MURA-207

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION*

ENERGY OF COLLIDING PROTON BEAMS AND ANGULAR DISTRIBUTION OF MESONS PRODUCED

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ABSTRACT: Numerical calculations were carried out for the energies available in the center of mass system of two protons of energy, 20 Bev each, colliding at various angles in the laboratory system, and the velocities of the center of mass for such collisions. Formulas for angular distributions of mesons produced in the laboratory system have been derived by assuming certain angular distributions for monoenergetic mesons in the center of mass system, corresponding to two cases of high energy proton-proton collisions (i) oblique collisions of two proton beams of same incident energy and (ii) head-on collisions of two proton beams of different incident energies. Numerical calculations for these meson angular distributions were confined to oblique collisions of two proton beams of 20 Bev each at an angle 120° and head-on collisions of two proton beams, one with 20 Bev while the other has incident energy 10 Bev in the laboratory system.

* Supported by Contract AEC #AT(11-1)-384

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I. Available Energy in the CM System and Velocity of the CM.

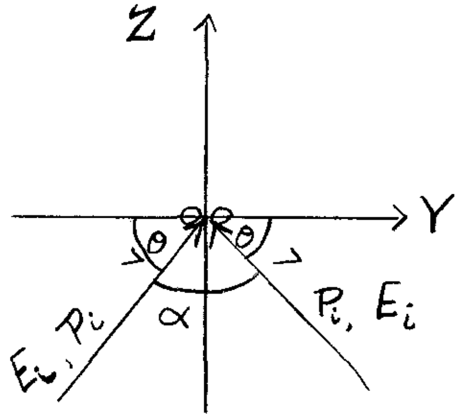


Fig. 1, Lab system

Consider two protons of same incident energy and momentum, E_i (including rest energy), P_i, colliding at an angle in the Lab system. (Fig. 1) The total energy and the total momentum are respectively:

$$E_T = 2E_i \quad (1)$$

$$|P_T| = 2P_i \sin\theta \quad (2)$$

When referred to another coordinate system with same orientation moving uniformly with velocity $\vec{v}$ these two quantities will be denoted by E_T' and P_T'. The well-known transformation is expressed in:

$$E_T = \frac{E_T' + (\vec{v}\cdot\vec{P}_T')}{\sqrt{1-\frac{v^2}{c^2}}} \quad (3)$$

$$\vec{P}_T = \vec{P}_T' + \frac{\vec{v}}{v^2}\,\frac{(\vec{v}\cdot\vec{P}_T')\left(1-\sqrt{1-\frac{v^2}{c^2}}\right) + E_T'\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (4)$$

If one chooses center of mass system as the above-mentioned moving coordinate system, $\vec{P}_T' = 0$. From the transformation relations one obtains,

the energy in CM system, $E_T' = \sqrt{1-\frac{v_{cm}^2}{c^2}}\,E_T$, (5) where

the velocity of CM is $\vec{v}_{cM} = \frac{c^2\vec{P}_T}{E_T}$. (6)

By substitution of (1) and (2), and through the relation $E_i^2 = p_i^2c^2 + m_0^2c^4$ where m_0 is the rest mass of a proton,

(6) gives $$|\vec{v}_{cM}| = c\sin\theta\sqrt{1-\frac{m_0^2c^4}{E_i^2}} \quad (7)$$

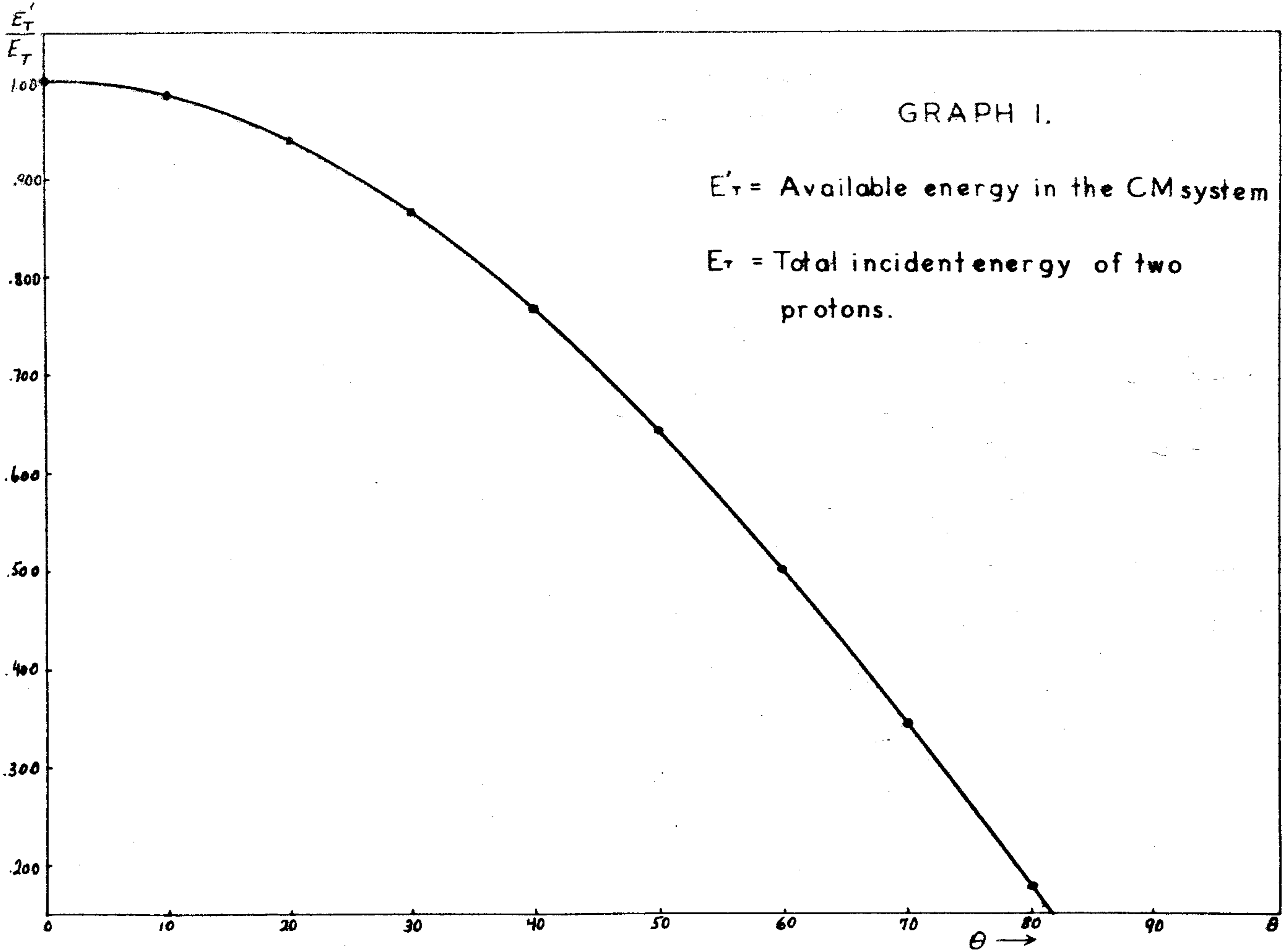
when $m_0c^2 \ll E_i$.

