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MURA-215

SURPRESSION OF BETATRON OSCILLATION EXCITATION (R.F. KNOCKOUT) BY THE R.F. ACCELERATING SYSTEM OF A FIXED FIELD ACCELERATOR.

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ABSTRACT: Two recent suggestions for avoiding radio frequency knock-out of a beam due to R.F. excitation of betatron oscillations are discussed. The first is the use of two cavities tipped to give electric fields at angles $\Phi_1 = \pm \Phi_2$ with the equilibrium orbit and spaced so that ζ radians of betatron oscillation can occur between them with $\tan \Phi_1 = -\frac{\nu_x}{k+1}\left(\frac{1+\cos\zeta}{\sin\zeta}\right)$. The R.F. phase must travel from one cavity to the other with the speed of the stacked particles. The second suggestion is that it is possible to turn off the cavity voltage briefly while it is passing the R.F.K.O. frequency without losing an appreciable number of particles and probably without enlarging the dimensions of the stacked beams seriously.

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Two possibilities of suppressing R.F.K.O. which grew out of recent discussions with K. R. Symon are explained here.

The first is the suggestion that with two R.F. cavities properly phased and tipped the radial betatron oscillations due to a tipped cavity (sine oscillations) and the betatron oscillations due to abrupt energy gain (cosine oscillations) might be simultaneously eliminated.

It is clear that if the electric fields of the two cavities are parallel to the orbit, sine oscillations will not be generated. If the spacing of the cavities is a half a radial betatron wave length and if the accelerating voltage is the same in each cavity when the particle passes through, then cosine oscillations would be eliminated too. The way to do this is to have the phase of the R.F. voltage propagate from the first cavity to the second cavity with the speed of the stacked particle. This calls for a fixed delay line between the drivers for the two cavities which are accelerating other particles at frequencies which would excite betatron oscillations in the stacked beam.

Some objection to this is that this pair of cavities would not accelerate slower particles of energy far below the stacking energy with the full available cavity voltages because if a slow particle passed the first cavity at full voltage, it would pass the second cavity at somewhat less than full voltage. Also spacing of the cavities a half a betatron wave length apart may be awkward if straight sections are not wanted at that spacing. The general solution at closer separations of cavities is needed to answer these objections so that cavities closer together can be used:

If τ is the time for a particle in the stacked beam to travel from the first cavity to the second, we want

$$a_1 \sin \omega_s \nu_x (t-t_1) + b_1 \cos \omega_s \nu_x (t-t_1) + a_2 \sin \omega_s \nu_x (t-t_1-\tau) + b_2 \cos \omega_s \nu_x (t-t_1-\tau) = 0 \quad (1)$$

in which a_1 and a_2 are the amplitudes of sine oscillations excited by tipped first and second cavities passed at t_1 and $t_1 + \tau$ respectively. b_1 and b_2 are amplitudes of cosines excited by the jumps in energy on passage through the cavities. ω_s is the angular velocity of a stacked particle around the accelerator. The superposition of these oscillations should be zero for all time after $t_1 + \tau$. This gives

$$\begin{aligned} a_2 &= -a_1 \cos \omega_s \nu_x \tau + b_1 \sin \omega_s \nu_x \tau \\ b_2 &= -b_1 \cos \omega_s \nu_x \tau - a_1 \sin \omega_s \nu_x \tau \end{aligned} \quad (2)$$

but

$$\begin{aligned} a_i &= (R/\nu_x)(\Delta p_i / p) \tan \Phi_i \\ b_i &= [R/(k+1)](\Delta p_i / p) \end{aligned} \quad (3)$$

where Δp_i is the gain in momentum parallel to the closed orbit on passing through the gap and Φ_i is the angle between the closed orbit and the direction of the R.F. electric field.

