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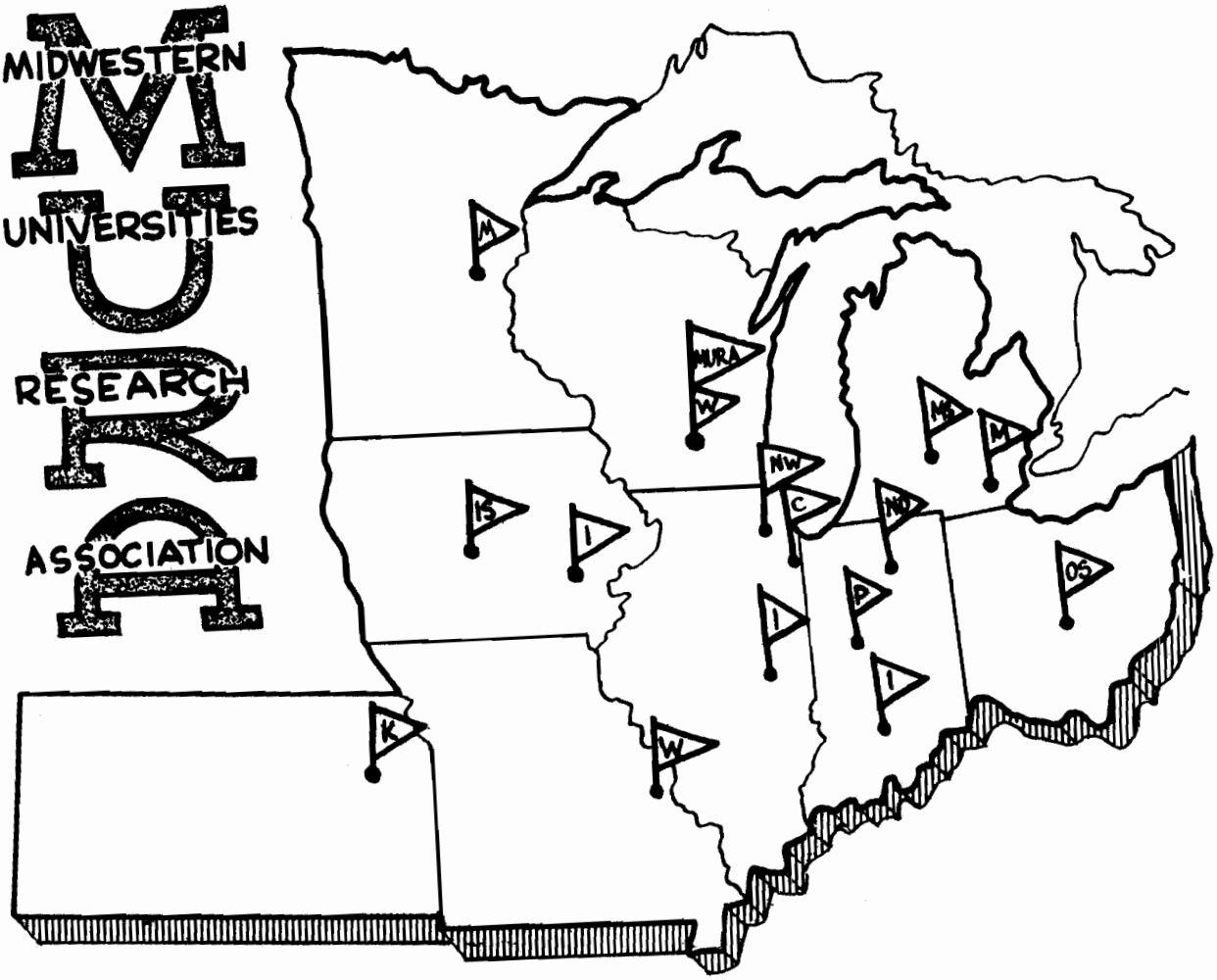

REPORT

NUMBER 213

MURA-213
Internal

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION*

COMPUTATIONAL CHECKS OF PARTICLE MOTION IN THE ILLINOIS SEPARATED-SECTOR FFAG MODEL, WITHOUT PERTURBATIONS

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January 23, 1957

ABSTRACT: As a check of the computational methods which have been employed in the analysis of FFAG structures, a comparison is made of the results obtained with the IBM 704 FOROCYL program and with the ILLIAC SCAPOCYL program for the Illinois separated-sector model. A similar structure, with magnetic fields of simplified Fourier composition, was also studied to determine the possible influence of mesh-size on the results. These studies are summarized in the present report.

* Supported by Contract AEC #AT(11-1)-384

**On leave from Iowa State College, Ames, Iowa

I. INTRODUCTION:

The properties of FFAG accelerators,[1,2,3] including those of the spirally-ridged[4] and separated-sector[5] variety, have appeared to warrant[6,7] sufficient detailed study that computational programs and electron models have been designed to permit acquisition of a detailed familiarity with the particle orbits and other aspects of the operation of such structures. A recent transfer of the computational work from the ILLIAC computer of the Graduate College of the University of Illinois to the larger-capacity IBM-704 facility of the MURA Computation Section has permitted a comparison between the results of orbit studies made with the computational programs of these respective computers.