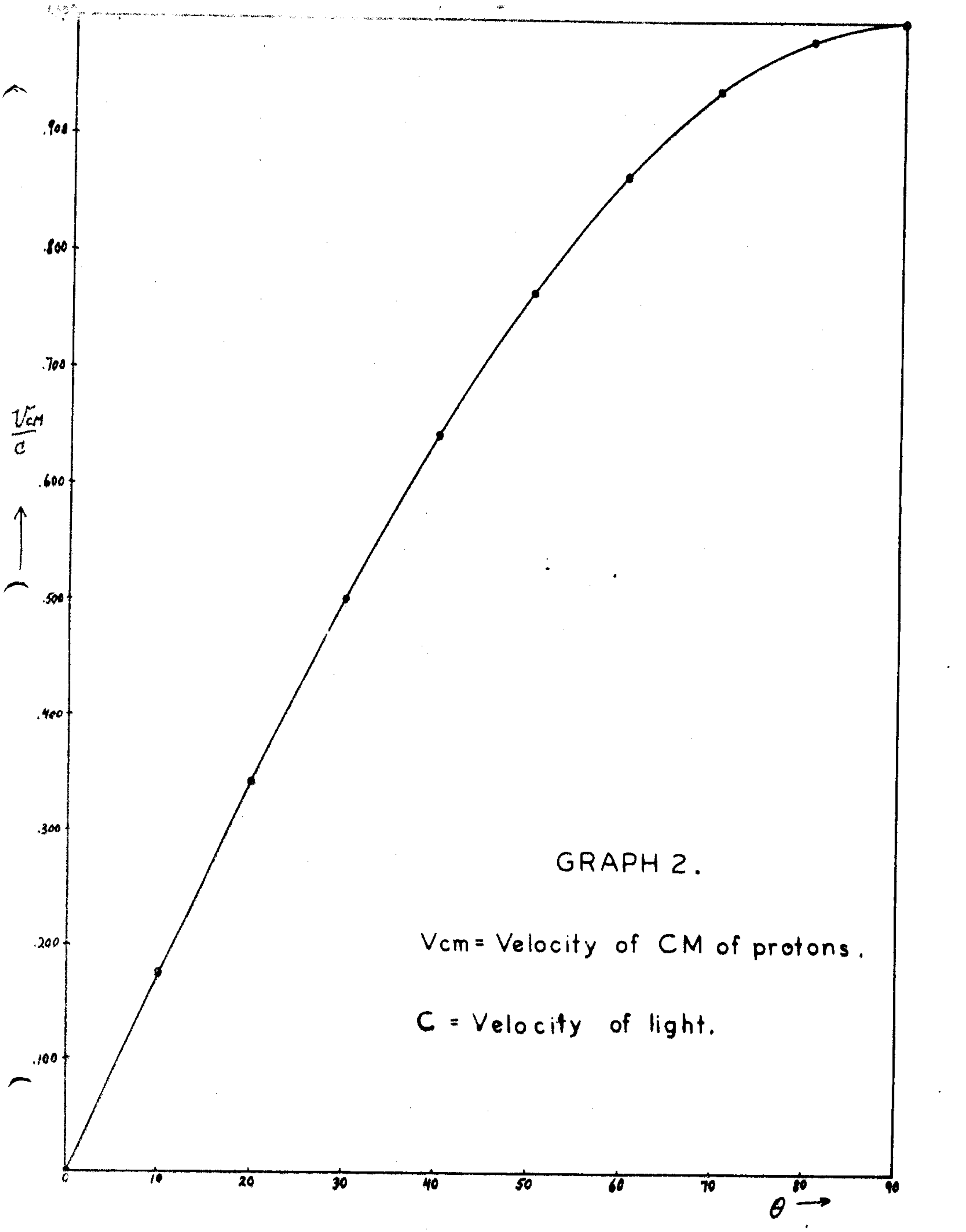
Then from (5) $E_T' = 2\sqrt{E_i^2 \cos^2\theta + m_0^2 C^4 \sin^2\theta}$

$$= 2 E_i \sqrt{\cos^2\theta + \frac{m_0^2 C^4}{E_i^2}\sin^2\theta} \qquad (8)$$

if $m_0 C^2 \ll E_i$, $E_T' \cong 2 E_i \cos\theta$

Graph I gives a plot of $\frac{E_T'}{E_T}$ against θ and Graph II a plot of $\frac{v_{CM}}{c}$ against θ for E_i = 20 Bev in both plots.


GRAPH I.
E'T = Available energy in the CM system
ET = Total incident energy of two protons.
ET'/ET
1.00
.900
.800
.700
.600
.500
.400
.300
.200
0
10
20
30
40
50
60
70
80
90
θ



GRAPH 2.

Vcm = Velocity of CM of protons.

C = Velocity of light.

II. Angular Distributions of Mesons:

We have assumed three kinds of angular distributions* for monoenergetic mesons in the CM system, namely: (a) $\cos^2\bar{\theta}$ for high energy mesons (e.g. mesons of energy around 2 Bev. say, (b) $\frac{1}{\sin\bar{\theta}}$ also for high energy mesons where $\bar{\theta}$ refers to the angle a moving meson makes with the incident direction of one of the colliding protons in the CM system, and (c) isotropic distribution for low energy mesons (e.g. mesons of energy around 500 Mev). The corresponding angular distributions of mesons in the Lab system for two cases of proton-proton collisions

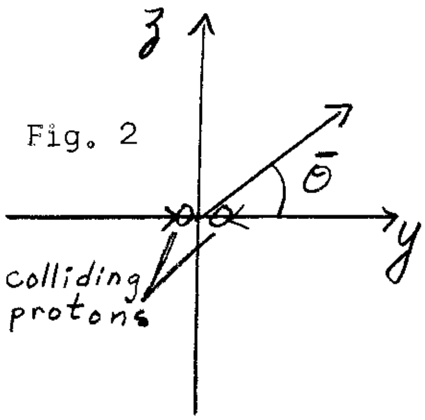


Fig. 2

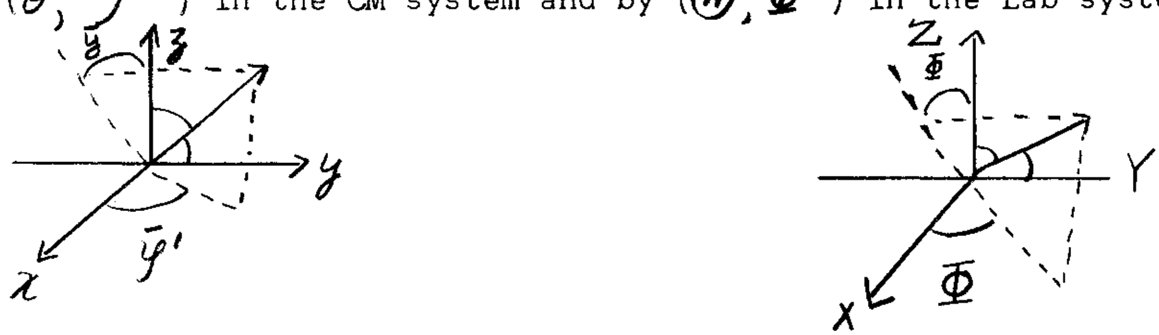
CM system

(i) oblique collisions of two protons of same incident energy and
(ii) head-on collisions of two protons of different incident energies
will be derived in the following sections.

Case (i) Oblique Collisions:

(a) $\cos^2\bar{\theta}$ high-energy meson distribution.

Let the direction of a moving meason be specified by $(\bar{\theta}, \bar{\varphi})$ in the CM system and by (Θ, Φ) in the Lab system.



CM system

Lab system

Fig. 3

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* The author is indebted to Dr. W. D. Walker, University of Wisconsin for his comments on the assumed angular distributions of mesons.

To find the corresponding angular distribution in the Lab system, the direct way would be to use the relation between the differential cross-sections $\sigma(\bar{\theta}, \bar{\varphi})$, $\sigma(\Theta, \Phi)$ in the CM and the Lab systems, i.e.

$$\sigma(\bar{\theta}, \bar{\varphi}) \sin\bar{\theta}\, d\bar{\theta}\, d\bar{\varphi} = \sigma(\Theta \Phi) \sin\Theta\, d\Theta\, d\Phi \tag{9}$$

where $\sigma(\bar{\theta}, \bar{\varphi})$ = Const. $\cos^2\bar{\theta}$, and to get the Jacobian $J\left[\frac{\partial(\cos\bar{\theta}, \bar{\varphi})}{\partial(\cos\Theta \Phi)}\right]$ involved in the transformation. However, due to the fact that the center of mass of the protons moves along the positive Z direction, this direct procedure seems to be mathematically complicated. We shall proceed in an indirect way as follows: first convert $\sigma(\bar{\theta}, \bar{\varphi})$ to $\sigma(\bar{\theta}', \bar{\varphi}')$ in the CM system, using z axis as a new polar axis; then transform $\sigma(\bar{\theta}', \bar{\varphi}')$ to $\sigma(\Theta', \Phi')$ in the Lab system with Z axis as a polar axis; finally convert $\sigma(\Theta', \Phi')$ to $\sigma(\Theta, \Phi)$ in the same Lab system with Y as a polar axis.

