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by

R. Stump, J. Pavlat, W. Reed, and D. Zaffarano

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INTRODUCTION

In most large accelerators now operating or being built, the radiofrequency electrical field is set up in ferrite loaded cavities which may be tuned by a d.c. biasing current. Such systems are electrically quite inefficient, and require fairly complex electronic instrumentation.

A more efficient and much simpler method for supplying high radiofrequency fields might involve the use of a "high Q" resonant cavity, with a single tube oscillator operating directly in and controlled by the cavity volume. This system would give very high voltages with only modest power dissipation. Frequency variation could be obtained by mechanically changing the shape of the cavity by a reentrant plunger, for example.

In this report are described some of the electrical properties of a model of such a tunable cavity, and the properties of the cavity with an oscillator tube in operation.

In general, the system is satisfactory: a frequency range of 7:1 and a gap voltage of 10,000 volts are obtained with less than 500 watts r. f. power going into the cavity. No breakdown or erratic behavior is observed when the cavity is operated with 500 watts input power, or while the plunger is moving. This indicates that at least at this power level, mechanical modulation introduces no difficulties. It is planned to increase the power into the cavity for a more critical test of this point.

I. Description of Cavity

A rough diagram of the cavity showing primarily the dimensions which determine the electrical properties is shown in Figure 1.

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* Work supported by the National Science Foundation through the Midwestern Universities Research Association.

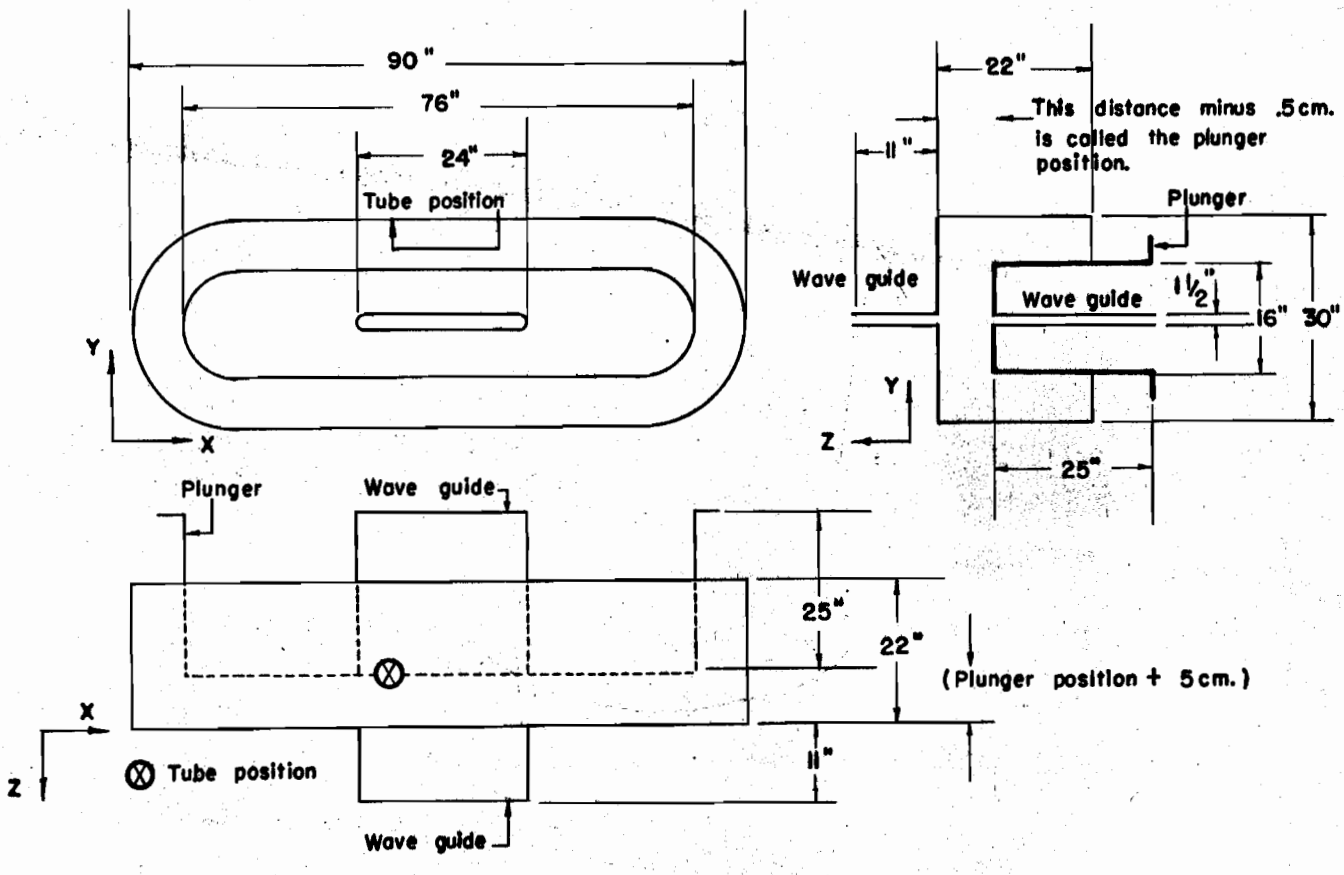

Figure 1 - Dimensions of Mechanically Tuned Model Cavity.

The plunger is free to slide from a position where the plunger face is .8 cm from the opposite face of the cavity to where the plunger face is 51.2 cm from the opposite face. Plunger positions are measured from a point where the face of the plunger is 0.5 cm from the face of the cavity. Thus the plunger position may be increased from .3 to 50.7 cm. The plunger is connected electrically to the cavity by 82 low resistance graphite brushes set as close together as possible around the plunger. (Sintered silver-graphite brushes could be used if even lower contact resistances were desirable.)

The entire interior of the cavity is silver plated.

The tube for supplying power to the cavity is located as shown.

There is a gap cut in the face of the plunger and the opposite face of the cavity which simulates the donut aperture in a F. F. A. G. synchrotron. The gap is 24" x 1 1/2". Extending out from the gap are metal sections having an inside shape which conforms with the gap, and are open at each end. These are called the waveguides, and are used as exponential attenuators for r. f. which leaks out of the cavity.

Coordinate directions are shown for later reference.