Such results are presented below for the separated-sector design adopted for the Illinois model now under construction in Madison, together with results for a similar structure characterized by magnetic fields of a simplified Fourier composition. The latter work was undertaken to examine the effect of mesh-size on the results of the computations, mesh-storage of the magnetic fields being employed in all the IBM trajectory studies cited in this report.

II. DESCRIPTION OF STRUCTURES:

A concise description of the pole-configuration, the parameters, and a summary of ILLIAC results for the Illinois model (operating point "Ω"), as considered in the ILLIAC calculations, was given[8] by Dr. Sessler in an informal memorandum. For this structure $k' = 1.70$ ($k = 0.70$), $l/w = 6.25$, $N = 6$. With the ILLIAC SCAPOCYL program (for problem 24, with a mesh 44 units wide) the RMS flutter of the median-plane field was $f_{eff} = 1.0873$, while the IBM FOROCYL results

(for problem 61, using a mesh 88 units wide with r = 1) gave $f_{eff} = 1.0839$.

The first and most prominent Fourier components of the normalized median-plane field, as obtained with the FOROCYL program, are as follows:

m	f_m (the coeff. of $\sin 2\pi m\xi$)	g_m (the coeff. of $\cos 2\pi m\xi$)
0		1
1	1.0688	-0.1312
2	-0.0258	-0.0875
3	0.0742	-0.0357
4	-0.0087	-0.0122
f_{eff}	1.0839	

For a test of the importance of mesh effects in use of the FOROCYL program, a series of 4 additional meshes was prepared. Each of the new meshes was intended to represent substantially the same field, the median-plane field being in each case similar to that for the Illinois model save for the omission of Fourier components above the third. For meshes 128-131, $\epsilon = 7.5 \times 10^{-9}$; for mesh 61, $\epsilon = 1.0 \times 10^{-9}$.

MESH	128	129	130	131	61
SIZE	45x 6	54 x 7	90 x 12	135 x 17	88 x 15 *
f_1	1.06373	1.06416	1.06378	1.06295	1.0688
g_1	-0.12269	-0.12277	-0.12265	-0.12685	-0.1312
f_2	-0.02566	-0.02568	-0.02566	-0.02694	-0.0258
g_2	-0.09082	-0.09061	-0.09083	-0.08881	-0.0875
f_3	0.07237	0.07243	0.07244	0.07269	0.0742
g_3	-0.03663	-0.03657	-0.03652	-0.03660	-0.0357
f_4	---	---	---	---	-0.0087
g_4	---	---	---	---	-0.0122
f_{eff}	1.07799	1.07841	1.07803	1.07758	1.0839

* In this table the second dimension of the mesh refers to σ_{PRPUE} -1.

We discuss below first the intercomparisons between meshes 128-131 and, second, the results of studies pertaining most directly to the Illinois Model.

III. THE STABLE FIXED POINTS, SMALL-AMPLITUDE BETATRON FREQUENCIES, AND STABILITY LIMITS AS A FUNCTION OF MESH SIZE

The coordinates of the stable fixed point, the values of σ for small-amplitude betatron oscillations, and the stability limits found by 100-sector searches are summarized below for meshes 128-131. In every case a phase plot of the largest amplitude stable motion obtained as a result of an x-search appeared to confirm the stable character of the motion. No evidence of y-growth was found for these amplitudes.

MESH	128	129	130	131
SIZE	45 x 6	54 x 7	90 x 12	135 x 17
x_f	-.02896	-.02902	-.02903	-.02915
p_{x_f}	-.17839	-.17846	-.17858	-.17829
σ_x/π	0.466	0.465	0.468	0.466
σ_y/π			0.370	
RADIAL LIMIT*	x_o u_o	x_o u_o	x_o u_o	x_o u_o
Unstable	-.139 -.110	-.139 -.110	-.139 -.110	-.139 -.110
Stable	-.134 -.105	-.134 -.105	-.134 -.105	-.134 -.105
AXIAL LIMIT	y_o	y_o	y_o	y_o
Unstable	.038	.038	.038	.036
Stable	.036	.036	.036	.034
DURATION OF LAST UNSTABLE y_o = runs				
y_o = .038	44	58	95	52 sectors
y_o = .036	-	-	-	79 sectors

*The values of x_o are the initial value of x (at $N\theta = 0$) with $p_{x_o} = p_{x_f}$.

The maximum $|y|$ at $N\theta = \pi$ (mod. 2π), for the first "stable" run encountered in a y=search, appeared to lie in the range 0.065 to 0.069. Intercomparison of the results obtained with the various mesh sizes shows no marked dependence on mesh size and a horizontal subdivision into some 90 intervals appears suitable.

IV. COMPARISON OF FOROCYL AND SCAPOCYL RESULTS FOR THE ILLINOIS MODEL

We tabulate below the general characteristics of particle motion, as determined by the FOROCYL-FORMESH program (mesh 61) and as determined[8] with the ILLIAC SCAPOCYL-STORMESH programs (operating point "Ω"):

PROGRAM	FOROCYL-FORMESH		SCAPOCYL-STORMESH
Stable Fixed-Points			
x_f	-.02955		-.02974
p_{xf}	-.17856		-.17908
σ_x/π	0.465_6		0.466
σ_y/π	0.373_6		0.375
RADIAL LIMIT	x_o	u_o	u_o
Unstable	-.1395	-.110	-.109
Stable	-.1345	-.105	-.092
AXIAL LIMIT	y_o		y_o
Unstable	.036		.039
Stable	.034		.034
Max. $\|y\|$ at $N\theta = \pi$ (mod. 2π) for 1st stable run	.0643		.075

The threshold for y-growth appears, with either program, to be just at, or else beyond, the radial stability limit cited here.