<u>first step</u>: Referring to Fig. 3 one easily finds the relations

$$\cos\bar{\theta} = \sin\bar{\theta}' \sin\bar{\varphi}' \tag{10}$$

$$\tan\bar{\varphi} = \tan\bar{\theta}' \cos\bar{\varphi}' \tag{11}$$

From $\sigma(\bar{\theta}, \bar{\varphi}) \sin\bar{\theta}\, d\bar{\theta}\, d\bar{\varphi} = \sigma(\bar{\theta}', \bar{\varphi}') \sin\bar{\theta}'\, d\bar{\theta}'\, d\bar{\varphi}'$

$$\sigma(\bar{\theta}', \bar{\varphi}') = \sigma(\bar{\theta}, \bar{\varphi})\, J\left(\frac{\partial(\cos\bar{\theta}, \bar{\varphi})}{\partial(\cos\bar{\theta}', \bar{\varphi}')}\right) \tag{12}$$

using relations (10) and (11),

$$J\left(\frac{\partial(\cos\bar{\theta}, \bar{\varphi})}{\partial(\cos\bar{\theta}', \bar{\varphi}')}\right) = 1$$

hence

$$\sigma(\bar{\theta}', \bar{\varphi}') = \text{Const} \sin^2\bar{\theta}' \sin^2\bar{\varphi}' \tag{13}$$

<u>second step</u>: Referring again to Fig. 3, one has, for mesons having same energy in CM system,

$$\sigma(\Theta', \Phi') = \sigma(\bar{\theta}', \bar{\varphi}')\, J\left(\frac{\cos\bar{\theta}', \bar{\varphi}'}{\cos\Theta', \Phi'}\right) \tag{14}$$

To get the relations between $(\bar{\theta}', \bar{\varphi}')$ and (Θ', Φ'), as usual, one first finds the connections between the velocity components of a meson in the CM and the Lab systems. Let $\vec{u}(u \sin\Theta' \cos\Phi', u \sin\Theta' \sin\Phi', u\cos\Theta')$ and $\underline{\vec{u}}(\underline{u}\sin\bar{\theta}'\cos\bar{\varphi}', \underline{u}\sin\bar{\theta}'\sin\bar{\varphi}', \underline{u}\cos\bar{\theta}')$ be the velocity vectors in both systems. Since the center of mass moves along positive Z,

$$u \sin\Theta' \cos\Phi' = \frac{\underline{u}\sin\bar{\theta}'\cos\bar{\varphi}'\sqrt{1-\beta^2}}{1+\frac{\underline{u}\cos\bar{\theta}' v_{CM}}{c^2}} \qquad \text{where } \beta = \frac{v_{CM}}{c}$$

$$u \sin\Theta' \sin\Phi' = \frac{\underline{u}\sin\bar{\theta}'\sin\bar{\varphi}'\sqrt{1-\beta^2}}{1+\frac{\underline{u}\cos\bar{\theta}' v_{CM}}{c^2}}$$

$$u \cos\Theta' = \frac{\underline{u}\cos\bar{\theta}' + v_{CM}}{1+\frac{\underline{u}\cos\bar{\theta}' v_{CM}}{c^2}}$$

From these,

$$\tan\Theta' = \frac{\underline{u}\sin\bar{\theta}'\sqrt{1-\beta^2}}{\underline{u}\cos\bar{\theta}' + v_{CM}} \tag{15}$$

$$\tan\Phi' = \tan\bar{\varphi}' \tag{16}$$

The relation (16) expresses the obvious fact that the azimuthal angle is invariant since the CM system moves only along the Z axis.

Rewriting (15) in the form

$$\alpha = \gamma\tan\Theta' = \frac{\sin\bar{\theta}'}{\cos\bar{\theta}' + \delta} \qquad \text{where } \gamma = \frac{1}{\sqrt{1-\beta^2}} \text{ and } \delta = \frac{v_{CM}}{\underline{u}}$$

and solving for $\cos\bar{\theta}'$ one obtains

$$\cos\bar{\theta}' = \frac{-\delta\alpha^2 \pm \sqrt{1-(\delta^2-1)\alpha^2}}{\alpha^2+1} \tag{17}$$

In (17), if $\delta < 1$ *, i.e., $v_{CM} < \underline{u}$, the plus sign should be used for $\Theta' < 90°$, and minus sign for $\Theta' > 90°$. For $\delta > 1$ i.e. $v_{CM} > \underline{u}$, Θ' will have a limiting value $< 90°$, which is given by $\tan\Theta' = \frac{1}{\gamma\sqrt{\delta^2-1}}$ and, moreover there will be two values of $\bar{\theta}'$ in the CM system corresponding to one angle Θ' in the Lab system. As the calculations reported here have been confined to the first case of $\delta < 1$, we shall essentially deal with this case.

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* See Marshak "Meson Physics" Pp 299-300, McGraw-Hill Book Co.(1952)

when $\delta < 1$, the Jacobian in (14) becomes

$$\text{for } \Theta' < 90^\circ;\ J\left(\frac{d(\cos\bar{\theta}')}{d(\cos\Theta')}\right) = \frac{\delta^2\left[\delta+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\cos^3\Theta'(\alpha^2+1)^2\sqrt{1-(\delta^2-1)\alpha^2}} \tag{18}$$

$$\text{for } \Theta' > 90^\circ\ J\left(\frac{d\cos\bar{\theta}'}{d\cos\Theta'}\right) = \frac{-\delta^2\left[\delta-\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\cos^3\Theta'(\alpha^2+1)^2\sqrt{1-(\delta^2-1)\alpha^2}}$$

Through (13), (16), (17) and (18), (14) gives,

$$\text{for } \Theta' < 90^\circ;\ \sigma(\Theta',\Phi') = \text{Const}\,\sin^2\Phi'\,\frac{\delta^2\left[\delta+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2\left[(\alpha^2+1)^2-\left(-\delta\alpha^2+\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{\cos^3\Theta'(\alpha^2+1)^4\sqrt{1-(\delta^2-1)\alpha^2}} \tag{19}$$

$$\text{for } \Theta' > 90^\circ;\ \sigma(\Theta',\Phi') = -\text{Const}\,\sin^2\Phi'\,\frac{\delta^2\left[\delta-\sqrt{1-(\delta^2-1)\alpha^2}\right]^2\left[(\alpha^2+1)^2-\left(-\delta\alpha^2-\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{\cos^3\Theta'(\alpha^2+1)^4\sqrt{1-(\delta^2-1)\alpha^2}}$$

<u>final</u> <u>step</u>: As a final step, we shall convert $\sigma(\Theta',\Phi')$ to $\sigma(\Theta,\Phi)$ in the same Lab system. The transformation here is an inverse of the one involved in the first step. With the relations:

$$\cos\Theta' = \sin\Theta\cos\Phi \tag{20}$$
$$\tan\Phi' = \cot\Theta\csc\Phi$$

one finds

$$J\left(\frac{\partial(\cos\Theta',\Phi')}{\partial(\cos\Theta,\Phi)}\right) = 1 \tag{21}$$

Hence $\sigma(\Theta,\Phi) = \sigma(\Theta',\Phi')$ (22)

Because of the two different expressions for $\sigma(\Theta',\Phi')$ in (19), one has to be careful to get the right expression for $\sigma(\Theta,\Phi)$. It is seen from the relation (20) that for the whole range of Θ, when $\Phi < 90^\circ$, $\Theta' < 90^\circ$, and when $\Phi > 90^\circ$, $\Theta' > 90^\circ$. For the case $\Phi = 90^\circ$, $\Theta' = 90^\circ$ (as seen also clear from Fig. 3). Thus we have obtained the following different expressions for $\sigma(\Theta,\Phi)$ corresponding to different values of Φ:

particularly

$$\Phi < 90^\circ;\ \sigma(\Theta,\Phi)^* = \frac{1}{1+\tan^2\Theta\sin^2\Phi}\,\frac{\delta^2\left[\delta+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2\left[(\alpha^2+1)^2-\left(-\delta\alpha^2+\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{\cos^3\Phi\sin^3\Theta(\alpha^2+1)^4\sqrt{1-(\delta^2-1)\alpha^2}} \tag{23}$$

$$\Phi > 90^\circ;\ \sigma(\Theta,\Phi)^* = -\frac{1}{1+\tan^2\Theta\sin^2\Phi}\,\frac{\delta^2\left[\delta-\sqrt{1-(\delta^2-1)\alpha^2}\right]^2\left[(\alpha^2+1)^2-\left(-\delta\alpha^2-\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{\cos^3\Phi\sin^3\Theta(\alpha^2+1)^4\sqrt{1-(\delta^2-1)\alpha^2}}$$

where $\alpha^2 = \delta^2(\csc^2\Theta\sec^2\Phi - 1)$

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* Constant coefficients in $\sigma(\Theta,\Phi)$ are omitted

(1) For $\Phi = 0$,

$$\sigma(\Theta,0) = \frac{\gamma^2\left[\delta+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2\left[(\alpha^2+1)^2-\left(-\delta\alpha^2+\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{\sin^3\Theta\,(\alpha^2+1)^4\sqrt{1-(\delta^2-1)\alpha^2}} \qquad (24)$$

(2) For $\Phi = 180°$

$$\sigma(\Theta,180°) = \frac{\gamma^2\left[\delta-\sqrt{1-(\delta^2-1)\alpha^2}\right]^2\left[(\alpha^2+1)^2-\left(-\delta\alpha^2-\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{\sin^3\Theta\,(\alpha^2+1)^4\sqrt{1-(\delta^2-1)\alpha^2}} \qquad (25)$$

In both (24) and (25)

$$\alpha^2 = \gamma^2 \cot^2\Theta .$$

These two expressions (23) and (24) give the Θ -distribution in the YZ plane.

