are examined. A count R is kept of the number of <u>quadrants</u> the ρ, p_ρ vector has turned through <u>clockwise</u>; a count M_F of the number of such examinations made is kept; a record of $\Sigma\rho^2$, $\Sigma\rho p_\rho$, Σp_ρ^2 is kept. The run is stopped when <u>either</u>

$R \geq R_o$

$M_F \geq M_{FO}$ (≤ 64 is necessary)

is reached. R_o and M_{FO} are specified on the agendum sheet. At this time there is printed:

R	$\frac{2^{-2}R}{M_F}$	$\frac{2^{-2}R}{M_F}$	$\frac{N_{RK}}{N_{RN}}$
M_F			
	(space)		
o	$2^{10}\langle\rho^2\rangle$	$2^{10}\langle\rho p_\rho\rangle$	$2^{10}\langle p_\rho^2\rangle$

the averages being over the aforementioned M_F observations.

A second run is now made by Part II of the program. This run is similar to the first described above except that the quantities

$$(p\rho_o)_y, (p\psi_o)_y, (\rho_o)_y, (\psi_o)_y, \left(\frac{N\theta}{2\pi}\right)_{oy}, (Q_o)_y$$

are used in the pertinent places. This time, however, $2^{10}\langle\psi^2\rangle$, $2^{10}\langle\psi p_\psi\rangle$, $2^{10}\langle p_\psi^2\rangle$ will be computed and printed.

It should be further noted that the axes of the small amplitude ellipses traversed in computing these averages should not exceed 2^{-6}. This does not imply that larger amplitudes may not be submitted in order to find rotation numbers for these large amplitudes, but in such cases the above averages will be nonsense numbers whenever the 2^{-6} limit is substantially exceeded.

At this point the programme stops. Hopefully, the programme will not encounter the various failure stops possessed in common with FORMESH (q. v.). If such be encountered, it can at least be predicted that the standard pertinent FORMESH failure mark will be printed. Additional possible effects are to be expected.

It should be further noted that for an ellipse

$$\langle\rho^2\rangle p^2 - 2\langle\rho p\rangle\rho p + \langle p^2\rangle\rho^2$$

is an invariant.

November 19, 1958

MEMO: To Computer Users

RE: FORFIXPOINT OVERWRITES: P-JUMPS and P-JUMPS DELUXE

FROM: Robert Morse

These program additions were written as the result of a verbal communication between F. Mills, T. Binford, and M. Storm. Their purpose is to provide a means for incrementing p_x at specified point during the course of a run, in the same manner as FORMESH P-JUMPS. They may be used either with FORFIXPOINT (NO. 55), or FORFIXPOINT SERIES (NO. 221). A p_x increment will take place before printing, assuming that printing has been scheduled for the integration step in question. (This is now also true for FORMESH P-JUMPS.)

FORFIXPOINT P-JUMPS will accept from 1 to 100 integers N_i which represent the number of Runge-Kutta steps between two successive insertions of a change in p_x. It will also accept from 1 to 100 fractions Δp_{x_i}, which represent the changes in p_x (positive or negative) which it is desired to produce. If Δp_{x_f} represents the last increment to be entered, one must load $N_{f+1} = 0$. This must be done even if $f = 100$. All values that are entered will be held from run to run, unless changed. The mesh size must be limited by $3ab \leq 7106$.

FORFIXPOINT P-JUMPS DELUXE provides for Δp_x to be computed by the 704 either as an arithmetic sequence:

$$p_{x_i} = (\Delta p_x)_o + (i - 1)\, d,$$

or as a geometric sequence:

$$p_{x_i} = (\Delta p_x)_o \cdot r^{i-1},$$

where $i = 1, 2, 3, \ldots$. It accepts two integers, $N_{initial}$ and $N_{general}$, which represent respectively the number of R-K steps before the first p_x increment, and the number of R-K steps between successive increments. It also will accept three fractions, $(\Delta p_x)_o$, d, and r/10, which may be positive or negative. $|r|$ has an upper limit of 9.9999999998. To obtain the arithmetic sequence, r must be entered as zero, and correspondingly for the geometric series. If both r and d are entered as zero, Δp_x will always be equal to $(\Delta p_x)_o$. The mesh used must have $3ab \leq 7277$.

MEMO from Robert Morse November 19, 1958

Agendum preparation: Use a standard FIXED POINT SEARCH Agendum sheet. Write a clear note on the top, as for example, "USE FORFIXPOINT P-JUMPS DELUXE OVERWRITE." Means must also be provided for the extra integer and fraction data to be punched and included with the usual FORFIX-POINT data.

