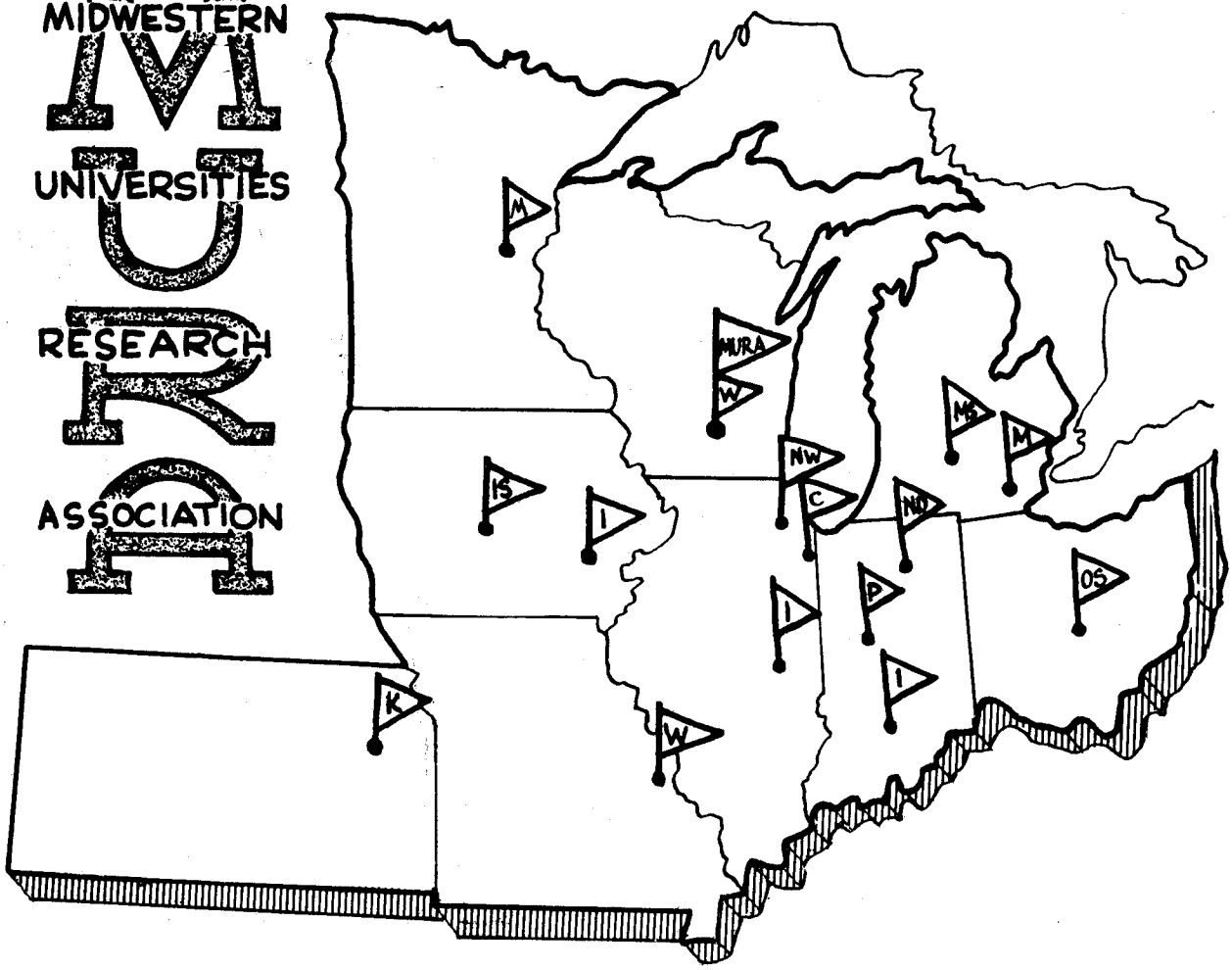
Fermilab Library
0 1160 0038539 7

~~ZGS LIBRARY~~ 226
H-1

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION



REPORT WELL TEMPERED FIVE
(Program 46)
February-March, 1957

NUMBER 226
Internal
(IBM Program)

MURA-226

WELL TEMPERED FIVE
(Problem 46)

L. D. Fosdick

The following notes are presented to guide the users of the program known as "Well Tempered Five." Programming details and methodology are not presented here.