II. Cavity Resonant Frequencies

The frequency of the lowest mode of oscillation of the cavity increases from about 30 mc to about 210 mc as the plunger is drawn out. There are of course an infinite number of modes of oscillation in a cavity. The frequency of the lowest mode and of some of the other modes which have been observed are shown in Figure 2.

In order to identify the modes shown in the figure, the corresponding modes for a rectangular box of the same dimensions as the cavity with the plunger flush with the back side are shown. The indices l m and n give the number of voltage (absolute value) maxima or minima in the three coordinate directions: l corresponds to those in the longest dimension of the cavity, m to those in the vertical direction, and n to those in a direction perpendicular to the face of the plunger (X Y and Z directions respectively in Figure 1.) When one of the indices is zero, the electric field is uniform and directed parallel to the corresponding coordinate. Thus all modes with n = 0 have an electric field perpendicular to the face of the plunger, i.e., in the Z direction.

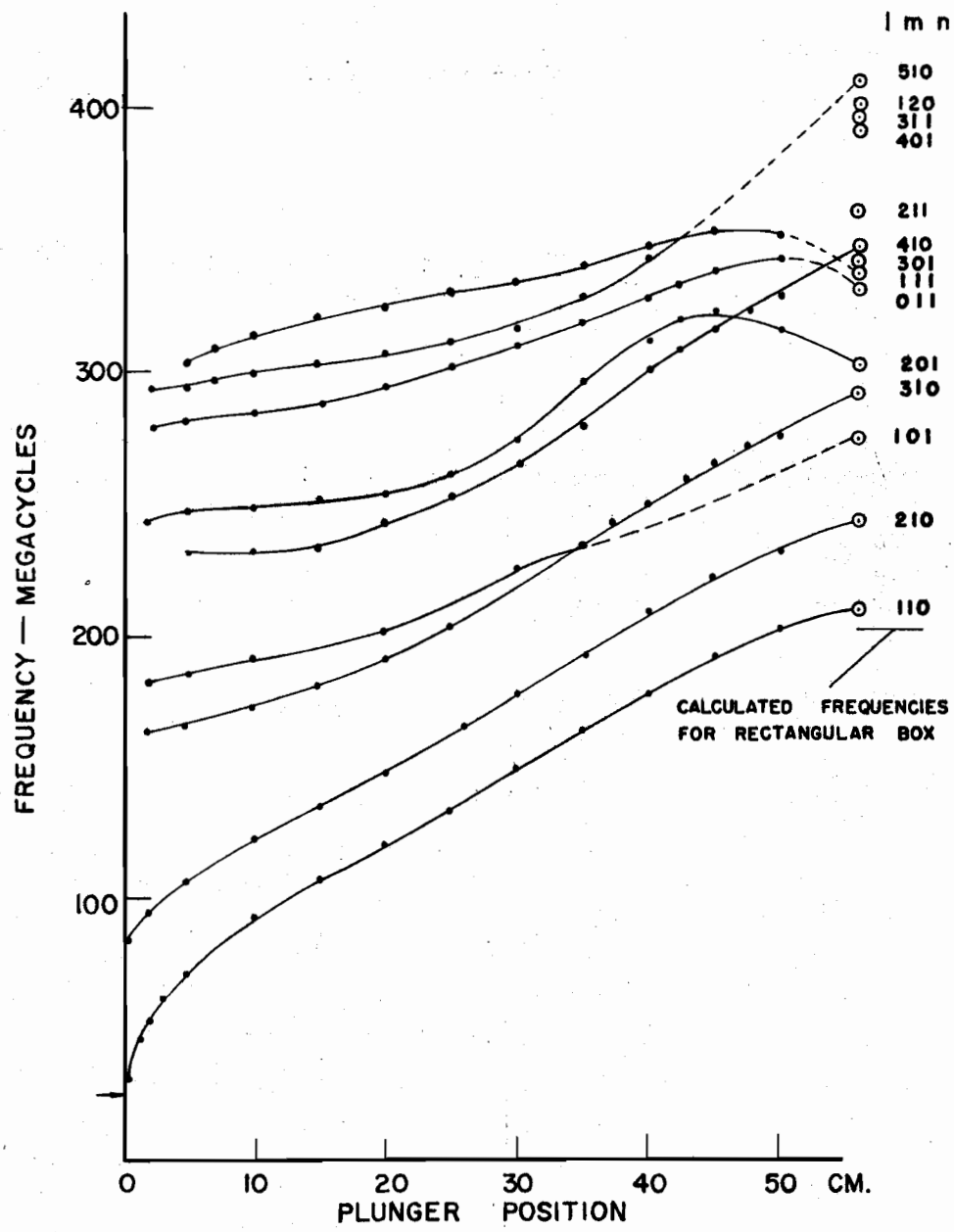

Figure 2 - Oscillation Modes Observed in Model Cavity.

When one index is zero, there is a single mode of oscillation at that particular frequency. When no index is zero, there is a three fold degeneracy in the modes of the rectangular box, so that in the cavity itself, where the degeneracy is resolved, there should be three modes observed. Note that not all modes have been observed.

When the cavity is excited as an oscillator, the current which flows in the exciting loop is composed of a series of narrow pulses at the resonant frequency of the cavity ("Class C" operation). Such a current is very rich in harmonics, so that if a harmonic of the resonant frequency coincides with any of the higher modes, part of the power put into the cavity will go into the higher mode. The amount of power which goes into the higher mode is just P_f, where

$$P_f = 1/2\ I_f^2\ Z_f$$

where I_f is the amplitude of the current at the particular frequency and Z_f the loop impedance at the higher frequency. Since both the loop impedance and the current amplitude fall with increasing frequency, only the lowest two or three harmonics need be considered. Also, at low frequencies, only in the modes where $n = 0$ will the magnetic field at the plane of the loop be in a direction to give high impedance. Table I gives the frequencies and plunger settings where appreciable power is expected to be found in modes above the lowest. Such interference has been observed at three of these positions: 7, 20, and 35 cm. With the particular loop being used, the other modes are apparently not appreciably excited. They may be excited with a different plate loop arrangement.

It may be of interest to note to what extent it is possible by fairly simple calculations to give the frequency of the fundamental mode of the cavity for various plunger settings.

