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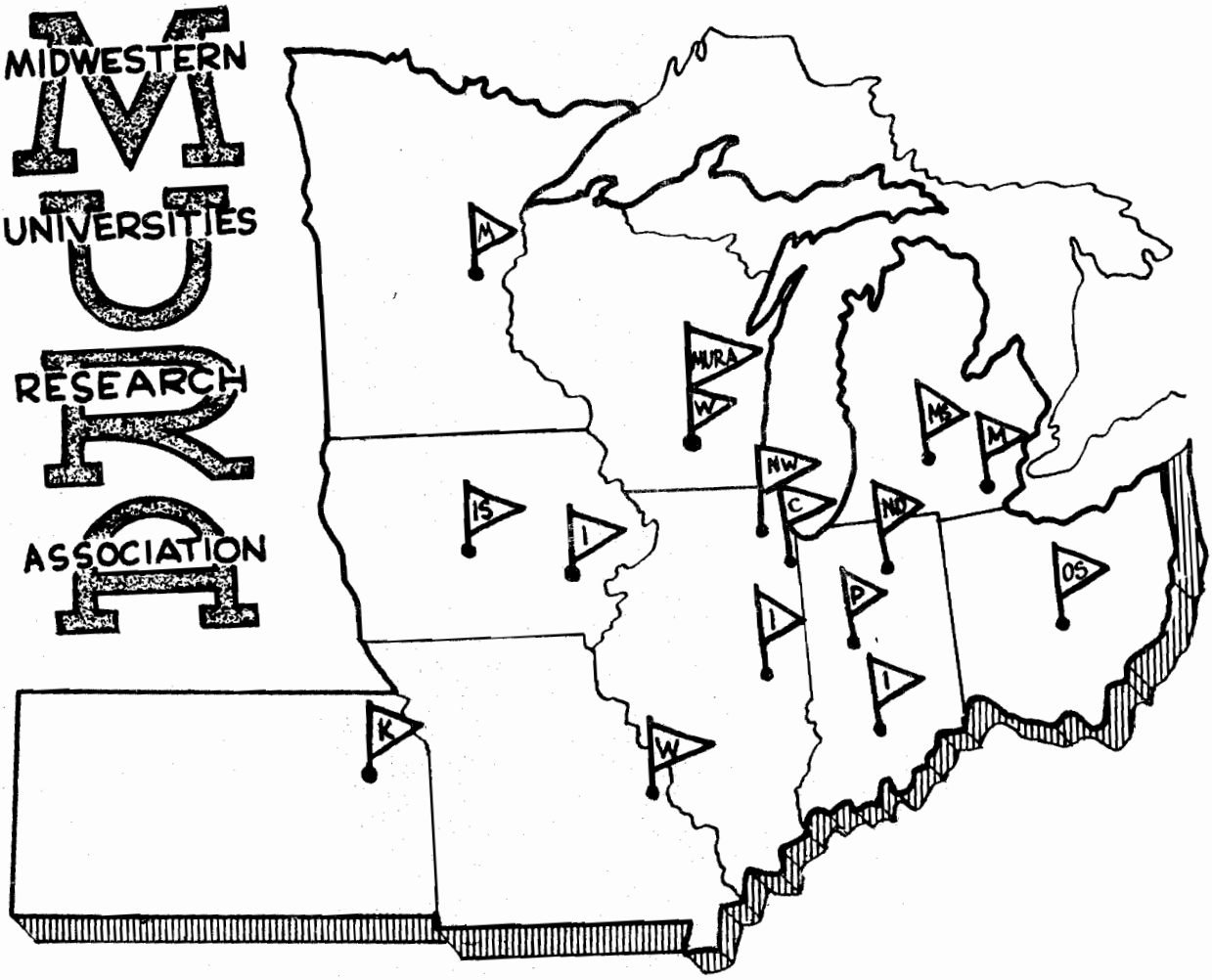


REPORT

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# STABLE EQUILIBRIUM ORBIT IN SEVERAL LARGE-SCALE SPIRAL-SECTOR FFAG ACCELERATORS

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ABSTRACT: The coefficients for a Fourier expansion of the closed equilibrium orbit are given for a number of representative spiral-sector FFAG accelerators. The FOROCYL-FORMESH I.B.M. programs formed the basis of the computations. In all the examples $k = 70$; $N = 32$; $f_{eff.}$ covered values in the neighborhood of 0.3, 0.7, and 1.2 to give $\mathcal{F}/(\omega N^2)$ close to 0.3; and meshes with $h = 1/a = 1/90$ (and $r = 1$) were used. Fourier coefficients of the normalized median-plane fields are also listed.

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## 1. PURPOSE:

FORMESH runs have been made with the MURA I.B.M. computer to depict the stable equilibrium orbit for each of several FFAG spiral-sector accelerators characterized by their appropriate FOROCYL fields. Location of the "fixed points" was aided by the FORFIXPOINT program and Fourier analysis of the equilibrium orbit obtained from the analysis of 32 coördinates per run by use of the FORANAL program.