The Well Tempered Five is a program for the IBM 704 to solve the equations of motion describing the betatron oscillations of particles in a fixed field alternating gradient accelerator. These equations are familiar and are copied below for convenience.

$$\frac{dx}{d\theta} = \frac{(1+x)\,p_x}{\sqrt{1-p_x^2-p_y^2}} ,$$

$$\frac{dy}{d\theta} = \frac{(1+x)\,p_y}{\sqrt{1-p_x^2-p_y^2}} ,$$

$$\frac{dp_x}{d\theta} = \sqrt{1-p_x^2-p_y^2} + \frac{er_1}{p}(1+x)\left[H_y - \frac{p_y}{\sqrt{1-p_x^2-p_y^2}} H_\theta\right], \tag{1}$$

$$\frac{dp_y}{d\theta} = \frac{er_1}{p}(1+x)\left[\frac{p_x}{\sqrt{1-p_x^2-p_y^2}} H_\theta - H_x\right] .$$

The program employs units which give $\frac{er_1}{p} = 1$.

The method of obtaining the three magnetic field components, H_x (radial), H_y (axial), and H_θ (azimuthal), is the feature which uniquely distinguishes this program. The field components are computed according to the following equations, described by Powell in MURA-JLP 6:

$$H_x = (1+x)^k \sum_{i=1}^{\infty} \sum_{m=0}^{M} (\alpha_{im} \cos m\Phi + \beta_{im} \sin m\Phi) \frac{(y/1+x)^{2i-1}}{(2i-1)!} ,$$

$$H_y = -(1+x)^k \sum_{i=0}^{\infty} \sum_{m=0}^{M} (\mu_{im} \cos m\Phi + \nu_{im} \sin m\Phi) \frac{(y/1+x)^{2i}}{(2i)!} , \tag{2}$$

$$H_\theta = (1+x)^k N \sum_{i=1}^{\infty} \sum_{m=0}^{M} m (\nu_{i-1,m} \cos m\Phi - \mu_{i-1,m} \sin m\Phi) \frac{(y/1+x)^{2i-1}}{(2i-1)!} ,$$

The coefficients μ i m, ν i m, α i m, and β i m appearing in the equations are obtained from the following recurrence relations:

$$\mu_{i+1,m} = \{m^2(K^2+N^2) - (k-2i)^2\}\mu_{im} - 2mK(k-2i)\nu_{im} ,$$

$$\nu_{i+1,m} = 2mK(k-2i)\mu_{im} + \{m^2(K^2+N^2) - (k-2i)^2\}\nu_{im} ,$$

$$\alpha_{i+1,m} = -(k-2i)\mu_{im} - mK\nu_{im} , \qquad \alpha_{om} = 0 ,$$

$$\beta_{i+1,m} = mK\mu_{im} - (k-2i)\nu_{im} , \qquad \beta_{om} = 0 . \tag{3}$$

The argument Φ in Eqs. (2) is

$$\Phi = K \ln (1 + x) - N\theta , \tag{4}$$

and K is perhaps more familiar when written as 1/w.

The user of this program characterizes the magnetic field for the problem to be solved by specifying its value on the median plane (y = 0). In particular he gives values for μ_{00}, μ_{01}, $\mu_{02} \cdots \mu_{0M}$, ν_{00}, $\nu_{01} \cdots \nu_{0M}$. The Well Tempered Five program then computes H_x, H_y and H_θ at the various coordinate values as they are needed in the solution of the equations of motion.

The highest harmonic accepted by the program is the 30th (i.e. $M = 30$ in Eqs. (2).) Any number of harmonics up to and including the 30th are permitted in a single problem.

Two methods of truncating the series expressions for H_x, H_y and H_θ are permitted by the program. In the first method the user simply specifies a number called i_{max}, then all terms in the series with $i > i_{max}$ are neglected in the field computation. In the second method the user specifies a number ϵ, then the program determines i_{max} by the requirement that the sum of the absolute magnitudes of the coefficients of the $y/1 + x$ factor for each of the three field components be less than ϵ when $i = i_{max}$; the common factor $(1 + x)^k$ is omitted in this determination and we assume $\sin m\Phi = \cos m\Phi = 1$. More precisely, the program forms for each i, starting with $i = 1$, the following summations:

$$\epsilon_{ix} = \sum_{m=0}^{M} \left\{ |\alpha_{im}| + |\beta_{im}| \right\} \frac{1}{(2i-1)!} \cdot \frac{1}{(\sqrt{K^2+N^2})^{2i-1}} ,$$

$$\epsilon_{iy} = \sum_{m=0}^{M} \left\{ |\mu_{im}| + |\nu_{im}| \right\} \frac{1}{(2i)!} \cdot \frac{1}{(\sqrt{K^2+N^2})^{2i}} , \tag{5}$$

$$\epsilon_{i\theta} = \sum_{m=0}^{M} \left\{ |\nu_{i-1,m}| + |\mu_{i-1,m}| \right\} \frac{Nm}{(2i-1)!} \cdot \frac{1}{(\sqrt{K^2+N^2})^{2i-1}} .$$