V. DETAILED FEATURES OF MOTION IN THE ILLINOIS MODEL

Some detailed features of particle motion in the Illinois model have been studied with the FOROCYL-FORMESH programs. The stable equilibrium orbit has been obtained and Fourier analysis of it has been made. The equilibrium orbit appears to be close to sinusoidal and we quote here only the major terms of its Fourier representation:

$$x_f = -.0211 \quad -.0290 \sin N\theta \quad -.0071 \cos N\theta$$
$$-.0011 \cos 2N\theta$$
$$-.0001 \sin 3N\theta \quad -.0002 \cos 3N\theta - \ldots .$$

The elements of the matrix $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ which serves to carry the vector $\begin{pmatrix} x - x_f \\ p_x - p_{xf} \end{pmatrix}$, characterizing a small-amplitude betatron oscillation, through one sector have also been calculated by Mr. Westlund (as a function of the starting point within the sector) from FOROCYL-FORMESH computations and the results plotted. The parameter β, defined as $B/\sin\sigma$, has been introduced by E. Courant and H. Snyder[9] for convenience in treating the response of an orbit to scattering, and for the present structure β_x varies between about 0.43 and 1.29. The value of β_x at $N\theta = 0$ (mod. 2π) is found to be 1.12. It may be of interst to compare this range of variation for β_x with that found[10] with the ILLIAC RIDGE-RUNNER program for radial motion in an erstwhile proposed spirally-ridged model ($k = 0.8$, $1/w = 23.0$, $N = 5$, $f = 1/4$, with $\sigma_x/\pi = 0.571$), for which β_x ranged between limits 0.390 and 1.465.

Corresponding results for axial motion in the Illinois model are also plotted. The value of β_y at $N\theta = 0$ (mod. 2π) is found to be 0.617 and at $N\theta = \pi$ (mod. 2π) is 1.40_4.

The unstable equilibrium orbit, characterized by four unstable fixed points has also been examined for mesh 61 (representing the Illinois model) and for mesh 130. The fixed points for $N\theta = 0$, mod. 2π, are situated as follows:

Mesh 61		Mesh 130	
x_f	p_{x_f}	x_f	p_{x_f}
+.004641	-.37076	+.0053625	-.372442
-.188104	-.11310	-.1897144	-.113569
-.154209	+.12219	-.1557207	+.126255
+.175982	-.14584	+.1789962	-.141352

We list below the first several terms of a Fourier representation of these unstable equilibrium orbits. The Fourier components were obtained from analysis of the coordinate values x, at 64 points along each orbit through an interval $4(2\pi)$ in $N\theta$, by use of the FORANAL program. Although ten sine and cosine components do not appear sufficient to give an accurate description of the orbits, the next fourteen sine and cosine components are individually less than about 1.21×10^{-4} in magnitude.

For Mesh 61:

x = -.038903	-.156057 sin s	+.062361 cos s
	+.000465 sin 2s	-.032536 cos 2s
	+.015917 sin 3s	+.021400 cos 3s
	-.023153 sin 4s	-.001637 cos 4s
	-.010137 sin 5s	-.004107 cos 5s
	-.000644 sin 6s	-.001309 cos 6s
	-.000585 sin 7s	+.000351 cos 7s
	-.000119 sin 8s	+.000004 cos 8s
	-.000370 sin 9s	-.000396 cos 9s
	-.000055 sin 10s	-.000400 cos 10s

+...,

For Mesh 130:

$$
\begin{aligned}
x = -.039173 \quad & -.158047 \sin s + .063549 \cos s \\
& +.000308 \sin 2s - .033159 \cos 2s \\
& +.016108 \sin 3s + .021426 \cos 3s \\
& -.022842 \sin 4s - .001360 \cos 4s \\
& -.010307 \sin 5s - .003957 \cos 5s \\
& -.000626 \sin 6s - .001367 \cos 6s \\
& -.000650 \sin 7s + .000350 \cos 7s \\
& -.000035 \sin 8s + .000046 \cos 8s \\
& -.000476 \sin 9s - .000433 \cos 9s \\
& +.000043 \sin 10s - .000358 \cos 10s \\
& \qquad\qquad\qquad\qquad\qquad\qquad + \ldots,
\end{aligned}
$$

where $s \equiv \frac{N\theta}{4}$.

It is noted that even, as well as odd, harmonics of $N\theta/4$ are present to a material degree.

VI. 400-SECTOR RUNS OF INTERMEDIATE AXIAL AMPLITUDE:

Motivated by the unexpected comparative behavior of three ILLIAC runs NN1386-1388 made with the SCAPOCYL program, twenty two FORMESH RUNS, each of 400 sectors duration, were made with Mesh 61. None of these runs gave evidence of instability and many gave reasonably clean p_y vs. y phase plots, of which some were characterized by a rotation number close to five. The initial conditions for the axial motion are listed below, the initial values for the radial motion being $x_0 = -.0295466$ and $p_{x_0} = -.1785605$ (representing the coordinates of the fixed point when axial motion is absent).

y	p_y	y	p_y
.0330	0	.0265	.0050
.0320	0	0	.0530
.0310	0	0	.0494
.0300 783	0 *	0	.0473
.0295	0	0	.0460
.0290	0	0	.0400
.0285	0	0	- .0420
.0280	0	0	- .0490
.0270	0	0	- .0550
.0265	0		
.0252	0		
.0242 567	0 *		
.0145 540	0 *		

* Also run on Mesh 12, which is identical to Mesh 61 save that the linear dimensions of the mesh cells are twice as great as for Mesh 61.

