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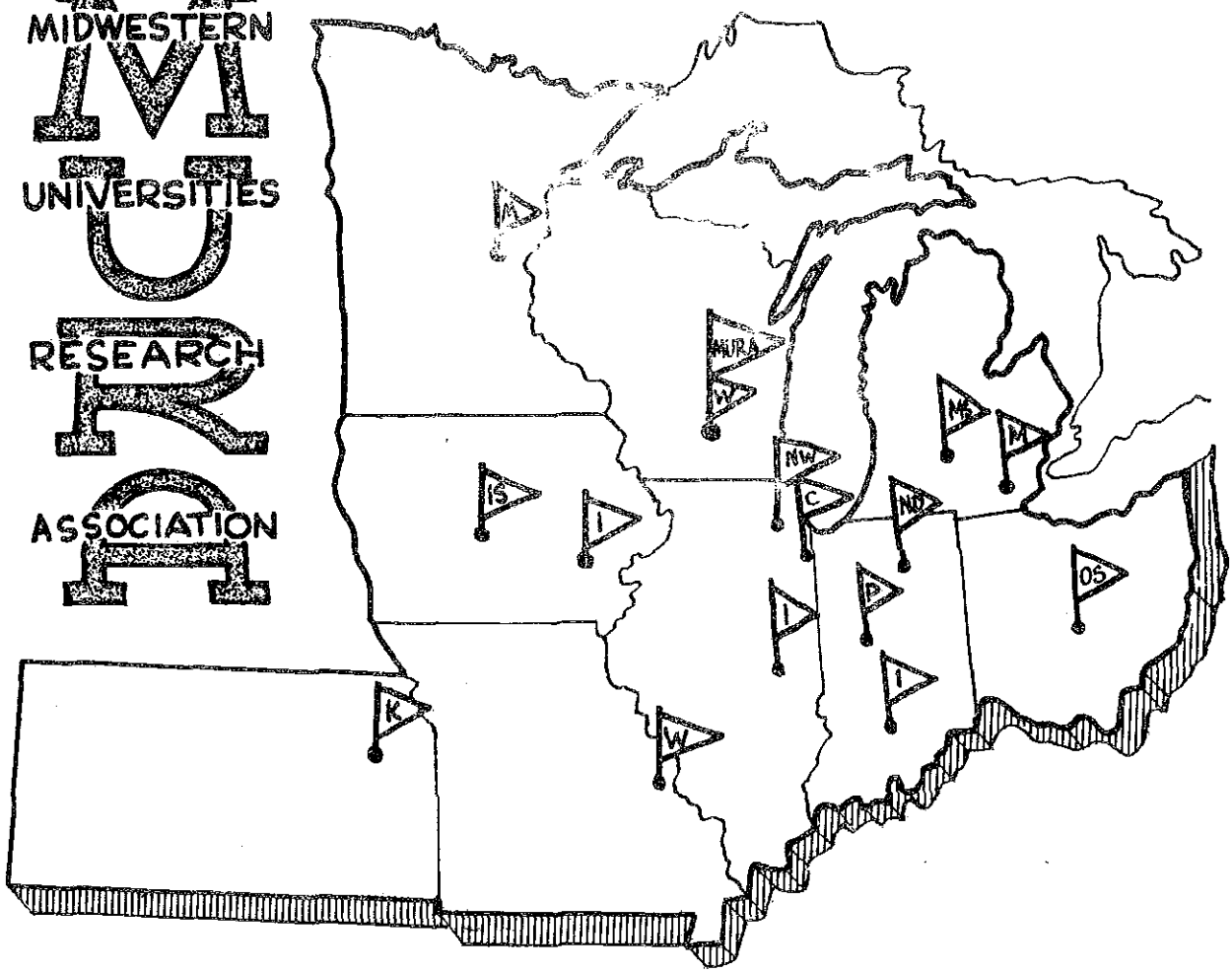


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# PRELIMINARY CONSIDERATIONS ON THE ACCELERATION OF PARTICLES BY REFLECTION OF ELECTROMAGNETIC WAVES*

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ABSTRACT: One of the acceleration methods proposed by Veksler, "the plasma linear accelerator", is discussed. A rough estimate of the magnitude of acceleration and conditions of getting a complete reflection of wave from plasma are given, but the stability of the plasma is not discussed.