$$\Delta p_1 \sim \sin(2\pi f t_1 + \beta_1)$$
$$\Delta p_2 \sim \sin(2\pi f [t_1 + \tau] + \beta_2)$$

where f is the frequency of the cavity, t_1 the time of passage of the first cavity, and β_i a phase adjustment of a cavity. Equations (2) become for a_2

$$\begin{aligned} \frac{1}{\nu_x} \sin[2\pi f(t_1+\tau) + \beta_2] \tan \Phi_2 = \\ -\frac{1}{\nu_x} \sin[2\pi f t_1 + \beta_1] \tan \Phi_1 \cos \omega_s \nu_x \tau \\ + \frac{1}{k+1} \sin[2\pi f t_1 + \beta_1] \sin \omega_s \nu_x \tau \end{aligned} \quad (4)$$

AND FOR b_2

$$\begin{aligned} \frac{1}{k+1} \sin[2\pi f(t_1+\tau) + \beta_2] = \\ -\frac{1}{k+1} \sin[2\pi f t_1 + \beta_1] \cos \omega_s \nu_x \tau \\ - \frac{1}{\nu_x} \sin[2\pi f t_1 + \beta_1] \tan \Phi_1 \sin \omega_s \nu_x \tau \end{aligned}$$

If we install a fixed time delay between the R. F. cavity voltages equal to the time for a stacked particle to travel from cavity 1 to cavity 2, then $\beta_2 + 2\pi \quad f\tau = \beta_1 + 2\pi \quad m$. If the integer m = 0, then

$$\tan\Phi_2 = \frac{\nu_x}{1+k}\sin\omega_s\nu_x\tau - \tan\Phi_1\cos\omega_s\nu_x\tau$$
$$1+\cos\omega_s\nu_x\tau = -\frac{k+1}{\nu_x}\tan\Phi_1\sin\omega_s\nu_x\tau \qquad (5)$$

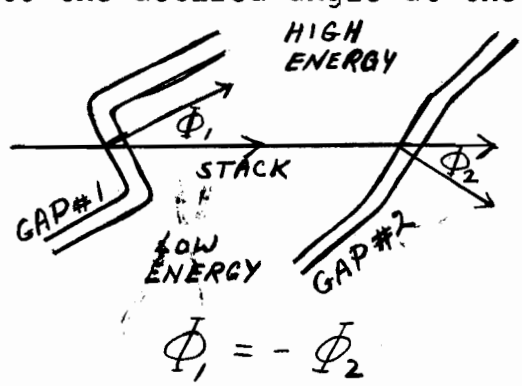
A solution of these equations (5) is obtained if the cavities are tipped at equal and opposite angles $\Phi_1 = -\Phi_2$.

Then $\tan\Phi_1 = -\tan\Phi_2 = -\frac{\nu_x}{1+k}\,\frac{1+\cos\omega_s\nu_x\tau}{\sin\omega_s\nu_x\tau}$ (6)

OR WITH THE SAME SIGN:

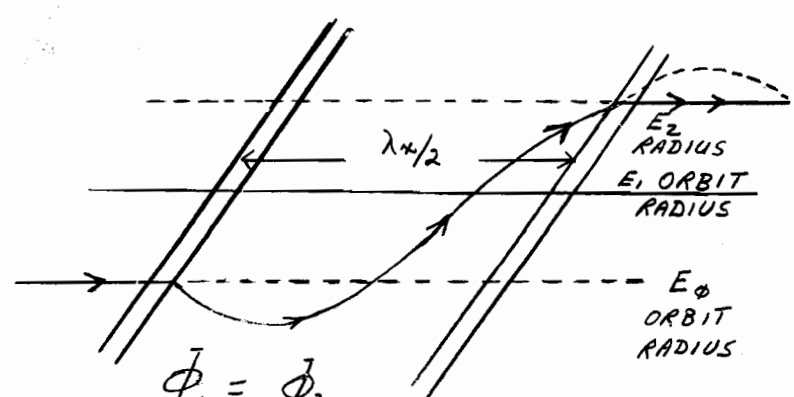
$\tan\Phi_1 = \tan\Phi_2$ or $\Phi_1 = \Phi_2$ AND $\omega_s\nu\tau = \pi$, $\Phi = 0$ OR $\Phi \neq 0$

This shows that we can use different cavity separations of amount $v\tau$ where v is the velocity of the stacked particle, if we tip the cavities oppositely at the angle Φ and let the voltage wave travel from one cavity to the other at the speed of the stacked particle. An accelerating electric field can have an adjustable angle by having a section of the gap lips adjusted to the desired angle at the stacking radius only thus:



$\Phi_1 = -\Phi_2$

$\omega_s\nu_x\tau < \pi$

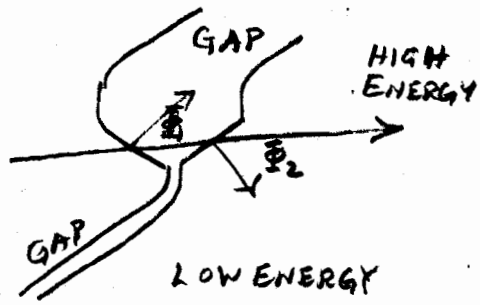


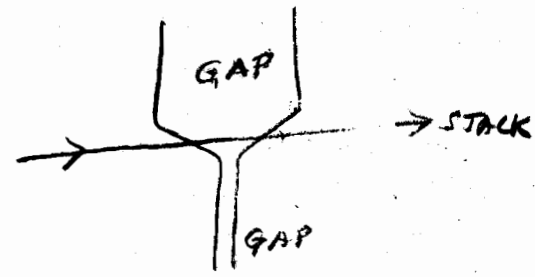
$\Phi_1 = \Phi_2$

$\omega_s\nu_x\tau = \pi$

Since the K.O. is a resonance phenomenon it might be difficult to get perfect elimination by this method. Two slanting R.F. cavities with $\Phi_1 = \Phi_2$ and 1 3/4 sectors apart but in the straight sections of the proposed 15 Bev MURA accelerators would suppress radial R.F.K.O.

In some cases it might even be possible to taper the gap widely in a single cavity to accomplish the desired result, thus

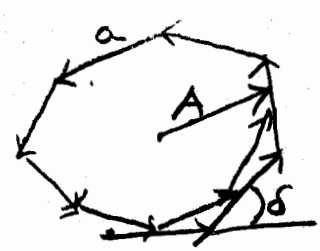




The second possibility is to avoid a large buildup of amplitude at resonance with a betatron frequency by jumping over the R.F.K.O. frequency. This can be done with very little beam loss if the width of the R.F.K.O. resonance is small compared to the bucket width. Then the R.F. voltage can be gradually decreased to zero before the R.F.K.O. band is reached and the voltage can be adiabatically built up after the frequency has been moved to the other side of the R.F.K.O. band. If the bucket was full before this operation was commenced and if the same voltage was later restored, the loss of particles would be approximately the fraction $\Delta\omega_\beta/(2\pi\Delta f)$. $\Delta\omega_\beta$ is the R.F.K.O. width in betatron oscillation circular frequency, Δf is the bucket width in R.F. frequency. If a higher voltage was restored on the cavity, all particles could be captured; but some empty phase space would then be brought into the stack in amount equal to twice the same fraction.

We calculate first $\Delta\omega_\beta$.

If the R.F. cavity gave an amplitude oscillation, a, to the orbit on each passage, then the net oscillation built up near resonance would be



$$A = \frac{Na}{2\pi}$$

where N is the number of passages to slipphase 2π . Since the cavity doesn't give full a on each passage, some average must be used. Symon has shown how to get this average and the result is $A = \frac{Na}{4\pi}$.

$$N\delta = 2\pi$$
$$\delta = (\omega_\beta - \omega_{KO})T = \Delta\omega_\beta T/2$$

where T is the revolution time of the stacked beam and ω_{KO} is the knockout circular frequency. So

$$A = \frac{a}{4\pi}\frac{2\pi}{\Delta\omega_\beta T/2} = \frac{a\,\omega_s}{2\pi\,\Delta\omega_\beta} \quad \text{AND} \tag{7}$$

$$\Delta\omega_\beta = \frac{a}{A}\frac{\omega_s}{2\pi} \tag{8}$$

A is the allowable amplitude (aperture or stability limit).