The greatest of the three quantities at each value of i, ϵ_{i_x}, ϵ_{i_y} and ϵ_{i_θ} is determined (it is called ϵ') and compared with ϵ and the first value of i encountered for which the inequality

$$\epsilon' < \epsilon \tag{6}$$

is satisfied is taken as i_{max}. The user is informed by the program, via a printed record called the initial print, of ϵ' and i_{max} determined by the

program. The quantity $\sqrt{\kappa^2+N^2}$ which appears in Eqs. (5) is a scale factor used in the field calculations, which except in this instance will not ordinarily concern the user. In either case i_{max} cannot exceed 24. If the user specifies an ϵ for which inequality (6) above cannot be satisfied for $i \leq 24$ the program will stop. (Restarting the program at this point will result in $i_{max} = 24$.)

The program will compute zero, one, or two invariants as requested by the user. Thus, if one invariant is desired the program will compute

$$K_x = \sqrt{\xi_x (x-x_f)^2 + \eta_x (x-x_f)(p_x - p_{xf}) + \zeta_x (p_x - p_{xf})^2}\ , \tag{7}$$

and

$$K_y = \sqrt{\xi_y (y-y_f)^2 + \eta_y (y-y_f)(p_y - p_{yf}) + \zeta_y (p_y - p_{yf})^2}\ .$$

If two invariants are requested the program will compute, in addition to the above, the quantities

$$K'_x = \sqrt{\xi'_x (x-x'_f)^2 + \eta'_x (x-x'_f)(p_x - p'_{xf}) + \zeta'_x (p_x - p'_{xf})^2}\ , \tag{8}$$

$$K'_y = \sqrt{\xi'_y (y-y'_f)^2 + \eta'_y (y-y'_f)(p_y - p'_{yf}) + \zeta'_y (p_y - p'_{yf})^2}\ .$$

The quantities $\xi_x, \eta_x, \zeta_x, x_f, p_{xf}, \xi_y, \eta_y, \zeta_y, y_f, p_{yf}$ and the corresponding primed quantities are specified by the user in a manner that will be described later. The values of the independent variable at which computation of the invariants is desired is specified by giving the number, N_{Ip}, of integration steps (i.e. Runge-Kutta Steps) between successive computations of the invariant, and a number, $(C_I)_o$, which gives the initial counter reading for the counter counting the number of integration steps between successive computations of the invariant. A corresponding pair of numbers N'_{Ip} and

$(C_{I'})_o$ must also be specified when a second invariant is called for by the user.

This completes the discussion of what the Well Tempered Five program does. Details of how the user requests these things to be done will now be given. Pages 6 and 7 comprise a standard agendum form which must be filled out and submitted by the user. All specifications pertinent to the solution of a problem with this program are given by making proper entry on the agendum sheets. The first column on the sheet, labeled Parameter, identifies the quantity whose value is to be specified on the corresponding line in the column headed Value. The second column, headed Address, is for the use of the key punch operator and need not concern the user. Remarks in the last column are inserted occasionally where it is thought that they might be helpful.

Entries are made by the user in the column labeled Value. Two types of numbers occur. The first eleven lines refer to parameters whose value is always an integer. To indicate this the words "FIXED POINT INTEGERS" appear in a block at the right which indicates the lines on which the integers are to be entered. Integers are written on the agendum in the standard fashion: leading zeros may be omitted, and $2^{35} - 1$ is the largest integer permitted. The other type of number is referred to as the floating point number and their locations are similarly indicated at the right by the words "FLOATING POINT NUMBERS." The value to be entered as a floating point number must first be expressed in the form

$$\text{Value} = n \times 10^{exp},$$

where $1 \leq |n| < 10$ or zero, and n may contain from 1 to 9 (inclusive) decimal digits and $|exp| \leq 37$. The quantities n and exp are then entered in the indicated portions of the Value column. The quantities n and exp are assumed

WELL TEMPERED FIVE AGENDUM (PROBLEM 46)