The results, which are given below in considerable detail, may be of value in checking theoretical formulae for the equilibrium orbit.

## 2. MESH CHARACTERISTICS:

The meshes employed were selected to be representative of large-scale separated-sector structures with the R.M.S. flutter covering values in the neighborhood of 0.3, 0.7, and 1.2. The fields of the first meshes were substantially free of harmonics beyond the third, while the ones examined later in the series are somewhat more representative of magnet structures which are technically convenient to realize.

We list below the machine parameters and the characteristics of the median-plane field. It is noted that in all cases $k = 70$ ($k' = 71$), $N = 32$, and $\mathcal{F}/(\omega N^2)$ is close to 0.3. In all cases the linear dimensions of the basic mesh cell were $h = \ell = 1/90$.

The Fourier coefficients $f_m$, $g_m$ refer to an expansion of the form

$$-h_z/h_0 = 1 + \sum_m \left( f_m \sin 2\pi m\xi + g_m \cos 2\pi m\xi \right),$$

where $\xi = \frac{1}{2\pi}\left[(1/\omega)\ln(1+x) - N\theta\right]$.

| MESH | 137 | 143 | 138 | 139 |
|---|---|---|---|---|
| k | 70 | 70 | 70 | 70 |
| N | 32 | 32 | 32 | 32 |
| $1/\omega$ | 1020 | 1020 | 465 | 256 |
| $f_{eff.}$ | 0.301499 | 0.301449 | 0.730380 | 1.185854 |
| $k/N^2$ | ... | ... | .068 359 375 | ... |
| $f/(\omega N^2)$ | 0.300321 | 0.300271 | 0.331667 | 0.296464 |
| $f_1$ | -.0440937 | -.0440693 | -.0819050 | -.0225268 |
| $g_1$ | .2979767 | .2979296 | .7171618 | 1.0676998 |
| $f_2$ | -.0022501 | -.0022501 | -.0241728 | -.0191325 |
| $g_2$ | .0127472 | .0127470 | .1082149 | .4754298 |
| $f_3$ | .0000756 | .0000756 | .0029726 | .0111283 |
| $g_3$ | -.0004125 | -.0004125 | -.0110451 | -.1980807 |
| Remaining individual $f_m$ or $g_m$ | ... | ... | ... | ... |
| Cumulative $f_{eff.}$ for entries shown | .3014996 | .3014496 | .7303799 | 1.1858539 |

| MESH | 140 | 144 | 141 | 142 |
|---|---|---|---|---|
| k | 70 | 70 | 70 | 70 |
| N | 32 | 32 | 32 | 32 |
| $1/\omega$ | 1020 | 1020 | 465 | 256 |
| $f_{eff.}$ | 0.303352 | 0.302871 | 0.737947 | 1.180897 |
| $k/N^2$ | ... | ... | ... | |
| $f/(\omega N^2)$ | 0.302167 | 0.301688 | 0.335103 | 0.295224 |
| $f_1$ | -.0443720 | -.0441245 | -.0813345 | -.0226679 |
| $g_1$ | .2998469 | .2993976 | .7258317 | 1.0743880 |
| $f_2$ | -.0020967 | -.0020967 | -.0227781 | -.0178281 |
| $g_2$ | .0118781 | .0118765 | .1022612 | .4430152 |
| $f_3$ | .0000783 | .0000781 | .0031519 | .0119295 |
| $g_3$ | -.0004274 | -.0004274 | -.0115379 | -.2052207 |
| $f_4$ | .0000004 | .0000003 | .0000996 | .0007506 |
| $g_4$ | -.0000016 | -.0000017 | -.0001478 | -.0109438 |
| $f_5$ | .0000002 | .0000001 | .0000687 | .0020056 |
| $g_5$ | -.0000002 | -.0000003 | -.0000841 | -.0257826 |
| $f_6$ | | | | -.0002243 |
| $g_6$ | | | | .0026588 |
| $f_7$ | | | | .0005872 |
| $g_7$ | | | | -.0066011 |
| $f_8$ | | | | -.0001132 |
| $g_8$ | | | | .0012307 |
| $f_9$ | | | | -.0004550 |
| $g_9$ | | | | .0048464 |
| $f_{10}$ | | | | .0000976 |
| $g_{10}$ | | | | -.0010281 |
| Remaining Individual $f_m$ or $g_m$ | ... | ... | $< 4 \times 10^{-5}$ | $< 5 \times 10^{-4}$ |
| Cumulative $f_{eff.}$ for entries shown | .3033523 | .3028721 | .7379472 | 1.1808973 |