At the high frequency limit, the cavity becomes approximately a rectangular box with easily calculated frequency. At the low frequency limit, the plunger and cavity form a coaxial cable, terminated in the capacity between the end of the plunger and the wall of the cavity. The resonant frequency is then determined by the characteristic impedance of the coaxial line, and the terminating capacity. The characteristic impedance of the line is given most easily by estimating the capacity per unit length of the line, and then finding the impedance from the relation between the characteristic impedance, capacity and velocity of light. The low frequency limit calculated in this manner was 25 mc, in satisfactory agreement with the observed value of 31 mc, considering the variability of the spacing between the plunger and the end of the cavity and the estimates involved in the calculation.

Table I

Plunger Positions and Frequencies at which Higher Modes May be Excited

Approx. Plunger Position	Frequency	Harmonics	Interfering Mode (ℓ)
1.7	47 mc	2	2
7.3	85	2	3
20	122	2	4
35	165	2	5
2.2	56	3	3
6.0	77	3	4
12.5	101	3	5
1.0	41	4	3
2.6	58	4	4
6.0	74	4	5

The variation in frequency with plunger position may be derived from the formula for the change in frequency due to perturbation of the boundary

$$\frac{\omega^2}{\omega_o^2} = 1 + \int (H^2 - E^2)dv$$

where:

ω_o = frequency of unperturbed cavity

ω = frequency of perturbed cavity

H = magnetic field strength

E = electric field strength

$\int dv$ = integral over volume removed from cavity

[dv positive if volume decreases].

H and E are normalized so

$$\int E^2 dv = \int H^2 dv = 1$$

where the integration is performed over the volume of the entire cavity.* The formula was used to extrapolate from the high frequency and low frequency limits. At the high frequency end, E and H were assumed to be just those in a rectangular box; the excluded volume was the volume excluded by the plunger. At low frequencies, the electric field was assumed to be uniform over the end of the plunger and zero at the sides. The change in volume is just that due to moving the plunger.

Figure 3 shows the results of these calculations. The solid curve is the observed frequency curve. The circled points are frequencies calculated as indicated above.

III. Field Plots

It has been shown** that the electric and magnetic fields in a cavity may be determined by the frequency shift caused by a small sphere in the cavity.

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* J. C. Slater, "Microwave Electronics," D. Van Nostrand, New York, 1950.

** Kitchen and Schelberg, Journ. Appl. Phys. 26, 618 (1955).

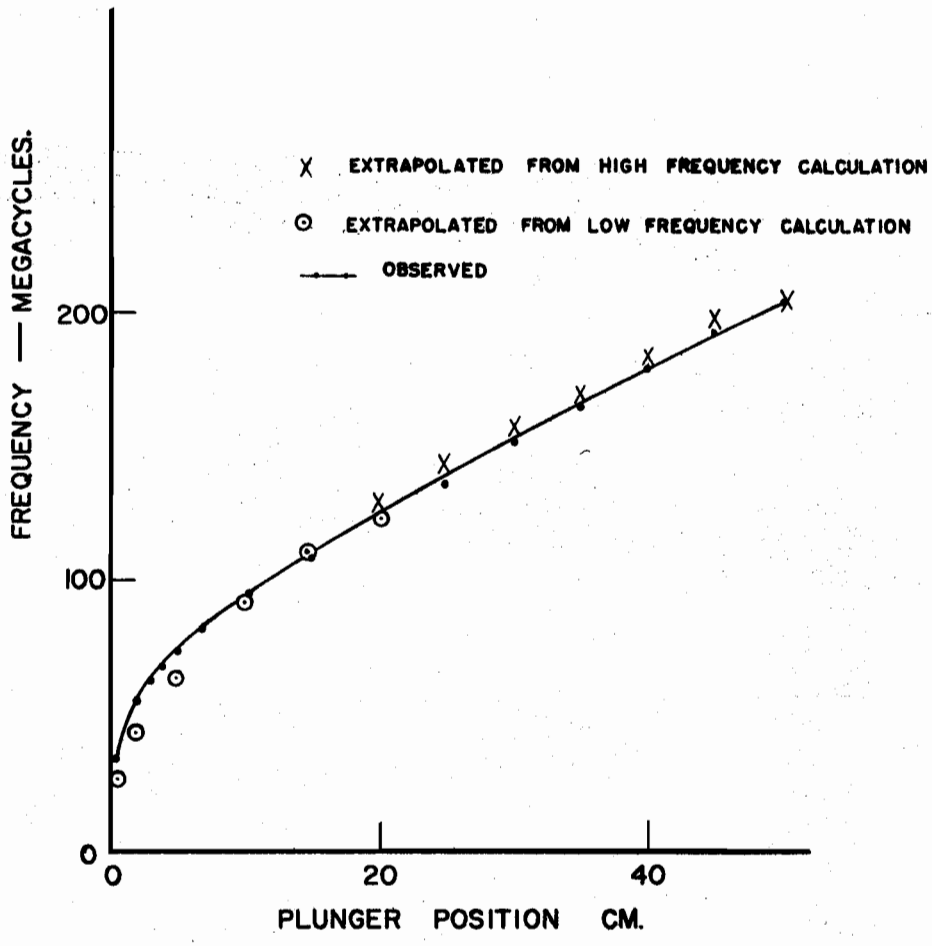

Figure 3 - Calculated and Observed Frequencies for Varying Cavity Volumes.

For these measurements, the cavity is operated as a self excited oscillator, at low power. The frequency is measured. A small dielectric sphere is then placed at the point where the field is to be measured, and the frequency is remeasured. The change in frequency, $\delta\omega$, is given by

$$\frac{\delta\omega}{\omega} = -\frac{3}{4}\frac{\delta v}{W}\left[\frac{\varkappa-1}{\varkappa+2}\varepsilon_0 E^2 + \frac{\mu-1}{\mu+2}\mu_0 H^2\right]$$

where:

$\delta\omega$ is the frequency shift

ω is the unperturbed frequency

δv is the volume of the sphere

W is the total stored energy in the cavity

$\varkappa$ is the dielectric constant of the sphere

ε_0 is permittivity of free space

μ_0 is permeability of free space

μ is relative permeability of the sphere

E is the field at the sphere

H is the magnetic field at the sphere.

In the measurements made here, only relative values of E for various points in the cavity were obtained. A 7/8" diameter lucite sphere was used: $\varepsilon \doteq 2.5$, $\mu = 1$. Then E^2 is proportional to $\frac{\delta\omega}{\omega}$, or $E = \text{const}\sqrt{\frac{|\delta\omega|}{\omega}}$.
This is the quantity plotted in the accompanying Figure 4.