VII. REFERENCES:

1. K. R. Symon, Phys. Rev. 98, 1152(A)(1955). One possible design of an FFAG accelerator was also suggested independantly at an earlier date by T. Ohkawa of the University of Tokyo, Tokyo, Japan (Phys. Soc. Japan, Symposium on Nuclear Physics) and by H. Snyder of the Brookhaven National Laboratory (private communication).

2. K. R. Symon, et al., Phys. Rev. 103, 1837 (1956).

3. L. J. Laslett, Science 124, 781 (1956).

4. D. W. Kerst, et al., Phys. Rev. 98, 1153(A)(1955).

5. Ref. 3, Fig. 5.

6. H. R. Crane, cited in Sci. Digest 39, #5, p. 32 (May 1956).

7. Pop. Mech. 106, #1, p. 94 (July, 1956).

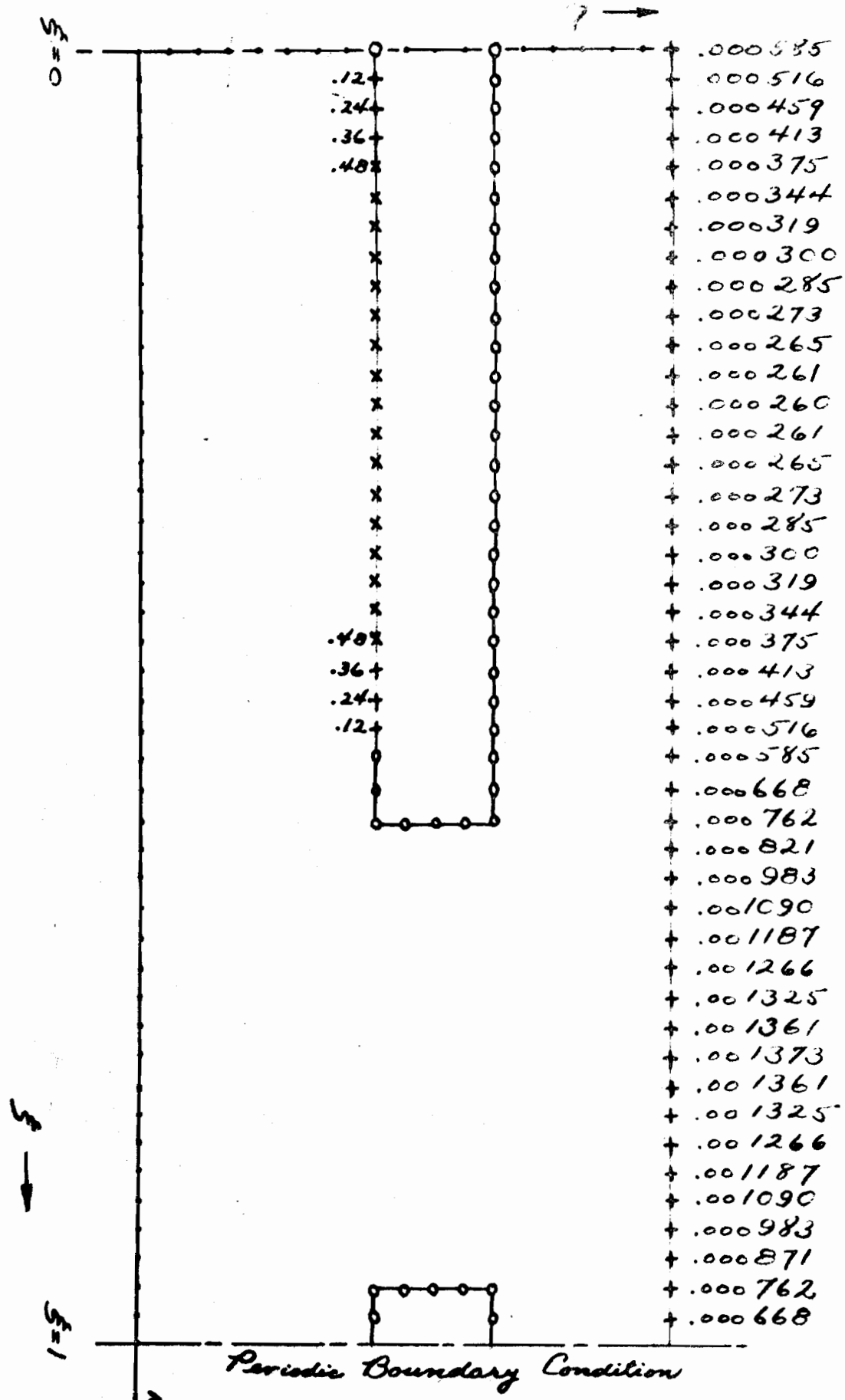
8. A. M. Sessler, "Summary of Results", informal memorandum dated September, 1956.