Parameter	Address	Value		Remarks	
I. D.	3097				
N_p	3068			The number of RK steps between prints of p_x, p_y, x, y, $N\Theta/2\pi$	FIXED POINT INTEGERS
N_e	3069			The number of RK steps to end of problem	
N_I	3070			The number of invariants: 0, 1 or 2	
N_{Ip}	3091			The number of RK steps between prints of 1st invariant	
N'_{Ip}	3092			The number of RK steps between prints of 2nd invariant	
$(C_p)_o$	3175			Counter counts RK steps modulo N_p; print p_x, etc. when C_p = zero	
$(C_e)_o$	3176			Counter counts RK steps; problem ends when $C_e = N_e$	
$(C_I)_o$	3178			Counter counts RK steps modulo N_{Ip}; print K_x, K_y when C_I = zero	
$(C'_I)_o$	3179			Counter counts RK steps modulo N'_{Ip}; print K'_x, K'_y when C'_I = zero	
i_{max}	3094			Last term in power series for H_x, H_y, H_z: $0 < i_{max} \leq 24$, need to specify only when SS3 UP	
		n	exp		
1/w	3064				FLOATING POINT NUMBERS
k	3065				
N	3066				
n_{RK}	3067			The number of RK steps per sector	
p_{xo}	3105			Initial coordinate values	
p_{yo}	3106				
x_o	3107				
y_o	3108				
$(N\Theta/2\pi)_o$	3109				
$p_{x\,max} = p_{y\,max}$	3039			(1) momentum upper bound	
$x_{max} = y_{max}$	3040			(1/2) position upper bound	
ϵ	3093			need to specify only when SS3 DOWN	
ξ_x	3071			Parameters for K_x	
η_x	3072				
ζ_x	3073			$K_x = \sqrt{\xi_x (x-x_f)^2 + \eta_x (x-x_f)(p_x - p_{xf}) + \zeta_x (p_x - p_{xf})^2}$	
x_f	3074				
p_{xf}	3075				
ξ_y	3076			Parameters for K_y	
η_y	3077			$K_y = \sqrt{\xi_y (y-y_f)^2 + \eta_y (y-y_f)(p_y - p_{yf}) + \zeta_y (p_x - p_{xf})^2}$	
ζ_y	3078				
y_f	3079				
p_{yf}	3080				
ξ_x'	3081			Parameters for K_x'	
η_x'	3082				
ζ_x'	3083			$K_x' = \sqrt{\xi_x' (x-x_f)^2 + \eta_x' (x-x_f')(p_x - p_{xf}') + \zeta_x (p_x - p_{xf}')^2}$	
x_f'	3084				
p_{xf}'	3085				
ξ_y'	3086				
η_y'	3087			Parameters for K_y'	
ζ_y'	3088			$K_y' = \sqrt{\xi_y' (y-y_f')^2 + \eta_y' (y-y_f')(p_y - p_{yf}') + \zeta_y' (p_y - p_{yf}')^2}$	
y_f'	3089				
p_{yf}'	3090				

WELL TEMPERED FIVE AGENDUM

Fourier coefficients of the median plane field

$$A_y = -(1+x)^k \sum_m \{\mu_{0,m} \cos m\Phi + \nu_{0,m} \sin m\Phi\}$$

Parameter	Address	Value n	Value exp	Parameter	Address	Value n	Value exp	
$\mu_{0,30}$	3113			$\nu_{0,30}$	3144			FLOATING POINT NUMBERS
$\mu_{0,29}$	3114			$\nu_{0,29}$	3145			
$\mu_{0,28}$	3115			$\nu_{0,28}$	3146			
$\mu_{0,27}$	3116			$\nu_{0,27}$	3147			
$\mu_{0,26}$	3117			$\nu_{0,26}$	3148			
$\mu_{0,25}$	3118			$\nu_{0,25}$	3149			
$\mu_{0,24}$	3119			$\nu_{0,24}$	3150			
$\mu_{0,23}$	3120			$\nu_{0,23}$	3151			
$\mu_{0,22}$	3121			$\nu_{0,22}$	3152			
$\mu_{0,21}$	3122			$\nu_{0,21}$	3153			
$\mu_{0,20}$	3123			$\nu_{0,20}$	3154			
$\mu_{0,19}$	3124			$\nu_{0,19}$	3155			
$\mu_{0,18}$	3125			$\nu_{0,18}$	3156			
$\mu_{0,17}$	3126			$\nu_{0,17}$	3157			
$\mu_{0,16}$	3127			$\nu_{0,16}$	3158			
$\mu_{0,15}$	3128			$\nu_{0,15}$	3159			
$\mu_{0,14}$	3129			$\nu_{0,14}$	3160			
$\mu_{0,13}$	3130			$\nu_{0,13}$	3161			
$\mu_{0,12}$	3131			$\nu_{0,12}$	3162			
$\mu_{0,11}$	3132			$\nu_{0,11}$	3163			
$\mu_{0,10}$	3133			$\nu_{0,10}$	3164			
$\mu_{0,9}$	3134			$\nu_{0,9}$	3165			
$\mu_{0,8}$	3135			$\nu_{0,8}$	3166			
$\mu_{0,7}$	3136			$\nu_{0,7}$	3167			
$\mu_{0,6}$	3137			$\nu_{0,6}$	3168			
$\mu_{0,5}$	3138			$\nu_{0,5}$	3169			
$\mu_{0,4}$	3139			$\nu_{0,4}$	3170			
$\mu_{0,3}$	3140			$\nu_{0,3}$	3171			
$\mu_{0,2}$	3141			$\nu_{0,2}$	3172			
$\mu_{0,1}$	3142			$\nu_{0,1}$	3173			
$\mu_{0,0}$	3143			$\nu_{0,0}$	3174			

