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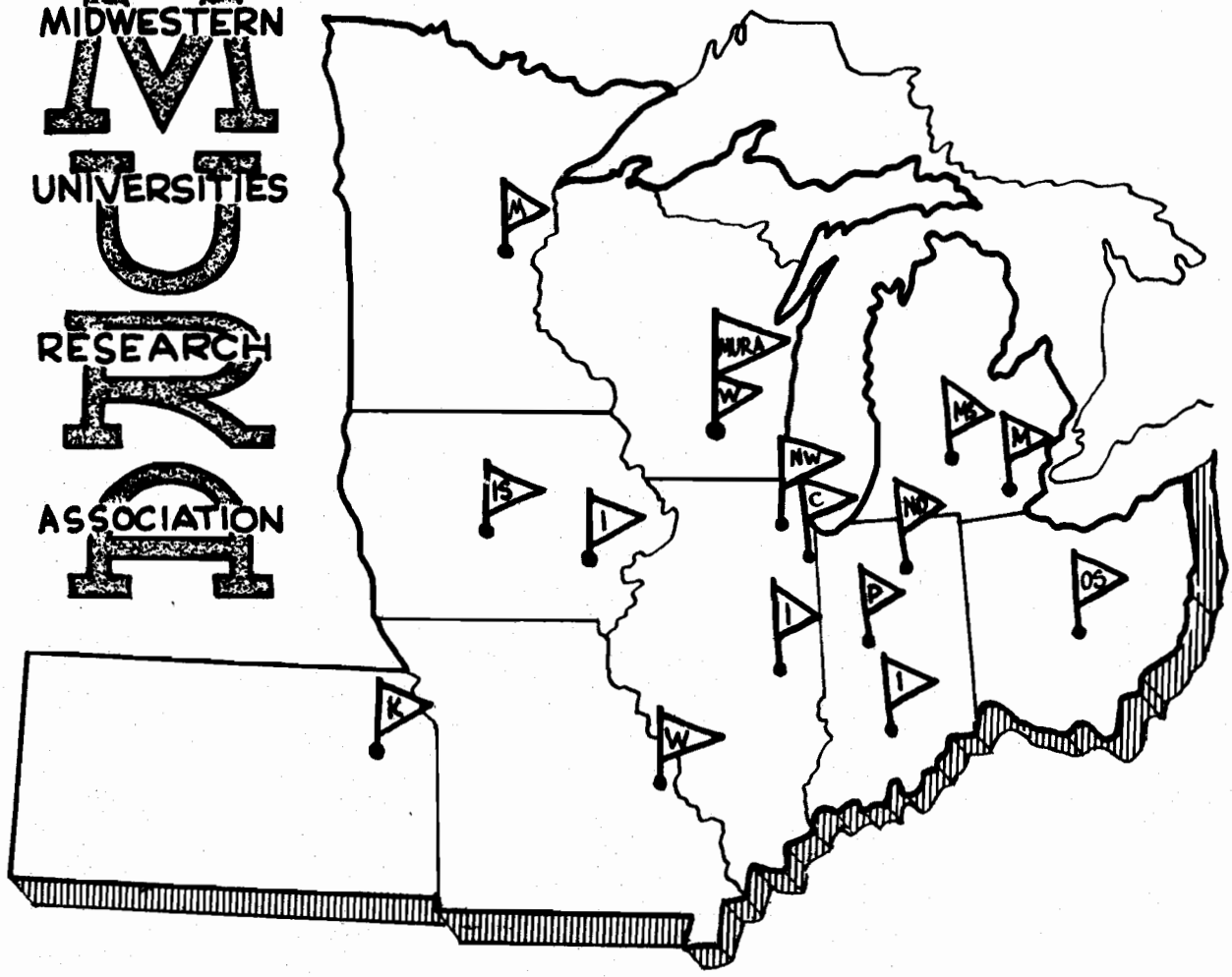

REPORT SCOFFLAW PROGRAM
(Program 30)
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SCOFFLAW PROGRAM

(Program 30)

F. C. Cole and J. N. Snyder

1. Purpose and Equations

The SCOFFLAW program integrates the coupled linear first-order differential equations

$$\begin{cases} \dfrac{dy}{ds} = y' = p \\ \dfrac{dp}{ds} = p' = F_p p + F_y y \end{cases} , \tag{1}$$

which are of course equivalent to the linear second order equation

$$y'' = F_p y' + F_y y \tag{2}$$

The functions F_p and F_y are to be given numerically.

SCOFFLAW is a refined version of COLESLAW, an ILLIAC program to integrate the equations(1). COLESLAW arose from the need to find frequencies of oscillations about equilibrium orbits known only numerically.* This is the case when it is desired to calculate motions in measured magnetic fields.

SCOFFLAW integrates (1) through an arbitrary number N_E (less than 1000) of steps by the Runge-Kutta Method with two different sets of initial conditions $y^{(1)}(0)$, $p^{(1)}(0)$, $y^{(2)}(0)$ and $p^{(2)}(0)$. It prints the values of $y^{(1)}(s)$, $p^{(1)}(s)$, $y^{(2)}(s)$, $p^{(2)}(s)$ every N_p steps, together with the Wronskian

$$W(s) = y^{(1)}(s)p^{(2)}(s) - y^{(2)}(s)p^{(1)}(s) \tag{3}$$

as a check on the integration. W has the value

$$W(s) = W(0)\, e^{-\int_0^s F_p ds} \tag{4}$$

SCOFFLAW automatically calculates $\cos \sigma$ in the special case where F_p is odd and F_y even about $s = 0$ as in a radial sector FFAG and where N_E steps constitute a half-period of (1) (so that F_p is odd and F_y even about $s_o = N_E h$.) Then

$$\cos \sigma = \frac{1}{W} \left\{ y^{(1)}(s_o)p^{(2)}(s_o) + y^{(2)}(s_o)p^{(1)}(s_o) \right\} \tag{5}$$

The program calculates the right hand side of (5) in every case, though the equality is true only in the special case above.