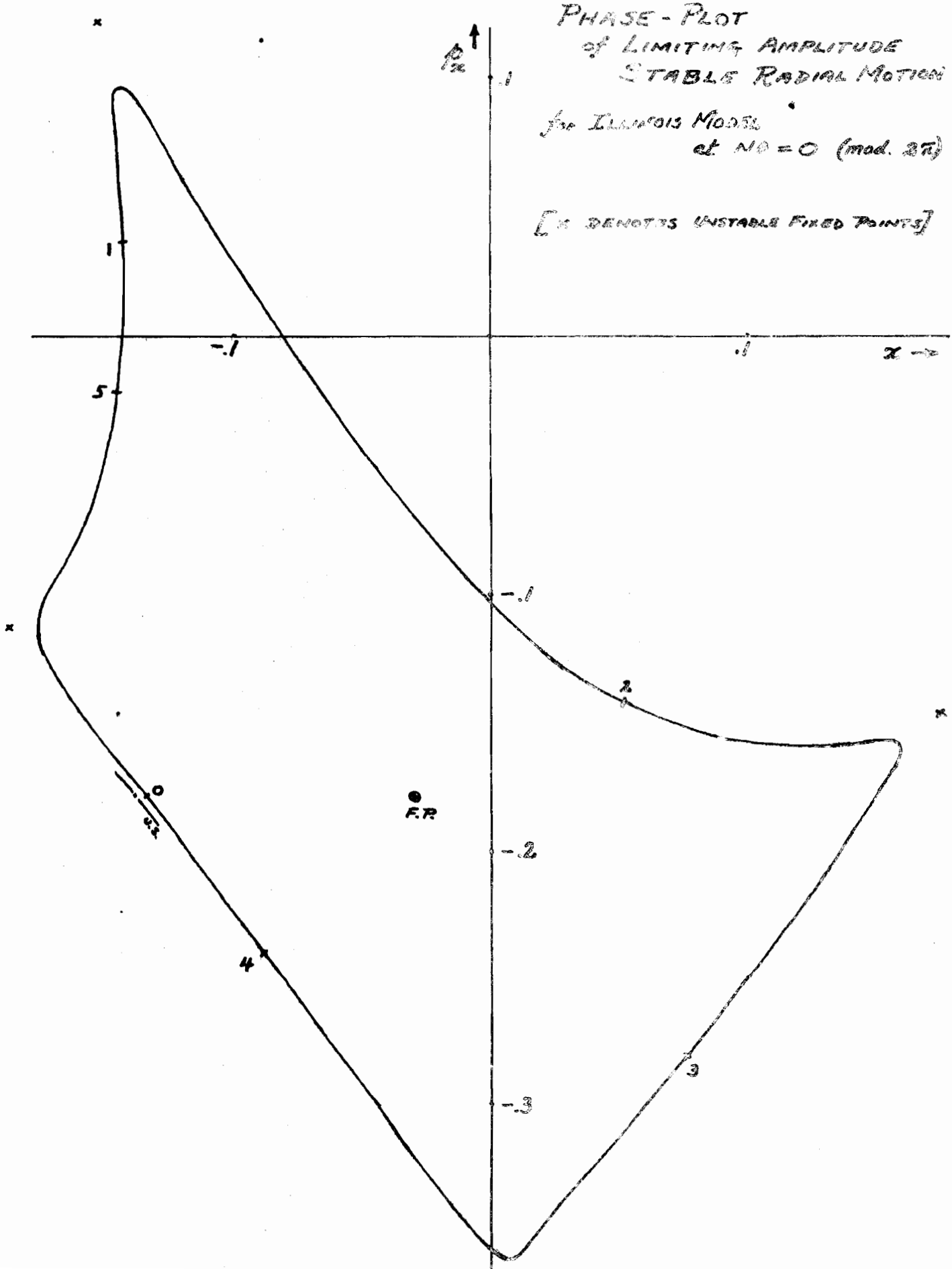
9. E. Courant and H. Snyder, Brookhaven Report EDC/HSS-1.