SENSE SWITCH

i_{max} PLANTED	SS3 UP	
COMPUTE i_{max}	SS3 DOWN	

NOTE:

1. Value = n x 10^{exp}, where $1 \leq |n| < 10$, or zero, and n may contain from 1 to 9 decimal digits. n and exp are assumed positive unless indicated by -. exp is understood zero when blank; $|exp| \leq 37$.
2. If this is one run of a series, only those values differing from those of the preceding run need be specified, EXCEPT for PRINT COUNTER, END COUNTER, 1st INV. COUNTER and 2nd INV. COUNTER, which are set equal to zero unless specified otherwise in the agendum.
3. If this run does not belong to a series, then all quantities not specified are set equal to zero, EXCEPT for $p_{x\,max} = p_{y\,max}$ and $x_{max} = y_{max}$ which have the values given in parentheses in the remarks column when not specified in the agendum.
4. Agenda of a series should be submitted with sheets stapled together in order of running.

positive unless designated negative by -. exp is understood zero when blank. Examples below show the proper form for entry of floating point numbers on the Agendum.

			n	exp
The number	3105.67	is entered as	3.10567	3
"	.0310567	"	3.10567	-2
"	3.10567	"	3.10567	
"	-3.10567	"	-3.10567	
"	-.0310567	"	-3.10567	-2
"	-3105.67	"	-3.10567	-3
"	12345678.9	"	1.23456789	7
"	355×10^{-12}	"	3.55	-10
"	0	"	0	

Examples of the form of floating point numbers as they are to be entered on the agendum.

Although most of the parameters are readily identified by the symbol appearing on the agendum and hence need no discussion, some of them may be unfamiliar or their use not clearly understood. The parameter N_p has the same significance as in other MURA programs. It is the number of integration steps that are taken between successive prints of the five quantities p_x, p_y, x, y and $N\Theta / 2\pi$; thus, if $N_p = 32$ these five quantities will be printed at equal intervals of 32 integration steps. If it is desired to print p_x, p_y, etc., once every sector at homologous points, then one must set $N_p = n_{RK}$ (i.e. N_p = the number of Runge-Kutta steps in a sector). If it is desired to print p_x, p_y etc., once every time around the "accelerator" at the same azimuth, then one must set $N_p = n_{RK}N$. The parameter N_I specifies the number of invariants to be computed by the program and it may have one of three values, 0, 1, or 2 indicating 0, 1, or 2 invariants. N_{Ip} is the number of integration steps that are taken between successive prints of K_x and K_y; thus it bears the same relation to the printing of K_x and K_y as N_p does to the printing of p_x, p_y, etc.

Similarly, $N_{I'p}$ controls the printing of the second invariants K'_x and K'_y. In any case invariants are only computed and printed if called for via the parameter N_I. As is customary in former programs C_p is the print counter. This counter is incremented by 1 at the completion of each integration step and when it becomes equal to N_p then printing of p_x, p_y, x, y, and $N\Theta/2\pi$ occurs and C_p is reset to zero. $(C_p)_o$ is the initial value of the print counter, thus this parameter determines the "phase" at which printing takes place. For example, if it is desired to print p_x, p_y, etc., 1/4 of the distance through the sector, at successive sectors then one can set $(C_p)_o = 3/4\ n_{RK}$ and initiate computations at the beginning of the sector (i.e. $(N\Theta/2\pi)_o = 0$); alternatively one can initiate computations 1/4 of the way through a sector (i.e. $(N\Theta/2\pi)_o = 1/4$) and set $(C_p)_o = 0$. The parameter $(C_I)_o$ is the initial value of the first invariant counter and bears the same relation to the computation and printing of K_x and K_y that $(C_p)_o$ does to the printing of p_x, p_y, x, y and $N\Theta/2\pi$. In an equivalent manner $(C'_I)_o$ is related to the printing of K'_x and K'_y. The parameter i_{max} has been discussed earlier. The parameters 1/w, k and N are the familiar machine parameters. The parameter $p_{x\,max}$ is equal to the parameter $p_{y\,max}$ and this number is the upper bound on the momentum. When the magnitude of either p_x or p_y exceeds this number the computations on the problem cease and the user is notified in a characteristic fashion. The parameter x_{max} similarly describes the upper bound on x.* Unlike all other parameters on the agendum, which unless specified otherwise in the agendum are set equal