(3) For $\Phi = 45°$,

$$\sigma(\Theta,45°) = \frac{2\sqrt{2}}{1+\frac{1}{2}\tan^2\Theta}\;\frac{\gamma^2\left[\delta+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2\left[(\alpha^2+1)^2-\left(-\delta\alpha^2+\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{\sin^3\Theta\,(\alpha^2+1)^4\sqrt{1-(\delta^2-1)\alpha^2}} \qquad (26)$$

(4) For $\Phi = 135°$

$$\sigma(\Theta,135°) = \frac{2\sqrt{2}}{1+\frac{1}{2}\tan^2\Theta}\;\frac{\gamma^2\left[\delta-\sqrt{1-(\delta^2-1)\alpha^2}\right]^2\left[(\alpha^2+1)^2-\left(-\delta\alpha^2-\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{\sin^3\Theta\,(\alpha^2+1)^4\sqrt{1-(\delta^2-1)\alpha^2}} \qquad (27)$$

In both (26) and (27)

$$\alpha^2 = \gamma^2(2\csc^2\Theta - 1)$$

(26) and (27) give the Θ -distribution in a plane which makes an angle of

45° with the YZ plane.

(5) For $\Phi = 90°$

$$\sigma(\Theta,90°) = \frac{(1-\delta^2)^{3/2}}{\gamma}\cos^2\Theta \qquad (28)$$

(6) $\frac{1}{\sin\bar{\Theta}}$ high energy meson distribution.

The procedure in transforming this angular distribution in CM system to the corresponding one in the Lab system is the same as before.

We found the following expressions for $\sigma(\Theta,\Phi)$ in the Lab system:

$\Phi < 90°$,

$$\sigma(\Theta,\Phi) = \frac{1}{\sqrt{\frac{1-\left[(\alpha^2+1)^2-\left(-\delta\alpha^2+\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{(1+\tan^2\Theta\sin^2\Phi)(\alpha^2+1)^2}}}\;\frac{\gamma^2\left[\delta+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\cos^3\Phi\,\sin^3\Theta\,(\alpha^2+1)^2\sqrt{1-(\delta^2-1)\alpha^2}} \qquad (29)$$

$\Phi > 90°$

$$\sigma(\Theta,\Phi) = \frac{1}{\sqrt{1-\dfrac{(\alpha^2+1)^2-\left(-\delta\alpha^2-\sqrt{1-(\delta^2-1)\alpha^2}\right)^2}{(1+\tan^2\Theta\sin^2\Phi)(\alpha^2+1)^2}}}\;\frac{\gamma^2\left[\delta-\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\cos^3\Phi\sin^3\Theta(\alpha^2+1)^2\sqrt{1-(\delta^2-1)\alpha^2}}$$

where $\alpha^2 = \gamma^2(\csc^2\Theta\sec^2\Phi - 1)$

In particular,

(1) $\Phi = 0°$

$$\sigma(\Theta,0°) = \frac{\gamma^2\left[\delta+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\sin^3\Theta(\alpha^2+1)\sqrt{1-(\delta^2-1)\alpha^2}\sqrt{\left[-\delta\alpha^2+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}} \tag{30}$$

(2) $\Phi = 180°$

$$\sigma(\Theta,180°) = \frac{\gamma^2\left[\delta-\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\sin^3\Theta(\alpha^2+1)\sqrt{1-(\delta^2-1)\alpha^2}\sqrt{\left[-\delta\alpha^2-\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}} \tag{31}$$

In both (30) and (31)

$$\alpha = \gamma\cot\Theta.$$

(3) $\Phi = 45°$

$$\sigma(\Theta,45°) = \frac{2\sqrt{2}}{\sqrt{\left\{(\alpha^2+1)^2-\dfrac{\left[(\alpha^2+1)^2-\left(-\delta\alpha^2+\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{1+\frac{1}{2}\tan^2\Theta}\right\}}}\;\frac{\gamma^2\left[\delta+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\sin^3\Theta(\alpha^2+1)\sqrt{1-(\delta^2-1)\alpha^2}} \tag{32}$$

(4) $\Phi = 135°$

$$\sigma(\Theta,135°) = \frac{2\sqrt{2}}{\sqrt{\left\{(\alpha^2+1)^2-\dfrac{\left[(\alpha^2+1)^2-\left(-\delta\alpha^2-\sqrt{1-(\delta^2-1)\alpha^2}\right)^2\right]}{1+\frac{1}{2}\tan^2\Theta}\right\}}}\;\frac{\gamma^2\left[\delta-\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\sin^3\Theta(\alpha^2+1)\sqrt{1-(\delta^2-1)\alpha^2}} \tag{33}$$

In (32) and (33)

$$\alpha^2 = \gamma^2(2\csc^2\Theta - 1)$$

(5) $\Phi = 90°$

$$\sigma(\Theta,90°) = \frac{\sqrt{1-\delta^2}}{\gamma\sqrt{1-(1-\delta^2)\cos^2\Theta}} \tag{34}$$

(c) isotropic distribution for low-energy mesons.

Since the distribution here is assumed isotropic in the CM system, it is preferable to use the z-axis in the CM system and the Z-axis in the Lab system as polar axes.

Then $\sigma(\bar{\theta}',\bar{\varphi}') = \text{Const.}$ in the CM system.

The corresponding angular distributions in the Lab system will be, by (16) and (18)

for $\Theta' < 90°$;

$$\sigma(\Theta') = \frac{\gamma^2\left[\delta+\sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\cos^3\Theta'(\alpha^2+1)^2\sqrt{1-(\delta^2-1)\alpha^2}} \tag{35}$$

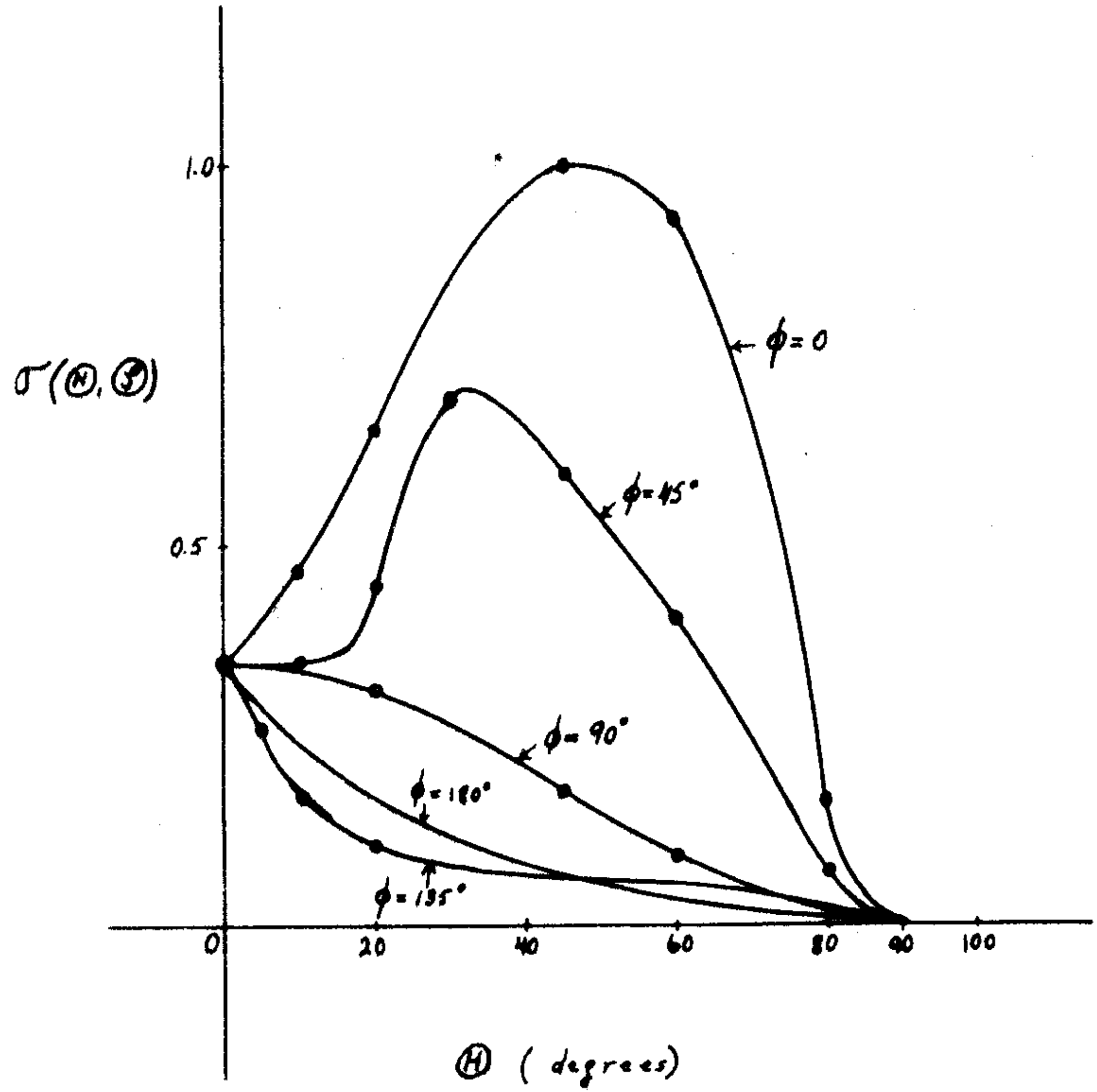
for $\Theta' > 90°$,

$$\sigma(\Theta') = -\frac{\gamma^2\left[\alpha - \sqrt{1-(\gamma^2-1)\alpha^2}\right]^2}{\cos^3\Theta'(\alpha^2+1)^2\sqrt{1-(\gamma^2-1)\alpha^2}}$$