Field measurements were made both across the gap (i.e. parallel to the face of the plunger - X direction in Figure 1) and through the gap (i.e. perpendicular to the plunger face-(-Z) direction.) The measurements were made at plunger settings and frequencies shown in Table II.

The general features of the field plots are the same in every case: The first mode has uniform field over the region measured, the second mode has a voltage node at the center, and the third mode has voltage nodes at each end of the gap. In each case measured, the field is uniform through the gap.

Figure 4 shows the field plots for a plunger setting of 15 cm.

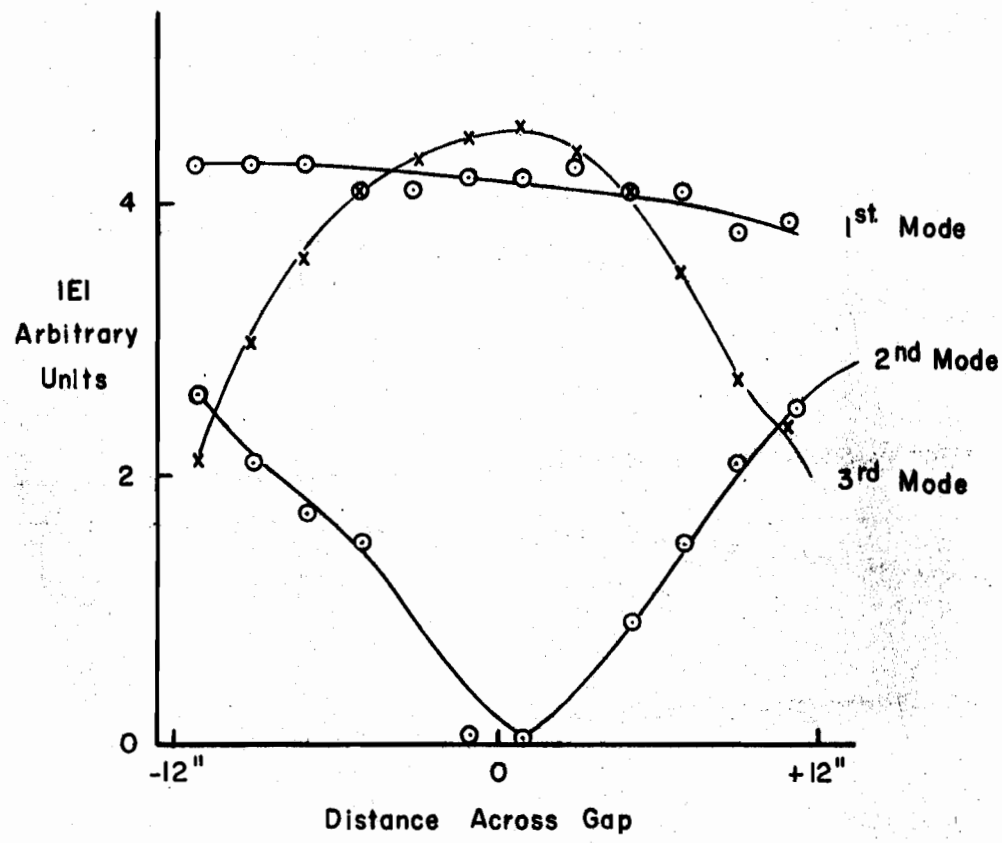

Figure 4 - Field Distribution Across Gap by Absorption Technique.

Table II

Tabulation of Field Plots Made

Plunger Position (cm)	Mode	Frequencies at which field plots were made (mc)
5	1	73.4
	3	167.9
7	1	90.0
12.3	1	100.5
15	3	182.8
	2	138.2
	1	109.3
26	3	205.8
	2	168.2
	1	140.3
38	1	177.2
50.7	1	207.0

If the cavity Q is known, these data may be used to predict the electric fields in the cavity for a given power input. Using

$$E^2 = \frac{-4}{3\varepsilon_0} \frac{W}{\delta V} \frac{\varkappa + 2}{\varkappa - 1} \frac{\delta\omega}{\omega}$$

and

$$Q = \frac{\omega W}{P}$$

where P is the power supplied to the cavity,

then

$$E = \left[\frac{4QP}{3 \varepsilon_0 \delta V} \frac{\varkappa + 2}{\varkappa - 1} \frac{|\delta\omega|}{\omega^2} \right]^{1/2}$$

or

$$\frac{V}{\sqrt{PQ}} = \left[\frac{4}{3 \varepsilon_0 \delta V} \frac{\varkappa + 2}{\varkappa - 1} \frac{|\delta\omega|}{\omega^2} \right]^{1/2} \chi$$

where χ is the distance across the cavity through the gap, and V is the total voltage through the gap.

This quantity $\frac{V}{\sqrt{PQ}}$ is plotted in Figure 5.

IV. Cavity Q

The Q of the cavity was measured at various plunger settings. The results are shown in Figure 6. These measurements are made by measuring the change in frequency between the 3 db points in the tramsmission of the cavity. Power is put into the cavity through a small loop. The voltage out is measured on another small loop. The frequencies at which the voltage out is 1/ 2 times maximum are measured. The resonant frequency, divided by the difference, gives the Q of the cavity. The loops are small enough that they do not appreciably lower the Q of the cavity.

For comparison, there is plotted on Figure 6 a curve of the Q of a reentrant cylindrical cavity.*

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* S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand, New York, 1943.