*After the agendum sheets were printed it was requested that x_{max} and y_{max} be separately settable. This feature is now in the program and the desired y_{max} is to be inserted in the margin of the agendum along side x_{max}, with address 3060. $y_{max} = 1/2$ unless specified otherwise.

to zero at the start of a problem or series of problems, $p_{x\,max} = p_{y\,max}$ is set equal to 1 and $x_{max} = y_{max}$ is set equal to 1/2 unless specified otherwise. The parameter ϵ has been discussed earlier.

On the second page of the agendum values of the Fourier coefficients for the median plane field are to be entered. The equation for the median plane field in which they appear is written above for convenience in remembering their significance.

The mode for truncating the power series for the field components H_x, H_y and H_s is specified by the setting of sense switch 3. The user must indicate by a check mark in the appropriate box which mode is to be used. If the user supplies i_{max} then a check is entered into the top box indicating that sense switch 3 is to be up. If the user supplies an ϵ then a check is entered into the bottom box indicating that sense switch 3 must be down.

As in other MURA programs a "series" of problems may be submitted by simply stapling the agendum sheets for the corresponding problems together in the desired order of running. In a series the parameter values are not reset to zero as each problem is completed and computations on the next one started. The parameters retain the values they had on the previous problem if no entry is made on the agendum. <u>There are four exceptions,</u> namely $(C_p)_o$, $(C_e)_o$, $(C_I)_o$ and $(C'_I)_o$ which are always reset to zero; if other values are desired they must be specified for each problem of the series. The sense switch 3 setting for every problem of a series must be the same.

The "search feature" is included in this program. Under the search feature successive problems of a series will be ingested by the computer and calculations will be carried out on each one until x, y, p_x, or p_y exceeds its

upper bound, then without interruption the computer will proceed to the next problem of the series. This will continue until a problem is completed, that is $C_E = N_E$, without x, y, p_x, or p_y exceeding its upper bound, at which time the computer will stop and succeeding problems of the series will be ignored. This feature is useful when exploring the phase plane for regions of stability. If this feature is desired the user must print SEARCH clearly at the top of the first agendum sheet of the series. (The search feature is put into operation by putting sense switch 6 up.)

By putting sense switch 2 up a particular problem of a series may be discontinued and computations begun on the next problem of the series without stopping the computer.* This feature is helpful in a search when the user is present at the time of computation and observes from the output that a particular run will be unstable even before one of the coordinates exceeds its upper bound.

We now turn our attention to the form in which the results of a calculation are produced by the Well Tempered Five. It will here be necessary to describe the form in which numbers are printed as it is different from that used in earlier MURA programs. In the Well Tempered Five program the numbers are printed in floating point form, with the exception of a few integers in the initial print which are printed in the conventional form. (The identifying I appears at the left of a line of integers.) A floating point number

$$\text{Number} = n \,.\, 10^{exp}$$

is printed

n exp

with the n always containing just 8 decimal digits and with the decimal point

*The computer will stop in this circumstance if sense switch 6 is also up.

between the 1st and 2nd decimal digits, counting from left to right. If n is positive the sign is omitted, and if n is negative the sign is printed. The exponent is a two digit decimal integer with the leading zero suppressed, and the sign of the exponent is printed whether positive or negative. If the floating point number is identically zero, then an eleven digit zero is printed. Examples appear below:

The number	3105.67	is printed as	31056700+ 3
"	.0310567	"	31056700- 2
"	3.10567	"	31056700+ 0
"	-3.10567	"	-31056700+ 0
"	-.0310567	"	-31056700- 2
"	-3105.67	"	-31056700+ 3
"	1234567.8	"	12345678+ 6
"	355×10^{-12}	"	35500000-10
"	0	"	00000000000

Examples of floating point numbers as they will appear on the printed output.