where $\alpha = \gamma \tan\Theta'$

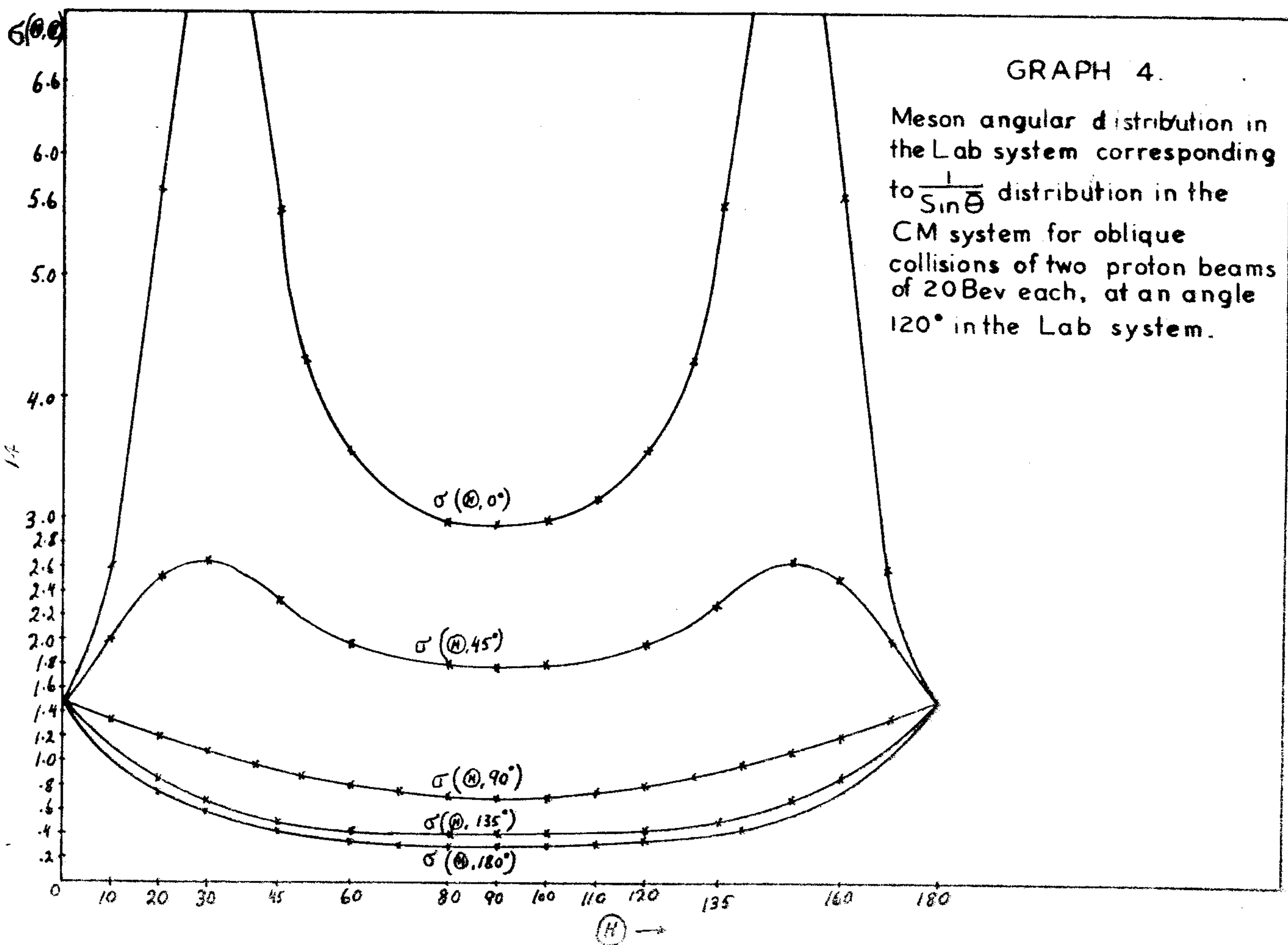
and the constant coefficients have been omitted.

Numerical calculations have been made for the above various differential cross-sections in the Lab system, all referring to collisions of two proton beams of 20 Bev at an angle of 120°. The corresponding plots are shown in graphs III, IV and V.



GRAPH 3

Meson angular distribution in the Lab system corresponding to $Cos^2 \bar{\Theta}$ distribution in the CM system for oblique collisions of two proton beams of 20 Bev each at an angle 120° in the Lab system.



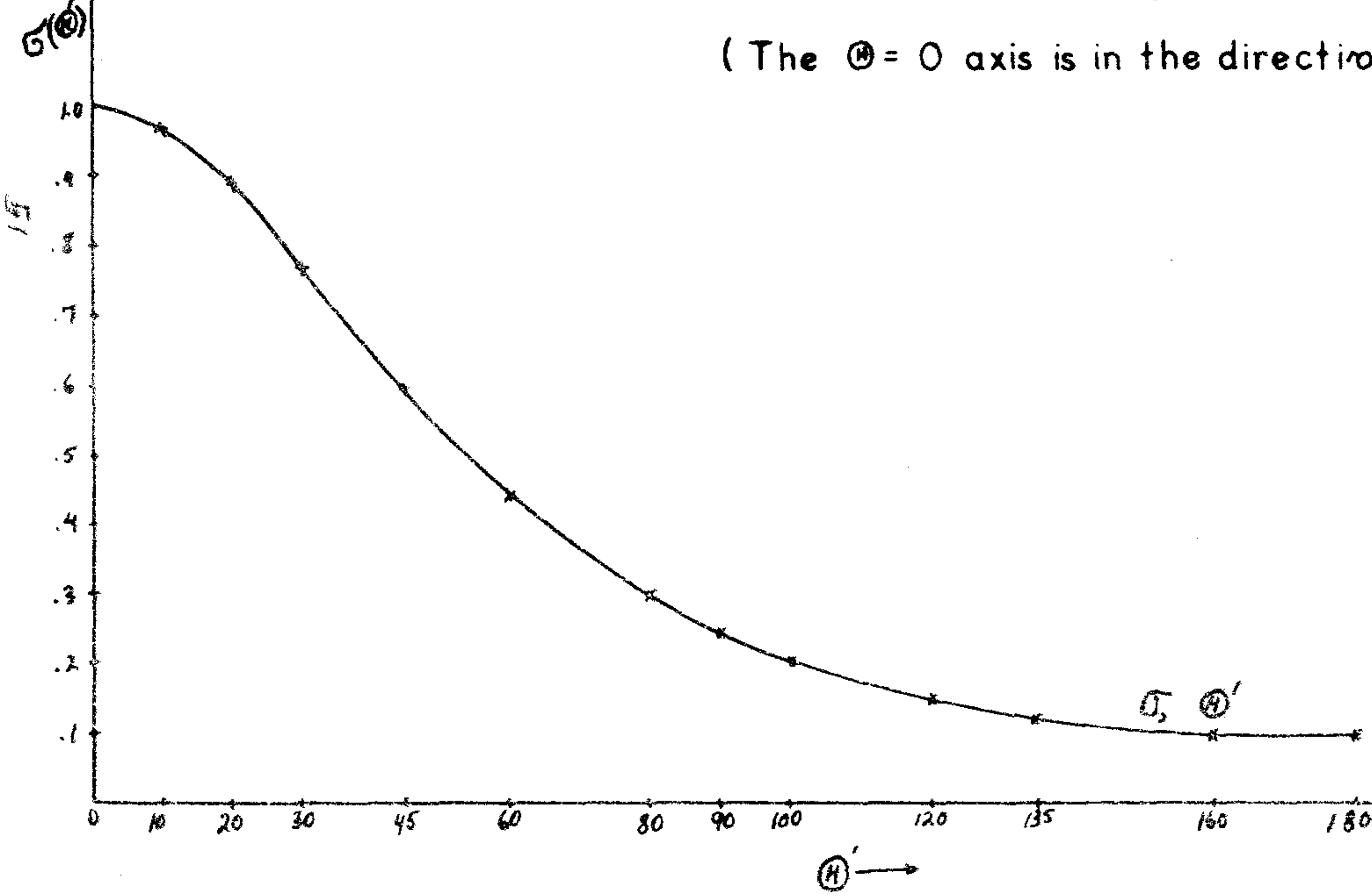
GRAPH 4.