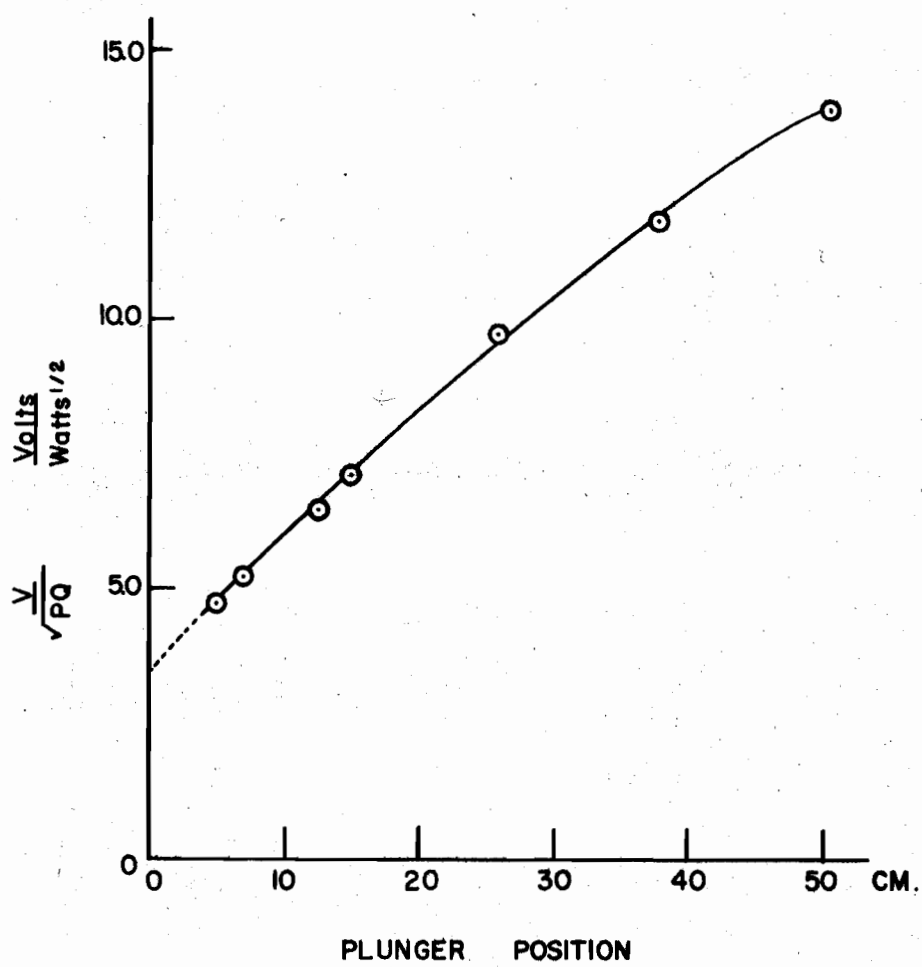

Figure 5 - Relation Between Cavity Power, Voltage, Q, and Volume.

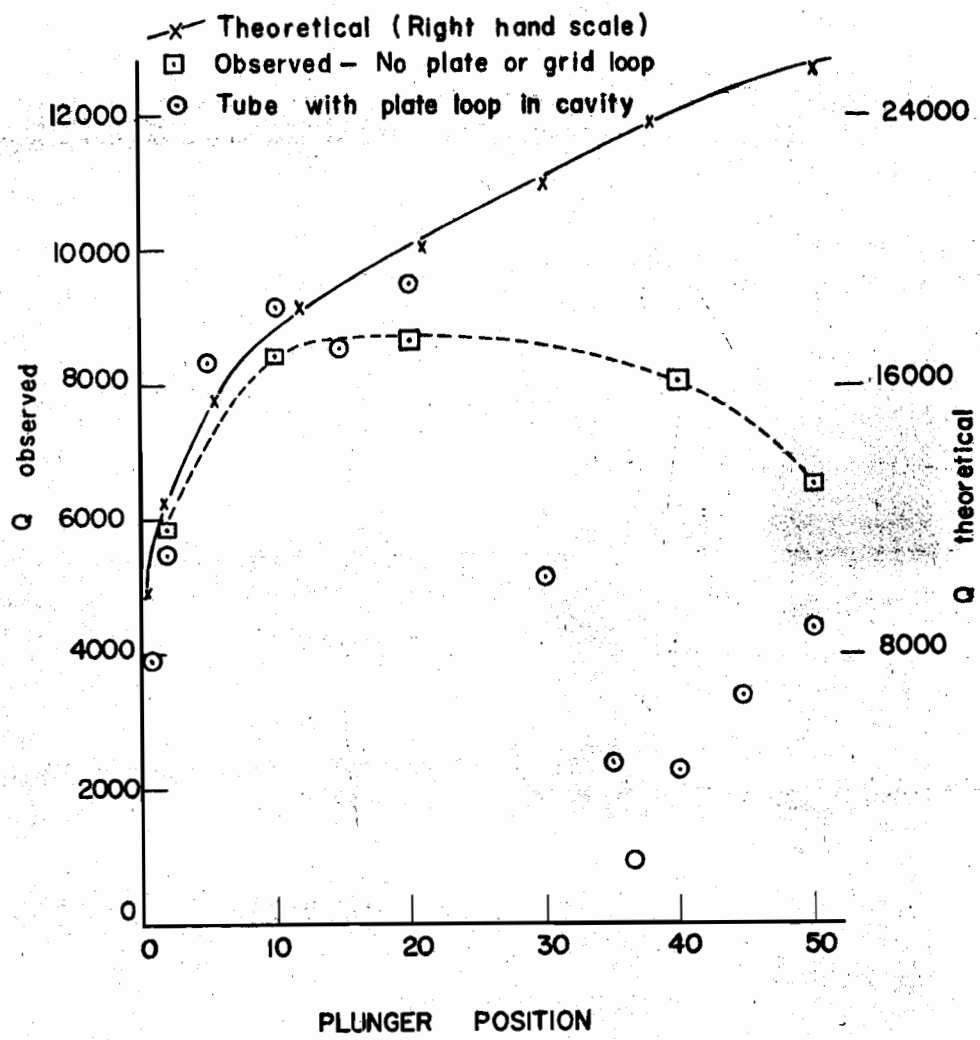

Figure 6 - Theoretical and Experimental Values of Q for Cavity.

When the oscillator tube and the coupling loops for plate and grid are in the cavity, the overall Q is lowered, particularly near the resonance points of the grid or plate loop. The circled points in Figure 6 show the Q of the cavity and tube with a plate loop resonance at about 170 mc. The sharp dip in Q at 36 cm is due to this resonance.

V. Coupling Loops

The requirement that the cavity oscillate properly over the entire range of frequencies necessitates use of coupling loops with resonant frequencies above the highest operating frequency. Assuming this to be true, the impedance of a loop in the cavity is given by:*

$$Z = \mu_0 \omega A^2 \bar{H}^2 Q$$

where: Z = loop resistance at resonance of cavity

ω = frequency x 2π

μ_0 = permeability of free space

A = area of loop

$\bar{H}$ = average magnetic field in loop perpendicular to plane of loop

Q = Q of cavity.