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It has been pointed out by Veksler[1] that there is a possibility of accelerating particles by transfering momenta from photons to particles in "coherent Thomson scattering." When the density of particles is sufficiently high, the scattered wave will interfere with the incident wave and finally a complete reflection of wave occurs. It is a well known fact that a plasma cuts the wave off at frequencies lower than the so-called plasma frequency; this has been observed by several investigators.[2]

If a cloud of particles is put in a wave guide, the cut-off frequency guide with particles depends on the particle density and is higher than the cut-off frequency of the guide alone. When an electromagnetic wave having a frequency between the two cut-off frequencies is fed into the guide, it will be reflected by the cloud. Therefore the momentum carried by the wave must be transferred to the cloud provided that the phase and the group velocities of the wave are higher than the velocity of the cloud. In the following, a rough estimate of the necessary density of particles is discussed.

Slow Particles

First we shall consider interaction of a plane wave and a plasma using the effective dielectric constant of the plasma.[3]

A cloud of particles behaves like a dielectric medium of dielectric constant $\varepsilon$, which is given by

$$\varepsilon = 1 - \frac{4\pi n e^2}{m\omega^2}$$

$$= 1 - \frac{\omega_p^2}{\omega^2} \qquad (1)$$

1. V. I. Veksler, CERN Symposium Proceedings Vol. 7 (1956).

2. S. C. Brown, Phys. Rev. 95 634 (1954).
L. Goldstein, Priv. Communication.

3. See L. Spitzer, Physics of Fully Ionized Gas Interscience Pub. 1956;
C. Longmire, Lectures on Physics of Ionized Gases (La2055)

where

n: numbers of particles per c.c.

m: mass of a particle

$\frac{\omega}{2\pi}$: frequency of the wave

$\omega_p = \frac{4\pi n e^2}{m}$ ; plasma frequency

In this medium, the dispersion relation for a plane wave $e^{i(\omega t - kx)}$ becomes

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2} \tag{2}$$

Hence the fraction of energy reflected at the boundary between the medium and the vacuum is given by

$$R = \left| \frac{\frac{\omega}{k} - c}{\frac{\omega}{k} + c} \right|^2$$

$$= \left| \frac{1 - \sqrt{1 - \frac{\omega_p^2}{\omega^2}}}{1 + \sqrt{1 - \frac{\omega_p^2}{\omega^2}}} \right|^2 \tag{3}$$

R increases with decreasing $\omega$ and reaches 1 at $\omega = \omega_p$. With $\omega < \omega_p$, $k$ becomes imaginary and the wave decays exponentially in the medium and is completely reflected at the boundary. The mean penetrating distance is given by

$$d = \frac{c}{\sqrt{\omega_p^2 - \omega^2}} \tag{4}$$

The depth of the cloud must be chosen greater than to stop the wave.

The Debye distance $h$ is much smaller than $d$ in this case :

$$h \sim \frac{\text{Velocity of particles}}{\omega_p} \ll \frac{c}{\sqrt{\omega_p^2 - \omega^2}} \tag{5}$$

In the wave guide without particles, the dispersion relation is

$$k_0^2 = \frac{\omega_0^2}{c^2} - \frac{\omega_{0c}^2}{c^2} \tag{6}$$

where

$k_0$ : wave number in the guide

$\omega_{0c}$ : cut-off frequency of the guide, which depends on the shape and size of the guide.

The phase and the group velocities are given by

$$v_0,\ \text{phase} = \frac{\omega_0}{k_0} = c\left(1 - \frac{\omega_{0c}^2}{\omega_0^2}\right)^{-1/2}$$

$$v_0,\ \text{group} = \frac{d\omega_0}{dk_0} = c\left(1 - \frac{\omega_{0c}^2}{\omega_0^2}\right)^{1/2} \tag{7}$$

In the guide, since the fields are not uniform, the effects of particles on the wave are not represented by a simple dielectric constant. However, in order to estimate the effects roughly, we might treat the clouds as a medium with $\epsilon$ .