Meson angular distribution in the Lab system corresponding to $\frac{1}{\sin\Theta}$ distribution in the CM system for oblique collisions of two proton beams of 20 Bev each, at an angle 120° in the Lab system.

GRAPH 5.

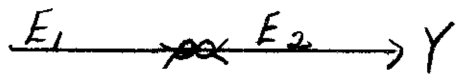
Meson angular distribution in the Lab system corresponding to the isotropic distribution in the CM system for oblique collisions of two proton beams of 20 Bev each at an angle of 120° in the Lab system.

(The Θ = 0 axis is in the direction of the CM velocity).



Case (ii) Head-on Collisions:

We consider head-on collisions of two high-energy proton beams of different incident energies, E_1 and E_2 in the Lab system.



The center of mass of two protons will move along Y axis with a velocity,

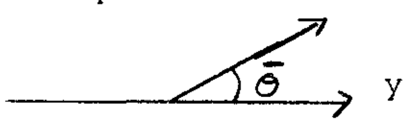
Lab system

$$V_{CM} = \frac{c}{E_1+E_2}\sqrt{E_1^2+E_2^2-2\sqrt{E_1^2-m_0^2c^4}\sqrt{E_2^2-m_0^2c^4}-2m_0^2c^4}$$

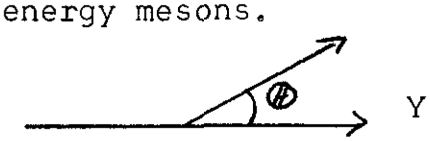
In cases of interest here, $m_0c^2 \ll E_1$ and E_2,

then $$V_{CM} \cong \frac{E_1-E_2}{E_1+E_2}\,c, \tag{37}$$

As before, we assume three kinds of angular distributions for meson production in the CM system: (a) $\cos^2\bar{\Theta}$ (b) $\frac{1}{\sin\bar{\Theta}}$ both for high energy mesons where $\bar{\Theta}$ is indicated in the figure below, and (c) isotropic distribution for low energy mesons.



CM system



Lab system

The corresponding distributions in the Lab system are found, by use of (17) and (18), as follows:

for (a) $\cos^2\bar{\Theta}$ distribution,

$$\Theta \leq 90°;\quad \sigma(\Theta) = \frac{\gamma^2\left[\gamma+\sqrt{1-(\gamma^2-1)\alpha^2}\right]^2\left[-\gamma\alpha^2+\sqrt{1-(\gamma^2-1)\alpha^2}\right]^2}{\cos^3\Theta\,(\alpha^2+1)^4\sqrt{1-(\gamma^2-1)\alpha^2}} \tag{38}$$

$$\Theta \geq 90°;\quad \sigma(\Theta) = \frac{-\gamma^2\left[\gamma-\sqrt{1-(\gamma^2-1)\alpha^2}\right]^2\left[-\gamma\alpha^2-\sqrt{1-(\gamma^2-1)\alpha^2}\right]^2}{\cos^3\Theta\,(\alpha^2+1)^4\sqrt{1-(\gamma^2-1)\alpha^2}} \tag{39}$$

for (b) $\frac{1}{\sin\bar{\Theta}}$ distribution,

$$\Theta \leq 90°;\quad \sigma(\Theta) = \frac{\gamma^2\left[\gamma+\sqrt{1-(\gamma^2-1)\alpha^2}\right]^2}{\cos^3\Theta\,(\alpha^2+1)\sqrt{1-(\gamma^2-1)\alpha^2}\sqrt{(\alpha^2+1)^2-\left[-\gamma\alpha^2+\sqrt{1-(\gamma^2-1)\alpha^2}\right]^2}} \tag{40}$$

$$\Theta \geq 90°;\quad \sigma(\Theta) = \frac{-\gamma^2\left[\gamma-\sqrt{1-(\gamma^2-1)\alpha^2}\right]^2}{\cos^3\Theta\,(\alpha^2+1)\sqrt{1-(\gamma^2-1)\alpha^2}\sqrt{(\alpha^2+1)^2-\left[-\gamma\alpha^2-\sqrt{1-(\gamma^2-1)\alpha^2}\right]^2}} \tag{41}$$

for (c) isotropic distribution.

$$\Theta \le 90°; \quad \sigma(\Theta) = \frac{\gamma^2\left[\delta + \sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\cos^3\Theta\,(\alpha^2+1)^2\sqrt{1-(\delta^2-1)\alpha^2}} \tag{42}$$

$$\Theta \ge 90°; \quad \sigma(\Theta) = \frac{-\gamma^2\left[\delta - \sqrt{1-(\delta^2-1)\alpha^2}\right]^2}{\cos^3\Theta\,(\alpha^2+1)^2\sqrt{1-(\delta^2-1)\alpha^2}} \tag{43}$$

In all these $\sigma(\Theta)'$ $\alpha = \gamma \tan \Theta$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{V_{CM}}{c}\right)^2}}$$

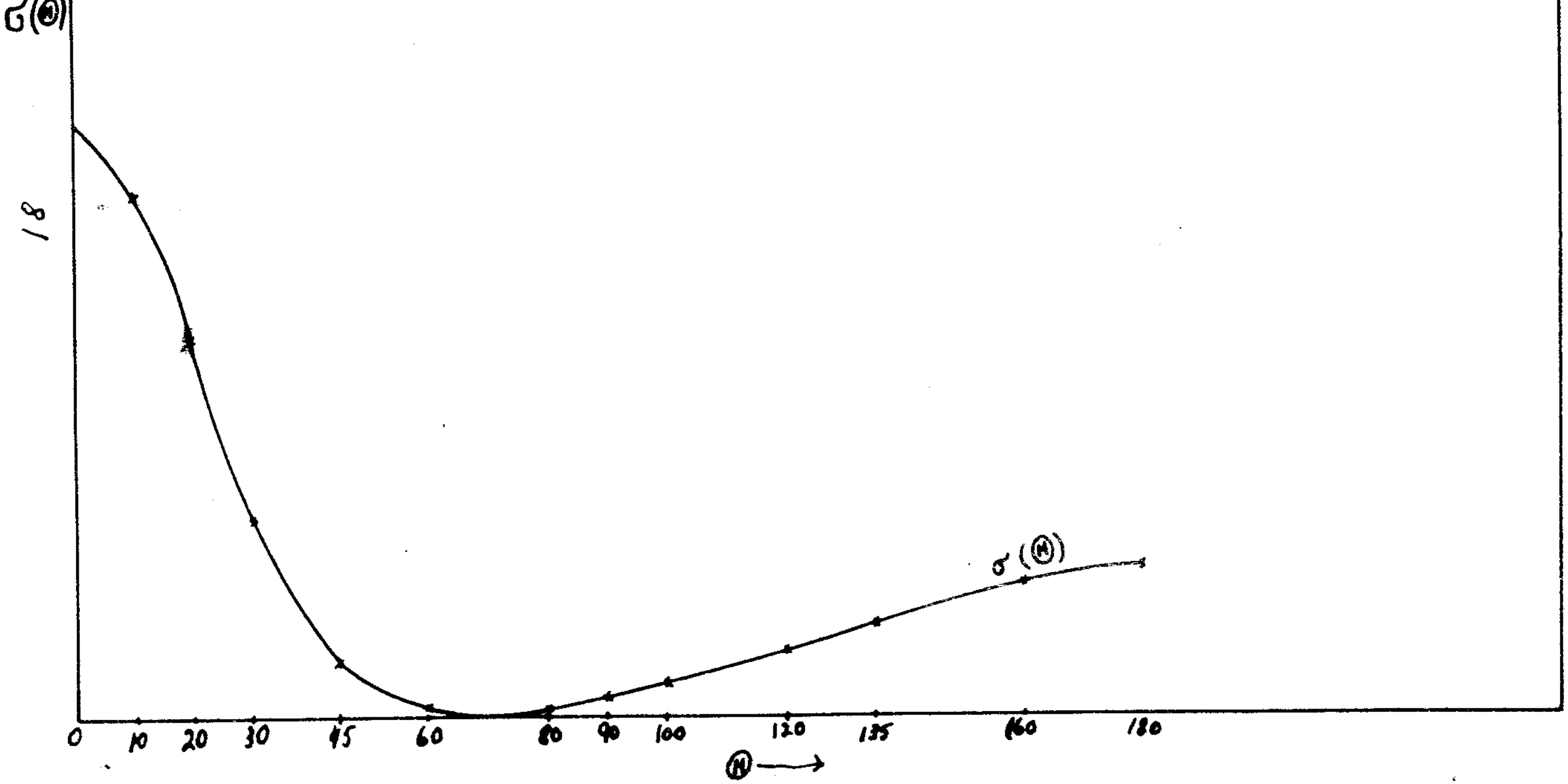
$$\delta = \frac{V_{CM}}{u}$$

where V_{CM} is given by (37) and u is the velocity of a meson in the CM system.