Before any answers are produced by the program an initial printing takes place, in which the working data for the problem about to be attacked by the computer is recorded. This initial print is on a page by itself and the numbers appear in five columns on lines numbered 0 through 26. Page 13 gives the format for this initial print. It is to be noted that line number 9 will be missing whenever sense switch 3 is up--that is, whenever i_{max} is inserted by the user. In line 24 of column 3 a quantity labelled print ctr. 2 appears. This is of no special value to the user and appears only because of a resultant convenience to the programming; it is the initial value of the counter that numbers the lines of printing of p_x, p_y, x, y and $N\Theta/2\pi$ and it should always have the value zero. For obvious reasons it is vital that this initial print be permanently kept with the problem results.

Initial Print Format

ID.	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
0	prob. #	run #			
1	$1/w$	k	N	n_{max}	
2	N_p	N_e	N_I		
3	ξ_x	η_x	ζ_x	x_f	p_{xf}
4	ξ_y	η_y	ζ_y	y_f	p_{yf}
5	ξ_x'	η_x'	ζ_x'	x_f'	p_{xf}'
6	ξ_y'	η_y'	ζ_y'	y_f'	p_{yf}'
7	N_{Ip}	N_{Ip}'			
8	i_{max}				
9	ϵ	ϵ'	this linemissing if SS3 UP		
10	$\mu_{0,30}$	$\mu_{0,29}$	$\mu_{0,28}$	$\mu_{0,27}$	$\mu_{0,26}$
11	$\mu_{0,25}$	$\mu_{0,24}$	$\mu_{0,23}$	$\mu_{0,22}$	$\mu_{0,21}$
12	$\mu_{0,20}$	$\mu_{0,19}$	$\mu_{0,18}$	$\mu_{0,17}$	$\mu_{0,16}$
13	$\mu_{0,15}$	$\mu_{0,14}$	$\mu_{0,13}$	$\mu_{0,12}$	$\mu_{0,11}$
14	$\mu_{0,10}$	$\mu_{0,9}$	$\mu_{0,8}$	$\mu_{0,7}$	$\mu_{0,0}$
15	$\mu_{0,5}$	$\mu_{0,4}$	$\mu_{0,3}$	$\mu_{0,2}$	$\mu_{0,1}$
16	$\mu_{0,0}$				
17	$\nu_{0,30}$	$\nu_{0,29}$	$\nu_{0,28}$	$\nu_{0,27}$	$\nu_{0,26}$
18	$\nu_{0,25}$	$\nu_{0,24}$	$\nu_{0,23}$	$\nu_{0,22}$	$\nu_{0,21}$
19	$\nu_{0,20}$	$\nu_{0,19}$	$\nu_{0,18}$	$\nu_{0,17}$	$\nu_{0,16}$
20	$\nu_{0,15}$	$\nu_{0,14}$	$\nu_{0,13}$	$\nu_{0,12}$	$\nu_{0,11}$
21	$\nu_{0,10}$	$\nu_{0,9}$	$\nu_{0,8}$	$\nu_{0,7}$	$\nu_{0,6}$
22	$\nu_{0,5}$	$\nu_{0,4}$	$\nu_{0,3}$	$\nu_{0,2}$	$\nu_{0,1}$
23	$\nu_{0,0}$				
24	$(C_p)_0$	$(C_e)_0$	print ctr. 2	$(C_I)_0$	$(C'_I)_0$
25	$p_{x\,max} = p_{y\,max}$	x max	y max		
26	p_{x0}	p_{y0}	x_0	y_0	$N\theta_0/2\pi$

The printed output will have two distinct forms depending on the presence or absence of invariants. When no invariants are called for, $N_I = 0$, then just the quantities p_x, p_y, x, y, and $N\Theta/2\pi$ appear in columns one through five, respectively, of each line with an index at the left of each line which simply counts the lines, starting with the first line numbered zero. A sample output of this type appears on page 15.