Then, the dispersion relation becomes

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_0^2}{c^2} - \frac{\omega_p^2}{c^2} \tag{8}$$

and the velocities are

$$v\ \text{phase} = \frac{\omega}{k} = c\left(1 - \frac{\omega_{0c}^2 + \omega_p^2}{\omega^2}\right)^{-1/2}$$

$$v\ \text{group} = \frac{d\omega}{dk} = c\left(1 - \frac{\omega_{0c}^2 + \omega_p^2}{\omega^2}\right)^{1/2} \tag{9}$$

Therefore the necessary density of particles to cut the wave off is given by

$$n > \frac{m(\omega^2 - \omega_{0c}^2)}{4\pi e^2} \tag{10}$$

The energy gain of the particles having a great enough density to reflect the wave completely is given by

$$N\frac{dT}{dt} \sim D\left(v_0\ \text{group} - v\right) S \tag{11}$$

where N: Total number of particles

T: Kinetic energy of a particle

D: Energy density of the wave

S: Cross section of the guide

$V_0$ group: Group velocity of the wave in the guide without particles

: Velocity of the particles

Since the power from the R.F. source is

$$W = v_{0g} \cdot S \cdot D, \tag{12}$$

(11) becomes

$$N \frac{dT}{dx} = W \left(1 - \frac{v}{v_{0.g}}\right)$$

$$\text{or} \quad N \frac{dT}{dx} = W \left(\frac{1}{v} - \frac{1}{v_{0g}}\right) \tag{13}$$

The following examples show the order of magnitude of the necessary densities of particles, the energy gain and so on.

① Electron cloud

- o Wave guide cut-off frequency $10^9$/sec.
- o Wave frequency $2\sqrt{3}$ $10^9$/sec.
- o Group velocity of the wave in the guide 0.5C
- o Density of electrons for cut-off $4 \times 10^9$ electrons/cm$^3$
- o $d \sim 5$ cm
- o Energy gain of an electron at $v = 0.25$ c.

  with R.F. power $1 M W$/cm$^2$ $\sim 8$ KV/cm.

② Neutral plasma consisting of protons and electrons (Assumed that electrons cut off the wave and protons are accelerated.)

- o Wave guide cut-off frequency $10^9$/sec.
- o Wave frequency $1.005 \times 10^9$/sec.
- o Group velocity 0.1 C
- o Density for cut-off $1.3 \times 10^8$ pairs/cm$^3$
- o $d \sim 5$ cm
- o Energy gain of a proton at $v = 0.05$ C

  with R.F. power 1 K$W$/cm$^2$ 34 KV/cm.

The necessary densities are rather small compared to the usual plasma densities.

Plasma

We shall consider the clouds of particles as a plasma and discuss the interaction between a plasma and a wave using the linearized equations derived by Watson[4].

4. K. M. Watson, Lectures on Physics of Ionized Gases. La-2055

A plasma having the unperturbed density and temperature distributions $n_0$ and $\Theta_0$ interacts with a wave and suffers perturbation. We separate all quantities into an unperturbed and a perturbed part and keep only the first order terms of the perturbation. Then the equations in case of uniform temperature become

$$\Box(\vec{E} - 4\pi e n_0 \vec{x}_e) = \mathrm{curl}\,[\mathrm{curl}\,(4\pi e n_0 \vec{x}_e)]$$

$$\frac{\partial^2 \vec{x}_e}{\partial t^2} = \frac{\Theta_0}{m n_0}\,\Gamma\,\mathrm{grad}\,[n_0\,\mathrm{div}\,\vec{x}_e] + \frac{\Theta_0}{m n_0}(n_0\,\mathrm{grad})\left[\frac{\vec{x}_e}{n_0}\cdot\mathrm{grad}\,n_0\right] - (\mathrm{div}\,\vec{x}_e)(\mathrm{grad}\,n_0) - \frac{e}{m}\vec{E} \tag{14}$$

where

$\Box \equiv \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$

$n_0$: $n_0(x, y, z)$

$\vec{x}_e$: displacement of electrons

$m$: rest mass of an electron

$\Gamma$: ratio of two specific heats of the system

$\Gamma \sim 1 + \frac{2}{f}$ ($f$: number of degree of freedom).