By (3) we get for a relativistic stack:

$$a = (R/\nu_x)(V\gamma/E_s)\,\text{TAN}\,\Phi$$

where γ is sin ϕ_s , the sine of the stable phase angle, E_s is the energy of the stack and V is the cavity voltage in the same units as E_s. Then

$$\Delta\omega_\beta = \frac{R}{\nu_x}\frac{V\gamma}{E_s}(\text{TAN}\,\Phi)\frac{\omega_s}{2\pi A} \quad . \tag{9}$$

To find Δ f convert the bucket width, ΔW, defined in MURA-KRS/AMS1 and supplement 1.

$$\Delta W = \Delta E/\omega = da/d\theta = a/2\pi = \frac{4}{\omega\pi}\left[\frac{2VE}{\pi h|K|}\right]^{\frac{1}{2}}\alpha_3(\gamma)$$

Where a is bucket or stable synchrotron phase space area, V is the cavity voltage, E is the <u>accelerated</u> (not stacked) particle's energy, h is the harmonic number, ω is the <u>accelerated</u> (not stacked) particles' angular velocity,

$$K = \frac{E}{\omega}\frac{d\omega}{dE} = \frac{(k+1)(E_0/E)^2 - 1}{(k+1)[1-(E_0/E)^2]}$$

where k is the field index and E_0 is the rest energy.

So for Δ f corresponding to ΔW

$$2\pi\Delta f = h\Delta\omega = h\frac{\omega}{E}K\,dE = h\omega K\Delta W/E$$

$$\text{so } 2\pi\Delta f = \frac{4\omega}{\pi}\sqrt{\frac{2h}{\pi}}\sqrt{\frac{V}{E}}\,|K|^{\frac{1}{2}}\,\alpha_3(\gamma) \tag{10}$$

Using the approximation $\alpha_3(\gamma) \cong 1-\gamma$ and (9) we find finally the ratio of widths:

$$\frac{\Delta\omega_\beta}{2\pi\Delta f} = \left(\frac{\gamma}{1-\gamma}\right)\frac{R}{A}\left(\frac{\omega_s}{\omega}\right)\frac{\sqrt{VE}}{E_s}\sqrt{\frac{\pi}{2h}}\;\frac{\tan\Phi}{8\nu_x\sqrt{K}} \tag{11}$$

As an example suppose V = .1 Mev, E = 300 Mev, E_s = 1000 Mev, R = 5 x 10^3 Cm, A = 5 Cm, h = 1, $\nu_x \cong 10$, $\omega_s/\omega \cong 2$, $K \cong 1$. $\Delta\omega_\beta/(2\pi\Delta f) \cong \Phi/8$ which means that $\Phi \sim 5^\circ$ tip of the cavity would cause a buildup to $\sim$ 5 Cm = A if we were willing to lose 1% of the accelerating beam.

Thus if similar cases are encountered in accelerators and if the cavities are lined-up better than 5°, which should be easy, we would lose less than 1% for each jump; and by some non-adiabatic gymnastics with the bucket after passing the resonance most of the spilled drops might be caught.

It would not do to have each passage of a bucket past the K.O. frequency shake up the stack by 5 cm since there would be many bunches brought up to the stack and since there may be several K.O. frequencies. This would cause the stacked beam to be too big for colliding beams. By letting the cavity voltage down and bringing it up again adiabatically this accumulation of amplitude could be

avoided, and the agitation of the stack would shrink to a small value as the oscillator recedes from the K.O. frequency.

This method could be used for vertical K.O. and for other resonances due to electric field gradients.

The following pertinent comments on this process have been made: Ohkawa points out that the R.F.K.O. band width can be wider than (9) shows by the addition of $\nu_x \Delta\omega_s$ where $\Delta\omega_s$ is the spread in angular velocities of particles of slightly differing energies in the stack. If the stack is where K is very large, then $\Delta\omega_s$ is very small and this increase would not be big.

Jones has pointed out that the cosine oscillations resulting from the momentum increase on passage through the cavity make the amplitude bigger. Symon showed that this can be included in equations (9) and (11) by replacing

$$\tan\Phi \text{ BY } \sqrt{\tan^2\Phi + \frac{1}{1+k}} \qquad (12)$$

The double cavity system discussed in the first part of this report can reduce this factor.