Line No.	p_x	p_y	x	y	$N\Theta/2\pi$
00000	-17843000- 1	00000000000	-13400000- 1	10000000- 5	00000000000
00001	-14934744- 1	17286679- 7	-13869780- 1	10002860- 5	31250000- 2
00002	-11974931- 1	25802585- 7	-14251783- 1	10009036- 5	62500000- 2
00003	-89810024- 2	34248631- 7	-14547036- 1	10017479- 5	93750000- 2
00004	-59583559- 2	40085125- 7	-14756322- 1	10028061- 5	12500000- 1
00005	-29351963- 2	31192499- 7	-14880354- 1	10038535- 5	15625000- 1
00006	20524673- 4	-88671591- 8	-14920681- 1	10042554- 5	18750000- 1
00007	28041053- 2	-94388201- 7	-14880802- 1	10029344- 5	21875000- 1
00008	52911531- 2	-23213038- 6	-14767040- 1	99850089- 6	25000000- 1
00009	73597315- 2	-41852291- 6	-14588951- 1	98949877- 6	28125000- 1

Sample Computer Output with $N_I = 0$

If one invariant is called for then the lines of print containing p_x, p_y, x, y, $N\Theta/2\pi$ will be mixed with lines of print of K_x and K_y which will appear in columns one and two, respectively. The order of printing on successive lines is in the order of monotonely non-decreasing $N\Theta/2\pi$. When p_x, p_y, x, y, $N\Theta/2\pi$ and K_x, K_y are to be printed for the same $N\Theta/2\pi$, then p_x, p_y, etc. is printed first. Lines of printing of the 1st invariant are separately numbered 10000, 10001, 10002, and so on; thus, the nth line of print of the 1st invariant is numbered 10,000 + n (n = 0, 1, 2...). Similarly if two invariants are called for, additional lines of print containing K'_x and K'_y in columns one and two, respectively, are interspersed among the lines of print of p_x, p_y, etc. and the lines containing K_x and K_y. As always the order of printing is in the order of monotonely non-decreasing $N\Theta/2\pi$. If p_x, p_y, etc., and K_x, K_y, and K'_x, K'_y are all to be printed at the same value of $N\Theta/2\pi$ then p_x, p_y, x, y, $N\Theta/2\pi$ is printed first, K_x, K_y is printed second and K'_x, K'_y is printed third. Lines of printing of the 2nd invariant are separately numbered 20000, 20001, 20002, and so on; thus the nth line of print of the 2nd invariant is numbered 20,000 + n (N = 0, 1, 2...)

Samples of outputs with just one invariant and with two invariants appear on pages 17 and 18.

Line No.	p_x or K_x	p_y or K_y	x	y	NΘ/2
00000	-17843000- 1	00000000000	-13400000- 1	10000000- 5	00000000000
10000	14144309- 1	16040573- 5			
00001	-14934744- 1	17286679- 7	-13869780- 1	10002860- 5	31250000- 2
10001	17902385- 1	10373212- 5			
00002	-11974931- 1	25802585- 7	-14251783- 1	10009036- 5	62500000- 2
10002	14285133- 1	66422058- 6			
00003	-89810024- 2	34248651- 7	-14547036- 1	10017479- 5	93750000- 2

Sample Computer Output with $N_I = 1$

Line No.	p_x or K_x or K'_x	p_y or K_y or K'_y	x	y	$N\Theta/2\pi$
00000	-17843000- 1	00000000000	-13400000- 1	10000000- 5	00000000000
10000	14144309- 1	16040573- 5			
00001	-14934744- 1	17286679- 7	-13869780- 1	10002860- 5	31250000- 2
20000	00000000000	21242983- 5			
00002	-11974931- 1	25802585- 7	-14251783- 1	10009036- 5	62500000- 2
10001	17902385- 1	10373212- 5			
00003	-89810024- 2	34248651- 7	-14547036- 1	10017479- 5	93750000- 2
20001	00000000000	90842230- 6			
00004	-59583559- 2	40085125- 7	-14756322- 1	10028061- 5	12500000- 1
10002	14285133- 1	66422058- 6			
00005	-29351963- 2	31192499- 7	-14880354- 1	10038535- 5	15625000- 1
20002	00000000000	12006727- 5			

Sample Computer Output with $N_I = 2$

When a problem comes to the end, that is to say when $C_E = N_E$, then the word END is printed below the last line of the answer sheet on the right side of the page. Whenever sense switch 2 is set up to stop a problem END will also be printed.

When p_x, p_y, x, or y exceed their bounds, as set by $p_{x\,max} = p_{y\,max}$, x_{max} and y_{max}, the computations on the problem are stopped and a characteristic number is printed in the line numbering column after the last line of answers. These numbers are

00001 for p_x overflow,
00002 for p_y overflow,
00003 for x overflow,
00004 for y overflow.

In a case of overflow the word END is not printed.

The approximate running time for a problem may be computed from the following formula,

$$\text{Running time (seconds)} = N_e \left[.01\,67\, i_{max}\,(m_{max} + 1) + 0.197\right] + .430\ (\text{no. of lines of print})$$

where m_{max} is the largest m for which μ_{om} and ν_{om} are unequal to zero (i.e. the highest harmonic).