10. F. T. Cole and L. J. Laslett, unpublished manuscript (draft dated 21 October 1955).

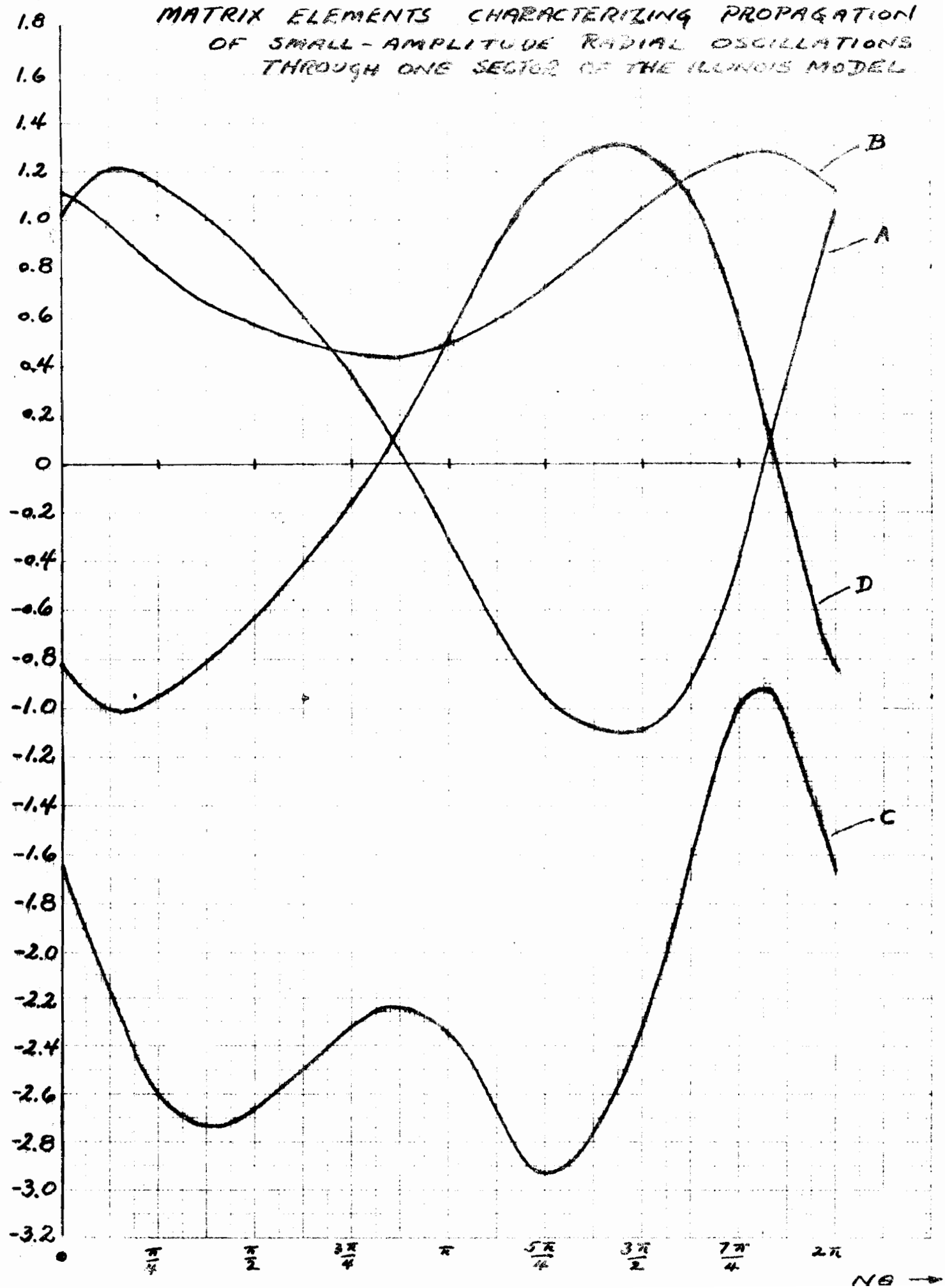
Scapocyl Boundary Values

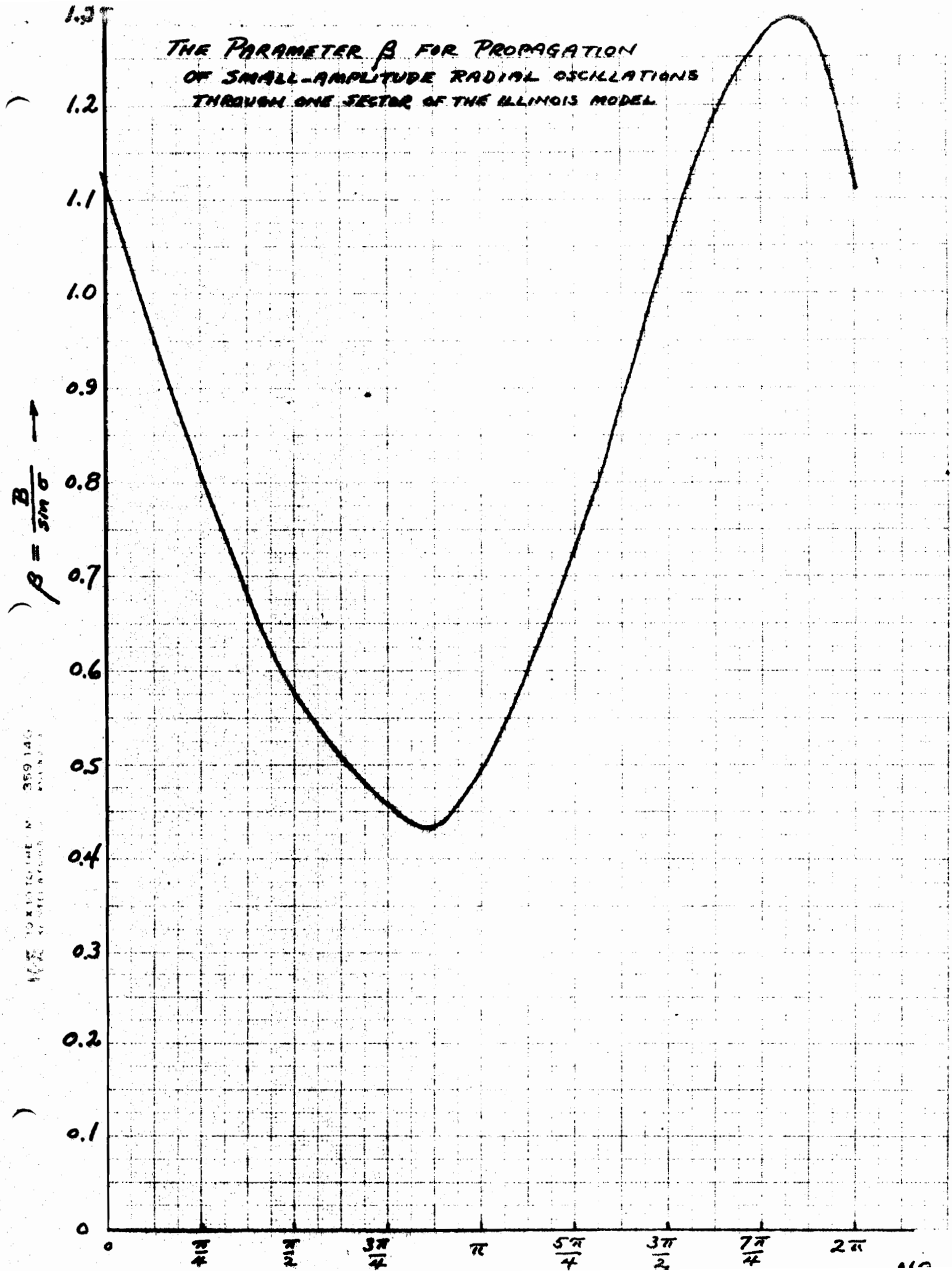
Point 2