Calculations have been done for two proton beams in head-on collisions, one of energy E_1 = 20 Bev, and the other of energy E_2 = 10 Bev in the Lab system. Graphs VI, VII and VIII give the plots.

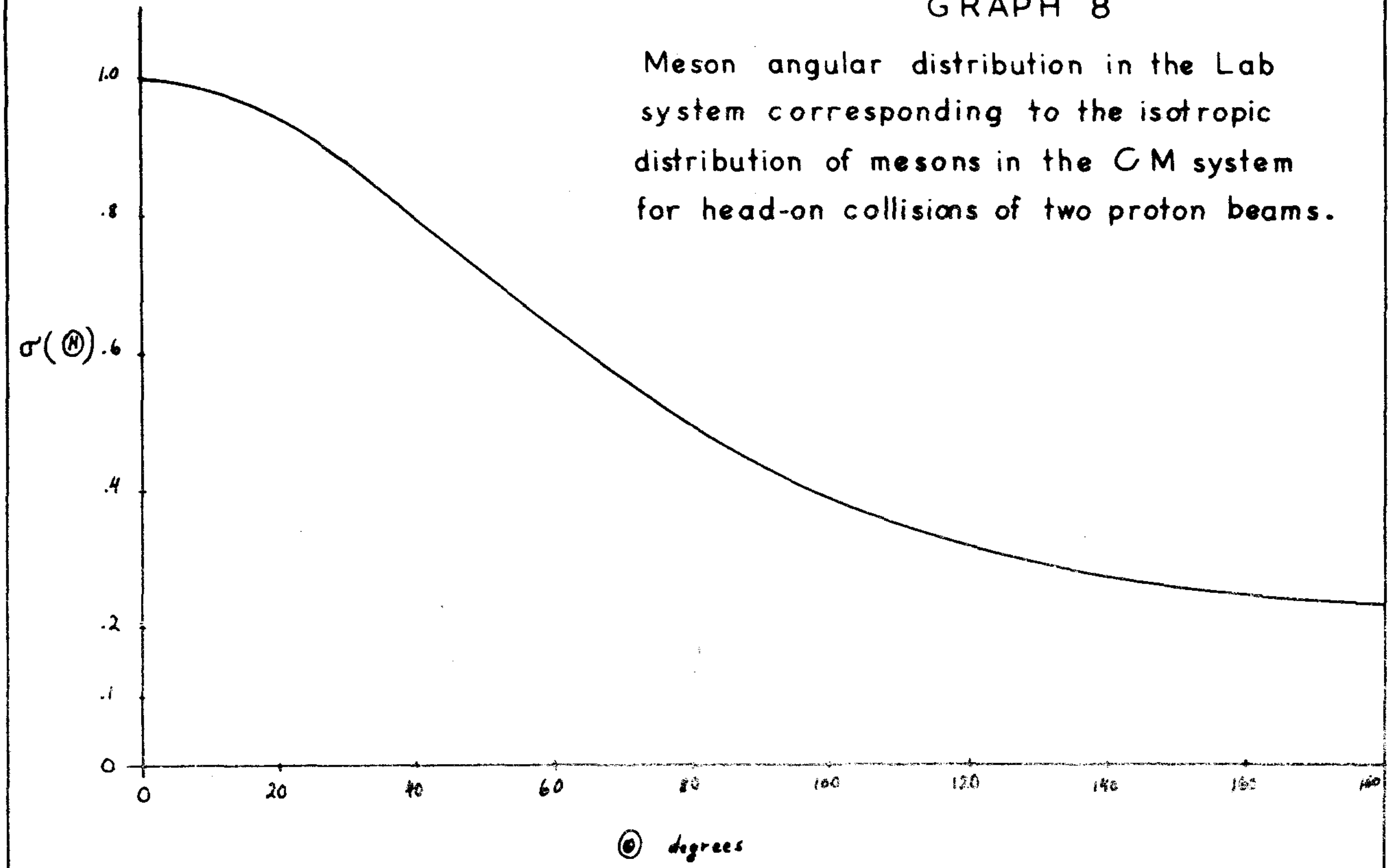
GRAPH 6

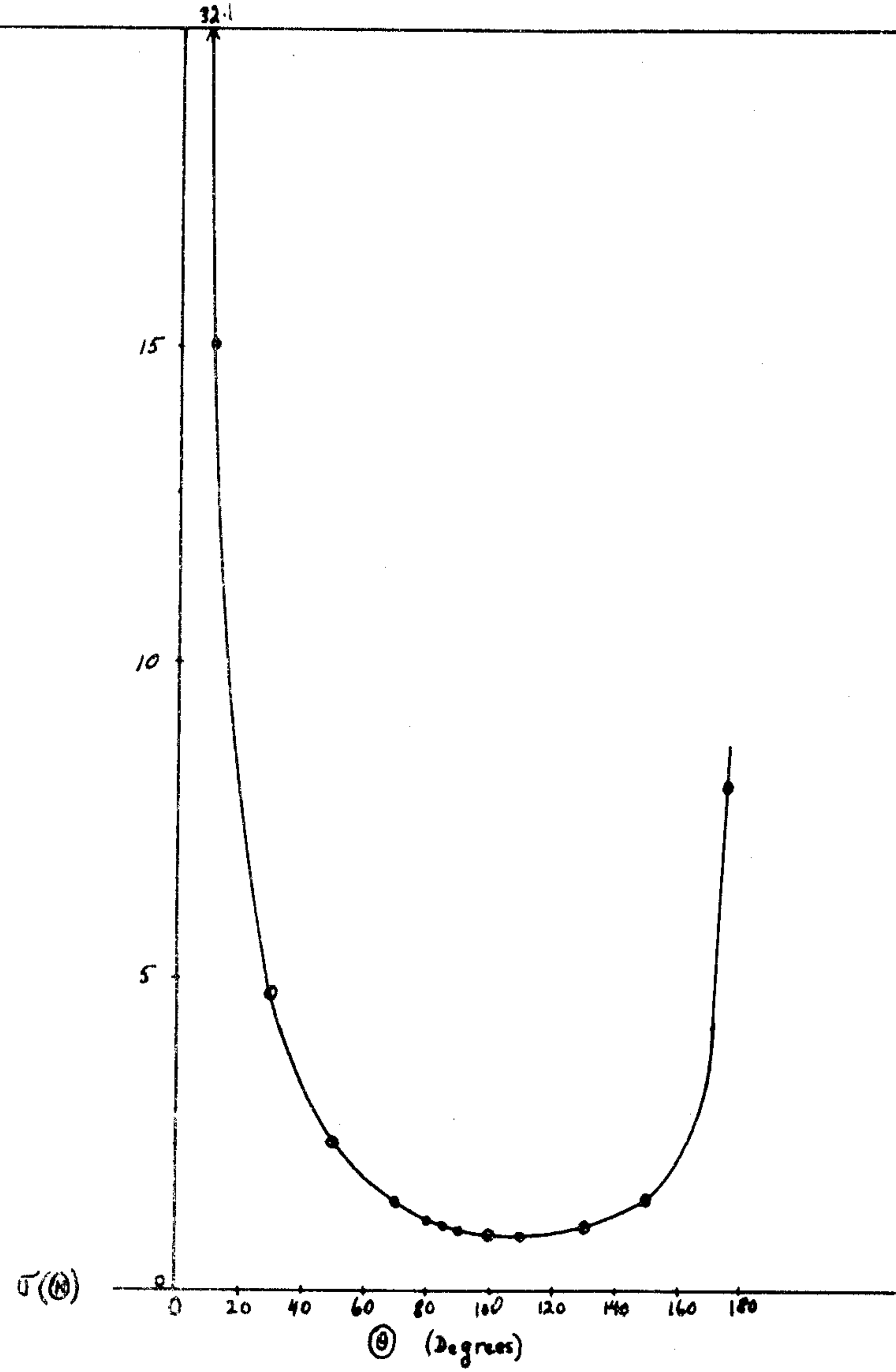
Meson angular distribution in the Lab system corresponding to $\cos^2\bar{\Theta}$ distribution in the CM system for head-on collisions of two proton beams.