Again, H is normalized so

$$\int H^2 dv = 1$$

with $\int dv$ over entire volume of cavity.

Since the oscillator is a tetrode, operating class C, the resistance of the coupling loop may vary over a wide range, i.e. 1000 to 10,000 ohms at least with efficiencies of 70 or 80%. Thus the size of the plate loop is not at all critical. Obviously, the operating point of the tube must be changed as the plate impedance changes, to get maximum power.

The area of the grid coupling loop is also not critical. By suitable bias arrangements, the oscillator may be made to operate with high efficiency over a wide range of grid excitation.

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* J. C. Slater, "Microwave Electronics," D. Van Nostrand, New York, 1950.

The calculation for the plate loop area is made using the assumption that the magnetic field is uniform in the space outside the plunger (i.e., in the volume remaining when the plunger is in as far as it will go). Then, the minimum area is that which will be large enough to give the required minimum loop impedance at low frequencies. Choosing this to be 1000 ohms at 30 mc,

$$1000 = \frac{2\pi \times 30 \times 10^6 \; A^2 \; H^2 \; 4000}{V \; H^2}$$

where:

$Q = 4000$ from Figure 6

$V \doteq .175 \times .56 \times 4.35 \; m^3$

so $A = 210 \; cm^2$.

The maximum loop area is that which will give the maximum allowed impedance at high frequencies. Taking a value of 10,000 ohms at 200 mc,

$$10{,}000 = \frac{2\pi \times 200 \times 10^6 \; A^2 \; H^2}{VH^2} \; 6500$$

where again $V \doteq .175 \times .56 \times 4.35 \; m^2$

and $Q = 6500$ from Figure 6,

so $A = 204 \; cm^2$.

Thus with a loop of area 207 cm^2, the loop impedance will be suitable at all plunger positions. If a more nearly constant impedance is required, a paddle could be inserted in the loop to cut out part of the area at high frequencies. The change in impedance caused by a paddle is not just that due to the change in area, since the magnetic flux crowds around the paddle, thus increasing the flux through the remaining area.

The area of the grid loop is determined by the RF grid voltage required. For a given energy in the cavity, the RF grid voltage increases with the frequency, so that if the grid loop has suitable area at low frequencies, it will be larger than needed at higher frequencies. The grid loop may be estimated from the following considerations:

The amplitude of the RF voltage on the grid is given by

$$V_G = \mu_0 A_G \omega \bar{H}_G$$

where A_G = area of grid loop

$\bar{H}_G$ = average magnetic field in grid loop

$\omega = 2\pi$ x frequency of oscillation.

For low level operation, the power output from the tube is then:

$$P = 1/2\ I_P^2\ Z_P$$

where I_P = amplitude of RF plate current

Z_P = impedance of plate loop.

Now the plate current is given by:

$$I_P = g_m V_G$$

where g_m is the transconductance of the tube.

Finally

$$Q = \frac{\omega W}{P}$$

where $W = 1/2\ \mu_0 \int H^2 dv$ is the energy contained in the cavity. Combining these equations gives:

$$QP = \omega W$$

$$1/2\ Q\ g_m^2\ \mu_o^2\ A_G^2\ \omega^2\ \bar{H}_G^2\ Z_P = 1/2\ \mu_o\ \omega \int H^2\ dv.$$

Using the same assumptions about the magnetic field as in the plate loop calculations and Z_P = 1000 ohms at 30 mc

$$A_G\ g_m = 2.1 \times 10^{-5}$$

or, using g_m = 5000 μ mhos

$$A_G = 42\ cm^2.$$

A schematic of the oscillator is shown in Figure 7, and the electrical circuit connections are shown in Figure 8. The oscillator is an air cooled tetrode, 4 x 500 A, with a plate loop of 224 cm^2