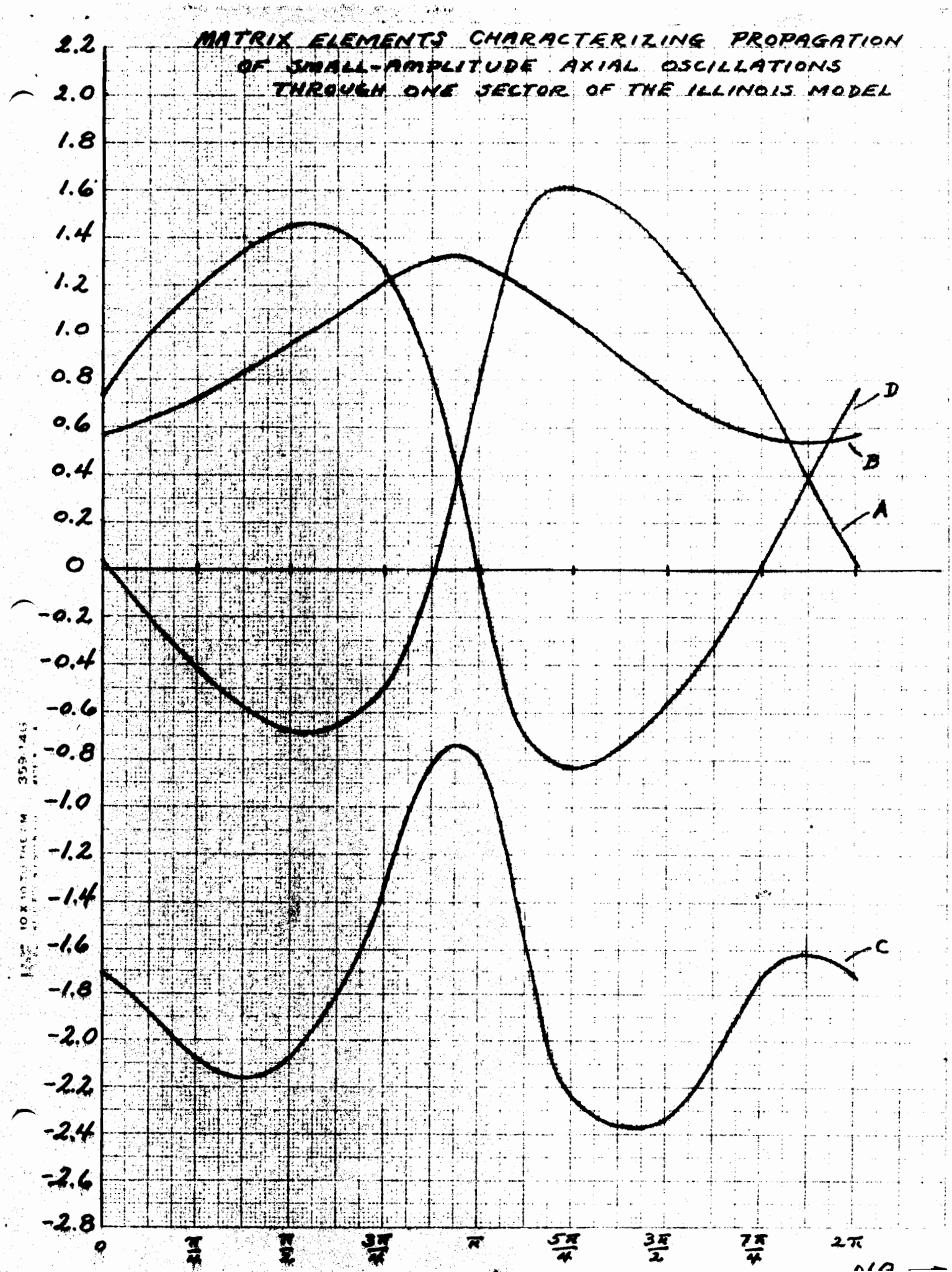



PHASE - PLOT
of LIMITING AMPLITUDE
STABLE RADIAL MOTION
for ILLINOIS MODEL
at NO = 0 (mod. 2π)
[x DENOTES UNSTABLE FIXED POINTS]
p_x
.1
-.1
.1
$x \rightarrow$
-.1
-.2
-.3
1
5
2
3
4
0
F.P.