GRAPH 8

Meson angular distribution in the Lab system corresponding to the isotropic distribution of mesons in the CM system for head-on collisions of two proton beams.



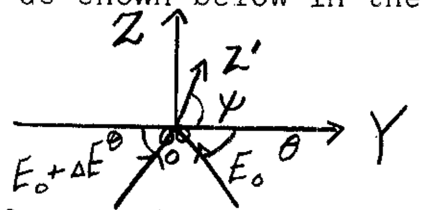


GRAPH 7.

Meson angular distribution in the Lab system corresponding to $\frac{1}{\sin\bar{\Theta}}$ distribution in the CM system for head-on collisions of two proton beams.

In the case of oblique collisions of two high energy proton beams, if the incident energies of the two proton beams are slightly different, there will be small deviations in the laboratory differential cross-sections for mesons as given before by the expressions from (23) to (35).

Let two protons with incident energies $E_1 = E_0 + \Delta E$ and $E_2 = E_0$ colliding as shown below in the Lab System



Lab system

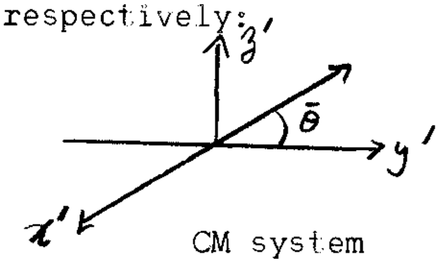
The center of mass will move along the Z' direction where ψ is slightly less than 90° and given by:

$$\tan\psi \cong \left[\frac{2(E_0^2 - m_0^2C^4)}{E_0 \Delta E}\right]\tan\theta \qquad (44)$$

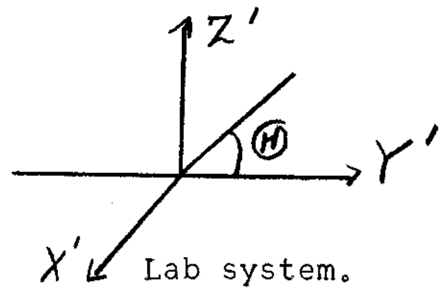
The deviation in V_{CM} is

$$\Delta V_{CM} = V_{CM}\frac{m_0^2C^4}{2E_0^3}\Delta E \qquad (m_0C^2 << E_0) \qquad (45)$$

In describing meson angular distributions, the CM and the Lab coordinates will be designated by (x', y', z') and (X', Y', Z') respectively:



CM system



Lab system.

The deviations in the Lab angular distributions due to ΔV_{CM} read as follows:

(a) $\cos^2\bar{\theta}$ distribution

$$\Delta\sigma(\Theta,\varphi) = \sigma(\Theta,\varphi)\left\{\frac{\left(-\delta\alpha^2 \pm\sqrt{1-(\delta^2-1)\alpha^2}\right)^2}{(\alpha^2+1)^2 - \left(-\delta\alpha^2 \pm\sqrt{1-(\delta^2-1)\alpha^2}\right)^2}\right\}\left[\left(\frac{+4\alpha}{\alpha^2+1} + \frac{4\alpha\delta \pm \frac{2(\delta^2-1)\alpha}{\sqrt{1-(\delta^2-1)\alpha^2}}}{-\delta\alpha^2 \pm\sqrt{1-(\delta^2-1)\alpha^2}}\right)\Delta\alpha + \frac{2\alpha^2 \pm \frac{2\delta\alpha^2}{\sqrt{1-(\delta^2-1)\alpha^2}}}{-\delta\alpha^2 \pm\sqrt{1-(\delta^2-1)\alpha^2}}\Delta\delta\right] + \left[\frac{-4\alpha}{\alpha^2+1}\right. \qquad (46)$$

$$+\frac{\alpha(\delta^2-1)}{1-(\delta^2-1)\alpha^2}\mp\frac{2\alpha(\delta^2-1)}{\left(\delta\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)\sqrt{1-(\delta^2-1)\alpha^2}}\Bigg]\Delta\alpha+\Bigg[\frac{2}{\delta\pm\sqrt{1-(\delta^2-1)\alpha^2}}$$

$$\mp\frac{2\delta\alpha^2}{\left(\delta\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)\sqrt{1-(\delta^2-1)\alpha^2}}+\frac{\delta\alpha^2}{1-(\delta^2-1)\alpha^2}\Bigg]\Delta\delta+\frac{2\Delta\gamma}{\gamma}\Bigg\}$$

where
$$\Delta\alpha=\gamma\sqrt{\cos^2\Theta\sec^2\varphi-1}\,\Delta\gamma$$

$$\Delta\delta=\delta\,\frac{\Delta V_{cM}}{V_{oM}}$$

$$\Delta\gamma=\gamma^3\,\frac{V_{cM}}{c^2}\,\Delta V_{cM}$$

upper sign is for $\varphi<90°$ and lower sign for $\varphi>90°$.

(b) $\frac{1}{\sin\theta}$ distribution

$\varphi\le 90°$

$$\Delta\sigma(\Theta,\varphi)=\sigma(\Theta,\varphi)\Bigg\{\Bigg[\frac{2\alpha}{\alpha^2+1}-\frac{2\alpha(\alpha^2+1)\tan^2\Theta\sin^2\varphi+\left(-\delta\alpha^2\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)\left(-2\alpha\delta\mp\frac{\alpha(\delta^2-1)}{\sqrt{1-(\delta^2-1)\alpha^2}}\right)}{\tan^2\Theta\sin^2\varphi(\alpha^2+1)^2+\left(-\delta\alpha^2\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)^2}\Bigg]\Delta\alpha$$

$$-\Bigg[\frac{\left(-\delta\alpha^2\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)\left(-\alpha^2\mp\frac{\alpha^2\delta}{\sqrt{1-(\delta^2-1)\alpha^2}}\right)}{\tan^2\Theta\sin^2\varphi(\alpha^2+1)^2+\left(-\delta\alpha^2\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)^2}\Bigg]\Delta\delta \qquad (47)$$

$$+\Bigg[-\frac{4\alpha}{\alpha^2+1}+\frac{\alpha(\delta^2-1)}{1-(\delta^2-1)\alpha^2}\mp\frac{2\alpha(\delta^2-1)}{\left(\delta\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)\sqrt{1-(\delta^2-1)\alpha^2}}\Bigg]\Delta\alpha$$

$$+\Bigg[\frac{2}{\delta\pm\sqrt{1-(\delta^2-1)\alpha^2}}\mp\frac{2\delta\alpha^2}{\left(\delta\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)\sqrt{1-(\delta^2-1)\alpha^2}}+\frac{\delta\alpha^2}{1-(\delta^2-1)\alpha^2}\Bigg]\Delta\delta$$

$$+\frac{2}{\gamma}\Delta\gamma\Bigg\}$$

where $\Delta\alpha$, $\Delta\delta$ and $\Delta\gamma$ are given as before.

(c) isotropic distribution

$$\Delta\sigma(\Theta')=\sigma(\Theta')\Bigg\{\Bigg(-\frac{4\alpha}{\alpha^2+1}+\frac{\alpha(\delta^2-1)}{1-(\delta^2-1)\alpha^2}\mp\frac{2\alpha(\delta^2-1)}{\left(\delta\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)\sqrt{1-(\delta^2-1)\alpha^2}}\Bigg]\Delta\alpha$$

$$+\Bigg[\frac{2}{\delta\pm\sqrt{1-(\delta^2-1)\alpha^2}}\mp\frac{2\delta\alpha^2}{\left(\delta\pm\sqrt{1-(\delta^2-1)\alpha^2}\right)\sqrt{1-(\delta^2-1)\alpha^2}}+\frac{\delta\alpha^2}{1-(\delta^2-1)\alpha^2}\Bigg]\Delta\delta \qquad (48)$$

$$+\frac{2\Delta\gamma}{\gamma}\Bigg\}$$

where Θ' is measured from Z' axis,

and
$$\Delta\alpha=\tan\Theta'\,\Delta\gamma$$

$$\Delta\delta=\frac{\delta}{V_{cM}}\,\Delta V_{cM}$$

$$\Delta\gamma=\gamma^3\,\frac{V_{cM}}{c^2}\,\Delta V_{cM}.$$

No numerical calculations have been made. It is a pleasure to appreciate discussions with Drs. K. R. Symon and P. M. Stehle.