## 3. ORBIT CHARACTERISTICS:

The characteristics of the stable equilibrium orbit for the meshes described in Section 2 are listed below. The specific values for the fixed point coördinates and momenta refer to azimuths for which $N\theta=0$, mod. $2\pi$; the Fourier coefficients for the corresponding orbit refer to an expansion of the form

$$x = G_o + \sum_m (F_m \sin mN\theta + G_m \cos mN\theta).$$

| MESH | 137 | 143 | 138 | 139 |
|---|---|---|---|---|
| $x_f$ | .000256764 | .000256737 | .00047673 | .000556307 |
| $p_{x_f}$ | .00185080 | .00185003 | .0029902 | -.0005259 |
| $G_o$ | -.00004782 | -.00004781 | -.00027001 | -.00062699 |
| $F_1$ | .00003395 | .00003393 | .00002583 | -.00004660 |
| $G_1$ | .00030402 | .00030398 | .00072209 | .00108730 |
| $F_2$ | .00001155 | .00001154 | .00002757 | -.00000988 |
| $G_2$ | .00000128 | .00000128 | .00002742 | .00010917 |
| $F_3$ | .00000035 | .00000035 | .00000463 | .00002111 |
| $G_3$ | -.00000068 | -.00000068 | -.00000222 | -.00001038 |
| $F_4$ | -.00000005 | -.00000005 | -.00000027 | -.00000258 |
| $G_4$ | -.00000004 | -.00000004 | -.00000058 | -.00000303 |
| $F_5$ | | | -.00000008 | -.00000058 |
| $G_5$ | | | .00000003 | .00000008 |
| $F_6$ | | | | -.00000006 |
| $G_6$ | | | | .00000014 |
| Remaining Individual $F_m$ or $G_m$ | $<5\times10^{-9}$ | $<5\times10^{-9}$ | $\leq10^{-8}$ | $<2.4\times10^{-8}$ |
| FORANAL Run No. | 15009 | 15010 | 15011 | 15012 |

| MESH | 140 | 144 | 141 | 142 |
|---|---|---|---|---|
| $x_f$ | .00025779 | .000257515 | .00047757 | .000549087 |
| $p_{x_f}$ | .0018485 | .0018407 | .0028402 | -.00097854 |
| $G_0$ | -.00004838 | -.00004823 | -.00027562 | -.00062933 |
| $F_1$ | .00003374 | .00003355 | .00002094 | -.00005431 |
| $G_1$ | .00030581 | .00030536 | .00072953 | .00109103 |
| $F_2$ | .00001166 | .00001163 | .00002797 | -.00000873 |
| $G_2$ | .00000108 | .00000110 | .00002646 | .00010294 |
| $F_3$ | .00000032 | .00000032 | .00000447 | .00001958 |
| $G_3$ | -.00000069 | -.00000069 | -.00000226 | -.00001119 |
| $F_4$ | -.00000005 | -.00000005 | -.00000029 | -.00000320 |
| $G_4$ | -.00000004 | -.00000004 | -.00000057 | -.00000396 |
| $F_5$ | | | -.00000008 | -.00000048 |
| $G_5$ | | | .00000003 | -.00000034 |
| $F_6$ | | | | -.00000042 |
| $G_6$ | | | | -.00000017 |
| Remaining Individual $F_m$ or $G_m$ | $\leq 5\times10^{-9}$ | $< 5\times10^{-9}$ | $< 1.2\times10^{-8}$ | $< 9\times10^{-8}$ |
| FORANAL Run No. | 15013 | 15014 | 15015 | 15016 |

Although not relevant to the purposes of the present report, it may be mentioned that the frequencies of the small-amplitude radial and axial betatron oscillations can be presumed to lie close to the values $\sigma_x = 0.58\pi$ and $\sigma_y = 0.3\pi$ which were found for meshes 137 and 143.

The equilibrium orbits for two structures with purely sinusoidal field variation have been exhibited previously as a result of RIDGE RUNNER computations with the ILLIAC (MURA notes, 7 December 1955), while the theoretical treatment of such cases has been given in a report by Cole [MURA/FTC-3 (January 19, 1956)] and in earlier reports cited therein. For the sake of completeness we list below the results of Fourier analysis for these two stable closed orbits as obtained with the FORANAL program, the results being in very good agreement with those reported previously as a result of hand computations.

| RIDGE RUNNER RUN | + 3344 | + 3350 |
|---|---|---|
| $k$ | 0.8 | 160 |
| N | 5 | 40 |
| $1/\omega$ | 23 | 2302 |
| $f$ | 1/4 | 1/4 |
| $x_f$ | $-.0022\ 106_5$ | $-.0000\ 307_5$ |
| $p_{x_f}$ | -.0558 827 | $-.0072\ 452_5$ |
| $G_o$ | -.0017 291 | -.0000 2159 |
| $F_1$ | -.0105 277 | -.0001 6510 |
| $G_1$ | -.0004 078 | -.0000 0780 |
| $F_2$ | -.0003 004 | -.0000 0732 |
| $G_2$ | -.0000 688 | -.0000 0123 |
| $F_3$ | -.0000 126 | -.0000 0046 |
| $G_3$ | -.0000 046 | -.0000 0013 |
| $F_4$ | -.0000 065 | -.0000 0003 |
| $G_4$ | -.0000 003 | -.0000 0001 |
| Remaining Individual $F_m$ or $G_m$ | $< 5 \times 10^{-8}$ | $< 2 \times 10^{-8}$ |
| FORANAL RUN NO. | 15017 | 15018 |