MATRIX ELEMENTS CHARACTERIZING PROPAGATION OF SMALL-AMPLITUDE RADIAL OSCILLATIONS THROUGH ONE SECTOR OF THE ILLINOIS MODEL




THE PARAMETER β FOR PROPAGATION
OF SMALL-AMPLITUDE RADIAL OSCILLATIONS
THROUGH ONE SECTOR OF THE ILLINOIS MODEL
β = B / sin σ
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
2π


MATRIX ELEMENTS CHARACTERIZING PROPAGATION
OF SMALL-AMPLITUDE AXIAL OSCILLATIONS
THROUGH ONE SECTOR OF THE ILLINOIS MODEL
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6
-1.8
-2.0
-2.2
-2.4
-2.6
-2.8
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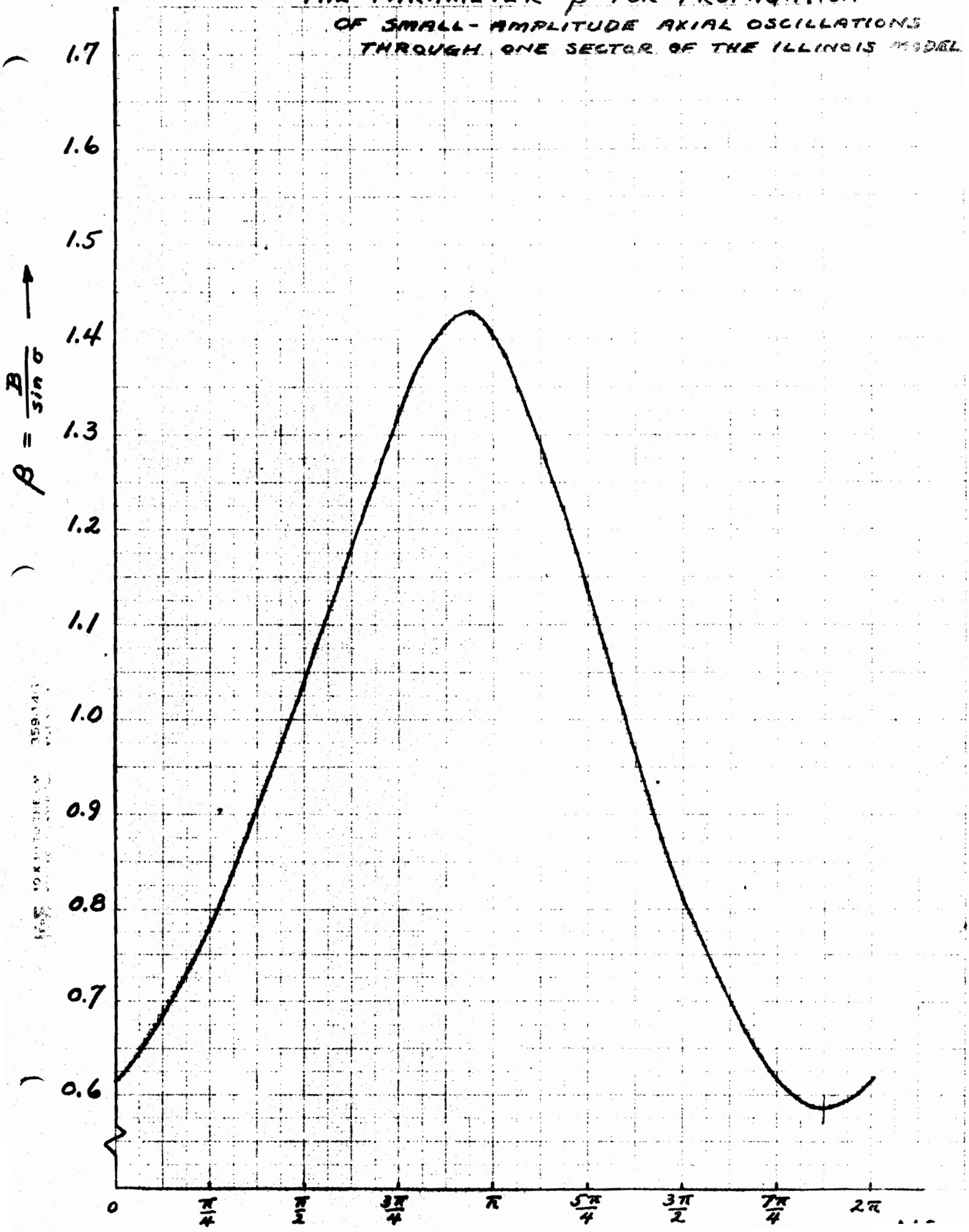
7π/4
2π
A
B
C
D

THE PARAMETER β FOR PROPAGATION OF SMALL-AMPLITUDE AXIAL OSCILLATIONS THROUGH ONE SECTOR OF THE ILLINOIS MODEL