In the simplest case where $\vec{E}$ is a plane wave and $n_0$ is constant, we have the following dispersion relations

$$c^2 k^2 = \omega^2 - \omega_p^2 \quad \text{(for transverse wave)}$$
$$\Gamma \frac{\Theta_0}{m} k^2 = \omega^2 - \omega_p^2 \quad \text{(for longitudinal wave)} \tag{15}$$

$k$ becomes imaginary with $\omega < \omega_p$ in both modes. When $n_0$ depends on x (direction of the propergation of the plane wave) and only the transverse part of the wave is considered, (14) reduces to

$$\frac{\partial^2 E}{\partial x^2} + \left(\frac{\omega^2}{c^2} - \frac{\omega_p^2(x)}{c^2}\right) E = 0. \tag{16}$$

Therefore the ratio of the reflected energy to the incident energy is given by

$$R = 1 - e^{-\frac{2}{c}\int\sqrt{\omega_p^2(x) - \omega^2}\,dx}. \tag{17}$$

The magnitude of disturbances of the plasma decreases exponentially in the direction of the wave from the boundary between the plasma and vacuum. This motion of charges reacts with the wave through the magnetic field and gives a forward push similar to the effects of the tail of a fish.

Since the plasma cuts the wave off, there is no oscillatory solution for the motion of the plasma. However, since the distrubances in the plasma propagate with the group velocity of the wave in the plasma, which is zero in our case, the plasma might be able to keep its shape during the short accelerating period. The acceleration here is due to the interaction between the perturbed motion of charges and the magnetic field and is small compared to the first order perturbation. Therefore in order to discuss the motion of the plasma, it may not be sufficient to use the linearized equations.

In the wave guide, we have the equations (14) with the boundary conditions for $\vec{E}$ and $n$ at the wall. Then $\vec{E}$ and $n_e$ have a dependence on the perpendicular direction to the propagation direction and the dispersion relation will be more complicated. The main effect is presumably decreasing $\omega_p$.

Relativistic Particles

Let us consider the case where particles are moving at relativistic velocities but the deviation from the average velocity is small and non-relativistic. Then we have the non-relativistic equations for the motion of plasma in the coordinate system moving with the plasma and Maxwell equations. The equations (14) become for a uniform plasma and a plane wave

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \frac{4\pi}{c^2} n e^2 \frac{\partial^2 \vec{X}_e}{\partial t^2}$$

$$\frac{\partial^2 \vec{X}'_e}{\partial t'^2} = \frac{e}{m}\vec{E}' \qquad (18)$$

when prime denotes quantities in the moving coordinates. Assuming the plasma and the wave are moving in the x-direction we have the following relations.

$$\begin{cases} x' = \gamma(x - c\beta t), \quad y' = y, \quad z' = z \\ t' = \gamma\left(t - \frac{\beta}{c}x\right), \quad \omega' t' - k' x' = \omega t - kx \\ E_y' = \gamma(E_y - \beta H_z), \quad E_z' = \gamma(E_z + \beta H_y) \end{cases} \tag{19}$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

From (18) and (19), we get the following dispersion relations

$$k^2 = \frac{\omega^2}{c^2} - \frac{4\pi n e^2}{c^2 m \gamma^3 \left(1 - \frac{c}{\omega}\beta k\right)}$$

$$\omega = \gamma(\omega' + k' c\beta)$$

$$k = \gamma\left(k' + \frac{\omega'}{c}\beta\right), \tag{20}$$

and reflection index:

$$R' = \left|\frac{\frac{\omega'}{k'} - c}{\frac{\omega'}{k'} + c}\right|^2 = \left|\frac{1+\beta}{1-\beta} \quad \frac{\frac{\omega}{k} - c}{\frac{\omega}{k} + c}\right|^2. \tag{21}$$

$R'$ becomes 1 when $k' = 0$ $\left(k = \frac{\omega}{c}\beta\right)$.

The density which gives $R' = 1$ is given by

$$n = \frac{m\omega^2}{4\pi e^2 \gamma^3} \tag{22}$$

The energy flow of the wave available for acceleration is given by

$$N\frac{dT}{dt} = D.S. \frac{1-\beta}{1+\beta} = \frac{W}{c}\frac{1-\beta}{1+\beta} \tag{23}$$

and is much smaller than non-relativistic case because the wave must fill up the space behind the fast moving plasma.

To use the wave guide for the relativistic particles, the frequency of the wave must be far above the cut-off frequency of the guide, since the group velocity of the wave has to be greater than the relativistic velocities of the particles. Therefore the significance of using the guide is less and is only for transporting the wave efficiently.