	P-JUMPS			P-JUMPS DELUXE	
Integers:	N_1	7230	Integers:	$N_{initial}$	7343
	N_2	7231		$N_{general}$	7344
	.	.			
	.	.			
	.	.			
Fractions:	Δp_{x_1}	7130	Fractions:	r/10	7345
	Δp_{x_2}	7131		d	7346
	.	.		$(\Delta p_x)_o$	7347
	.	.			
	.	.			

NOTE: If Δp_x becomes larger than $(1 - 2^{-35})$, an error signal 00006 will be printed. If the FORFIXPOINT SERIES program is being used, the run will be treated as a failure and the next run will be read in.

November 19, 1958

MEMO: To Computer Users

RE: FORFIXPOINT SERIES PROGRAMS (NOS. 221, 222, 223)

FROM: Robert Morse

The need for these programs was made known in a verbal communication between T. Binford and M. R. Storm. The new programs, which correspond respectively to the three original FORFIXPOINT programs, numbers 55, 66, and 67, permit the user to supply the computer with a series of runs. The computer will automatically read on a new run after discovering an instability in the present run. When a stable run is found, the computer will find a fixed point for it and then stop, ignoring all succeeding runs.

Rotation numbers and computation of invariants cannot be done with these programs. If they are needed, they must be found by means of a separate run with one of the original programs.

All quantities will be held from run to run, unless changed by the user. There is no further reduction in mesh size over that required by the corresponding original program.

Agendum preparation: An ordinary FIXED POINT SEARCH Agendum sheet may be used, together with a General Series Agendum sheet. To make sure that the correct program will be used, the old program number should be crossed off on the agendum sheet and the new one written in its place. In addition, one should write a note on top of the sheet, as for example, "USE FORFIXPOINT FUMBLE-BUMPS SERIES, PROGRAM 222."

FIXED POINT SEARCH AGENDUM

EMPLOY FIELDS FROM FOROCYL NO.		OR FORMERGE NO.	

USE FORFIXPOINT (PROGRAMME 55)		CHECK ONE
USE FORFIXPOINT FUMBLEBUMPS (PROGRAMME 66) (Attach FUMBLEBUMPS AGENDUM SHEET)		
USE FORFIXPOINT GRUMBLEBUMPS (PROGRAMME 67) (Attach GRUMBLEBUMPS AGENDUM SHEET)		

OMIT FIXED POINT SEARCH (SS3 UP)	
OMIT ROTATION NUMBER AND INVARIANTS COMPUTATION (SS6 UP)	

PARAMETER	ADDRESS	VALUE	REMARKS	
ID#	7597			INTEGERS
N_{EFP}	7598			
N_P	7599			
N_{RK}	7600			
m	7687		Needed only if $N \equiv 0$	
N_{FP}	7602			
RSR	7603		(+0)None;(-0)x, p_x;(+1)y, p_y;(-1)x, p_x, y, p_y	
b_{PHONY}	7626		To decrease y overflow limit	
$(Q_0)_x$	7436		(4 if not set)	
$(Q_0)_y$	7437		(4 if not set)	
N_{RN}	7438			
R_0	7439			
M_{FO}	7440			
HUMAN ID	7601			FRACTIONS
$(p_{x0})_0$	7591		(p_{xf}) entered if FP search omitted	
$(p_{y0})_0$	7592		(p_{yf}) entered if FP search omitted	
$(x_0)_0$	7593		(x_f) entered if FP search omitted	
$(y_0)_0$	7594		(y_f) entered if FP search omitted	
$\left(\frac{N\theta}{2\pi}\right)_{0FP}$	7421		(0 if not set)	
x_{max}	7595		(1/2 if not set)	
$(p_x)_{max} = (p_y)_{max}$	7596		(1/2 x 2^{-6} if not set) (2^{-6} scale factor)	
$2^m h$	7624		Needed only if $N \equiv 0$	
ϵ_x	7422		(.000005 if not set)	
ϵ_y	7423		(.000001 if not set)	
ϵ_{p_x}	7424		(.000005 if not set)	
ϵ_{p_y}	7425		(.000001 if not set)	
$(p_{\rho 0})_x$	7426		(-2^{-27} if not set)	
$(p_{\psi 0})_x$	7427		(0 if not set)	
$(\rho_0)_x$	7428		(.0005 if not set)	
$(\psi_0)_x$	7429		(0 if not set)	
$\left(\frac{N\theta}{2\pi}\right)_{0x}$	7430		(0 if not set)	
$(p_{\rho 0})_y$	7431		(0 if not set)	
$(p_{\psi 0})_y$	7432		(-2^{-27} if not set)	
$(\rho_0)_y$	7433		(0 if not set)	
$(\psi_0)_y$	7434		(.0001 if not set)	
$\left(\frac{N\theta}{2\pi}\right)_{0y}$	7435		(0 if not set)	