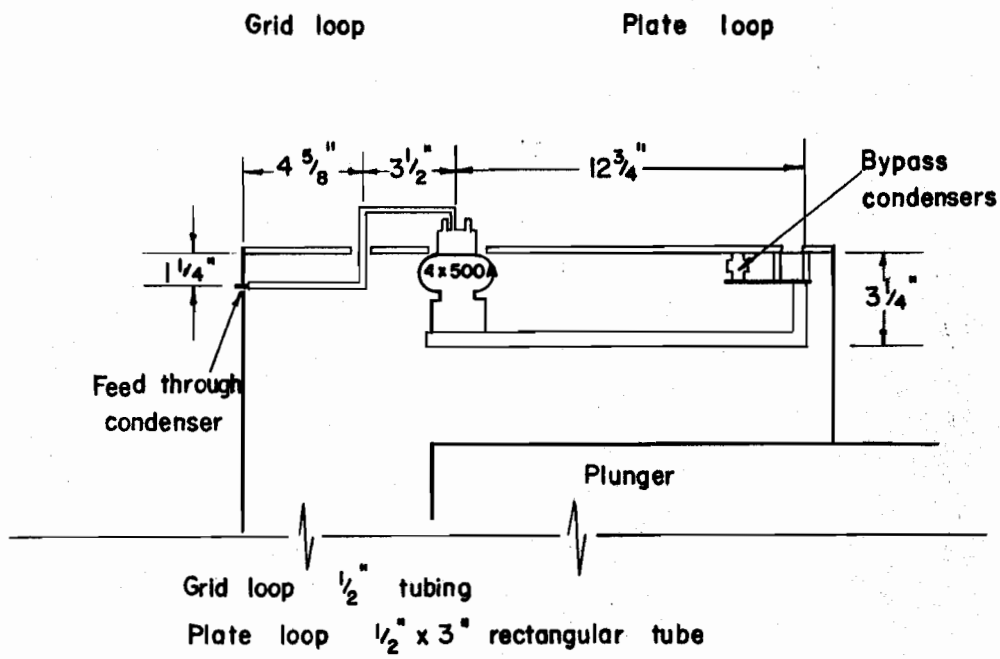

Figure 7 - Oscillator Schematic

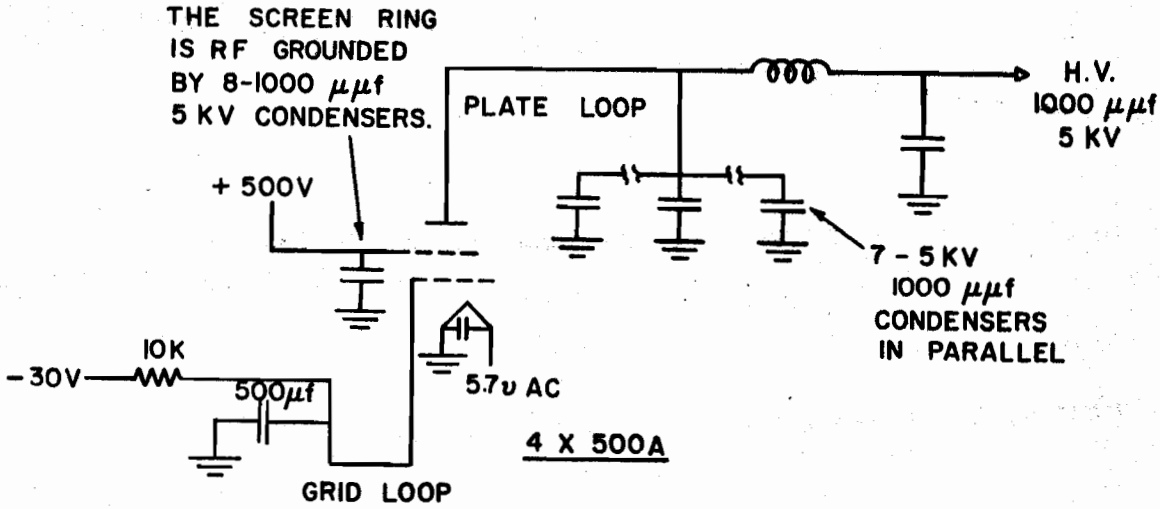

Figure 8 - Electrical Connection of Oscillator Tube.

and a grid loop of 37 cm^2. Oscillation in the proper mode is obtained over the entire frequency range with the exception of a region from about 105 to 120 megacycles.

The failure of the system to oscillate in one region is due to the resonance of the plate and grid loops in this region. The grid loop is resonant at about 105 mc, the plate loop at 120 mc. A simple calculation shows that in the case of the grid loop, when the frequency passes through resonance the phase of the grid voltage relative to the cavity flux changes by 180°; and similarly, when the frequency passes through the plate loop resonance the phase between the plate current and the cavity flux changes by 180°. Thus when the frequency is either below or above the resonant frequencies of both loops, proper oscillation is obtained--while at frequencies between the resonances of the plate and the grid the system will not oscillate.

The resonant frequencies of the grid and plate loops may be estimated as follows:

The loop is assumed to be a transmission line terminated in the input or output capacity of the tube. The first resonance of this system is at

$$\omega Z C \tan k\ell = \ell$$

where $\omega = 2\pi \times$ frequency

Z = characteristic impedance of line

C = input or output capacity of tube

$k = 2\pi$ / (wavelength in free space)

ℓ = length of the line.

In the present system this estimate gives a value of 90 mc for the first resonance of the grid loop, in satisfactory agreement with the observed resonance at 105 mc.

To remove the resonance from the frequency range of the cavity one may shorten the loop or lower the characteristic impedance. The first of these requires careful placement of the tube, the second may be achieved by using a very wide strap for the loop. In the present system, a grid loop 40 cm wide would be required to raise the frequency to 200 mc. In the tube used, 4 x 500 A, the grid structure itself is resonant at about 210 mc.

A somewhat simpler solution to the problem would be the use of probes for coupling. Since two nearby probes will have voltages

of the same phase, probes may not be used for both plate and grid in a grounded cathode oscillator. However, either grounded grid or grounded plate operation is possible.

Another solution would be the use of an oscillator tube which could be made more an integral part of the cavity, or perhaps construction of a special tube, if existing types are not suitable.

VI. Maximum Cavity Voltage

The electric field on the cavity was measured by the glo-ball method.*

Small glass spheres are filled with Helium to a pressure of about 1 cm. When in an oscillating electric field these balls go into a glow discharge at a field strength characteristic of the ball and the frequency. The ball is calibrated in a condenser at the same frequencies as are measured in the cavity.

Field measurements are made as follows: The ball is inserted in the cavity and the power into the cavity raised until the glow discharge starts. At the point just before the ball fires, the voltage, v_1, on a small loop is measured with a r.f. volt meter. The power into the cavity is reduced as soon as the ball fires and the ball is removed from the cavity, since a prolonged discharge causes a drift in the firing field. The power in the cavity is then raised to its full value (500 watts from the plate power supply) and the voltage, v_2, on the small loop remeasured. Then the total voltage through the gap is given by:

$$V = \frac{v_2}{v_1} E X$$

where V = total gap voltage

v_1 = voltage on small loop when ball fires

v_2 = voltage on small loop when maximum power is applied to cavity

E = electric field required to fire ball

X = distance through gap.

Results of a number of measurements are shown in Figure 9.

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* J. F.Steinhaus, University of California, Livermore Laboratory, Report UCRL-4559.