2. Agendum Format

One enters on the Agendum Sheet in the appropriate boxes the following quantities

1. I. D. Number (See Memo of J. N. Snyder for appropriate identification)

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*The equations of motion about such an equilibrium orbit are discussed in MURA-202 (Internal)

2. N_E - total number of R. K. steps.
3. N_p - number of R. K. steps per print.
4. m - the largest integer compatible with $2^m h < 1$.
5. $(C_p)_o$ - the initial setting of the print counter.
6. y_{max} - the largest allowable value of y subject to $|y| < 1$.
7. p_{max} - the largest allowable value of p subject to $|p| < 1$.
8. Initial conditions.

On sheets to be attached, values of $F_p/100$ and $F_y/100$ must be given. See the remarks under 2 on the agendum sheets. The restriction

$$2^m h \left[F_p p + F_y y\right] < 1$$

must be satisfied throughout.

3. Output Format

The output will have the following form

0030 (Program Number)
I. D. Number

00000	$y^{(1)}(0)$	$p^{(1)}(0)$	$y^{(2)}(0)$	$p^{(2)}(0)$	$W(0)$
00001	$y^{(1)}(h)$	$p^{(1)}(h)$	$y^{(2)}(h)$	$p^{(2)}(h)$	$W(h)$
etc.	.	. .	. .		
.	.				
.	.				
.	.				
.	.				
000..N_E	$y^{(1)}(N_E h)$	$p^{(1)}(N_E h)$	$y^{(2)}(N_E h)$	$p^{(2)}(N_E h)$	$W(N_E h)$
Decimal Marker	cos θ				

The decimal marker is used to fix the decimal point of cos θ. Count from the left the number of digits given in the decimal marker. For example, if cos θ is printed as -2000000, then cos θ = -0.200... if the D. M. is 00000 and cos θ = 2.000... if the D. M. is 00001, etc.

4. Accuracy

Several remarks about accuracy may be made. SCOFFLAW is much less accurate than other MURA programs, because the Runge-Kutta process needs values of F_p and F_y at points intermediate between s and s + h and these are found by interpolation. The interpolation as done in SCOFFLAW is better than in COLESLAW, but the errors of interpolation are much larger than the errors of the R. K. process (the factor appears to be of order 10^3). The error naturally depends on how rapidly F_p and F_y vary. With $N_E = 40$ and only the first harmonic present, comparison with the Laslett, Belford, Snyder Tables and Swansong runs (in neither of which is

there any interpolation), which agree with each other to about 1 part in 10^5, the errors in the $y^{(i)}(N_E h)$ and $p^{(i)}(N_E h)$ appear to be about 0.1% and the error in cos $\mathcal{O}$ about 0.2%. (This last should be compared with a 3% error in COLESLAW.) When a second harmonic 10% of the first was added, the errors increased by a factor 1.5. When the number of steps was doubled ($N_E = 80$) in the second case, the errors were reduced by an order of magnitude. This last result is probably somewhat fortuitous, since the interpolation is linear. Changing m has little effect on the accuracy, but round-off errors appear important. If the initial conditions are reduced, the accuracy suffers. The Wronskian appears in general to be more accurate than the functions or cos $\mathcal{O}$. It affords a valuable check on the correctness of the input data through (4).

In view of the limited accuracy attainable, care must be exercised in the use of SCOFFLAW. If questions of stability are under consideration (if $|\cos \mathcal{O}| \cong 1$), it is much wiser to use some other MURA program.

SCOFFLAW (Program 30)
Agendum Sheet

Parameter	Address	Value	Remarks	
ID	500			INTEGERS
N_e	501		40, unless set	
N_p	502		1, unless set	
m	503		2, unless set	
$(C_p)_o$	3534		0, unless set	

y_{max}	598		.9, unless set	FRACTIONS
p_{max}	599		.9, unless set	
$2^m h$	600		$\pi/80$, unless set	
$y_o^{(1)}$	601			
$p_o^{(1)}$	602			
$y_o^{(2)}$	603			
$p_o^{(2)}$	604			

Notes:

1. To omit a value, leave its line blank.

2. The F_p and F_y tables should be filled out on sheets provided and stapled in that order ahead of this sheet. Use a separate sheet for the F_p table and the F_y table. If one of these tables be identially zero, still devote a sheet to it filling in one value of zero.

3. The values given under "REMARKS" are those that obtain if no value be entered.

4. A series of runs on the same F_p and F_y tables may be made. If a series is in hand check this box vividly, fill out a sheet for each run, and staple entire series together with the pertinent F_p and F_y tables.

5. Only those parameters which change from run to run need be entered on subsequent sheets pertaining to a series. (Except that if $(C_p)_o \neq 0$, its value must be re-inserted for each run of the series.)

SCOFFLAW TABLES (Program 30)

Enter $F_{p/100}$ or $F_{y/100}$. This table is: [redacted] ($F_{p/100}$ or $F_{y/100}$)

Enter values from top to botto , then from left to right. (1000) (2000)