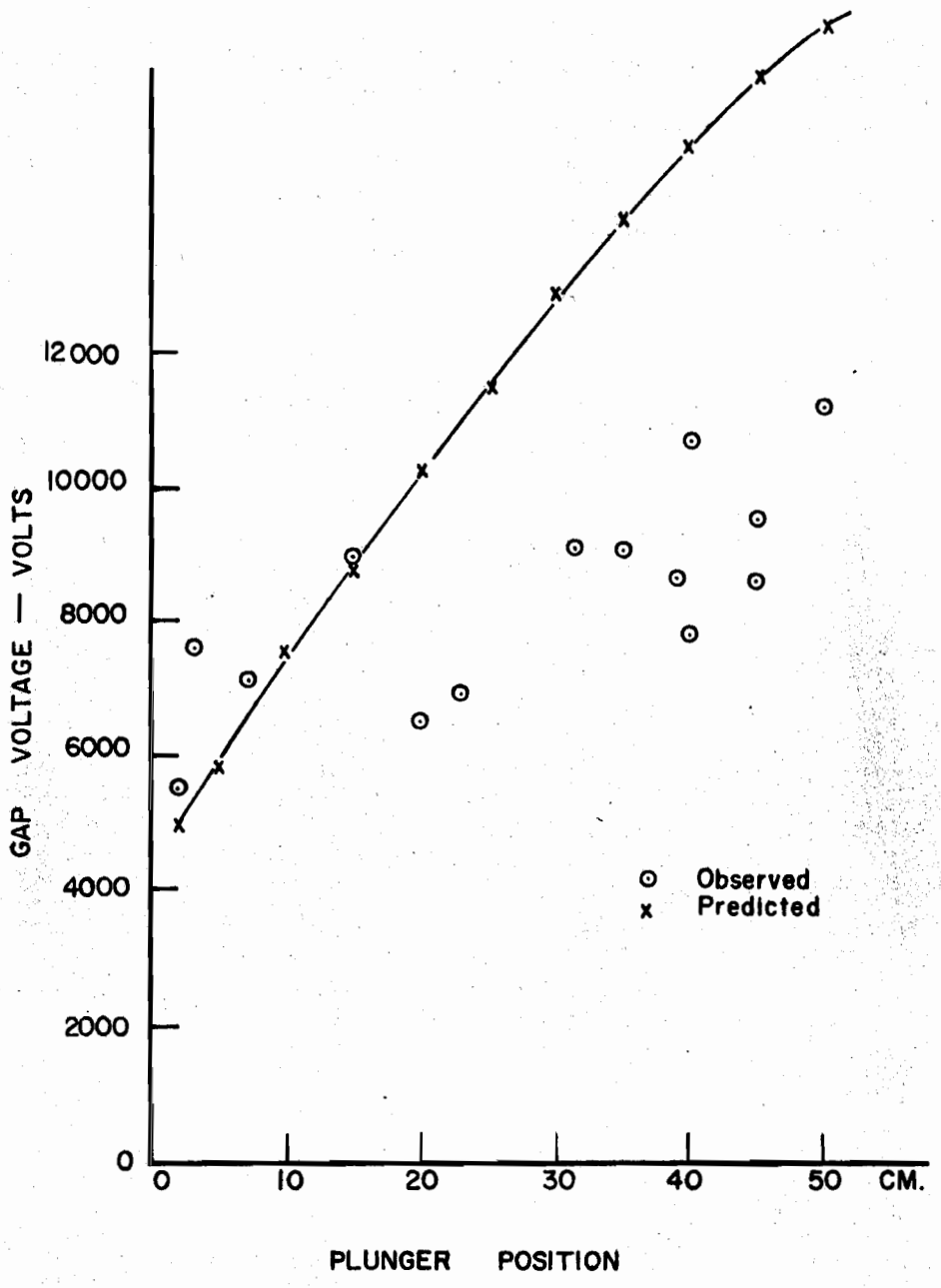


Figure 9 - Maximum Cavity Voltage as Obtained by Glo-Ball Technique.

To compare the actual voltage obtained through the gap and that expected from the properties of the cavity and the power input, it is necessary to know the Q of the cavity and the efficiency of the tube feeding the cavity.

For a basis of comparison, the cavity Q was assumed to be 4000 and the tube efficiency 75%. The efficiency estimate is probably fairly good, the Q is not so good - as indicated by the variation in Figure 6.

The results of Figure 5 then may be used to calculate the voltage. The results are shown in Figure 9 as the solid curve. The agreement is not too good. For some reason, which is not known, the experimentally found voltage trap through the gap is less than that predicted at high frequencies. One possible explanation is that the oscillator excites other modes of the cavity at harmonics of the main frequency. This draws power from the oscillator and reduces the efficiency. The higher modes do not contribute much to the cavity voltage, since the frequencies are so high that radiation through the gaps carries off energy very rapidly.

VII. Variation of Cavity Frequency by Paddles.

The frequency of the cavity may be varied over small limits by means of rotating paddles. The effect of such a paddle may be estimated from the formula for the frequency of a cavity with perturbed boundaries:

$$\omega^2/\omega_o^2 = 1 + \int (H^2 - E^2)dv$$

where the symbols are defined above. (See Section II.)

When a paddle is inserted near the edge of the cavity, the magnetic fields are much larger than the electric fields in the neighborhood of the paddle. When the paddle is parallel to the magnetic field, the frequency of the cavity is about the same as when the paddle is out. When the paddle is perpendicular to the undisturbed magnetic field, the lines of magnetic field spread out near the paddle and go around it. This has the effect of reducing the volume occupied by the magnetic field, and consequently raises the frequency.

To estimate the change in frequency which may be obtained with a paddle, the magnetic field is assumed constant in the volume of the cavity when the plunger is in all the way, and the maximum volume excluded by the paddle is estimated to be just the volume of a box the same area as the paddle and as long as the minimum dimension of the paddle.

A number of paddles were placed in the cavity, and the maximum percentage frequency changes were measured for various paddle settings. The results are shown in Figure 10. The paddle dimensions are shown at the right of each curve, with the frequency change as estimated above.

The variation obtained with a paddle as a function of angular position of the paddle is shown in Figure 11. The paddle is parallel with the magnetic field at about 145°.

VIII. Power Loss From Gap

The opening in the cavity at the gap would of course be necessary to allow particles in an accelerator to pass through the cavity. However, such openings allow radiation from the cavity with consequent lowering of the cavity Q. The use of extensions on the cavity with a cross section the same as that of the gap is only partly successful. The extensions form wave guides and the attenuation may be estimated.

The electric field inside the cavity is perpendicular to the plane of the gap (i.e. the z direction). Any field in this direction entering the wave guide is attenuated to 1/e of its original value in a distance of about one centimeter. However, in the wave guide, the electric field turns so that it is perpendicular to the faces of the guide which are closest together. (i.e. in the y direction in Figure 1) In this mode, the cut off frequency of the wave guide is only 250 mc. Thus there is only slow attenuation of the field going out the guide. This has two effects: there is radiation from the gap, and any particles coming out of the cavity will feel a vertical force.

A crude measurement indicates the problem: with a field inside the cavity of about 400 volts per inch, the field across the end of the guide, eleven inches out from the cavity itself, is about 100 volts per inch.

The turning of the electric field going into the wave guide would not occur in a completely symmetric system, and it would perhaps be possible to reduce it over the frequency range required by use of tuning probes set across the wave guide near the gap.

IX. Effect of Beam Loading

The effect of beam loading on the frequency of a resonant cavity can be investigated by a fairly simple equivalent circuit with lumped parameters.

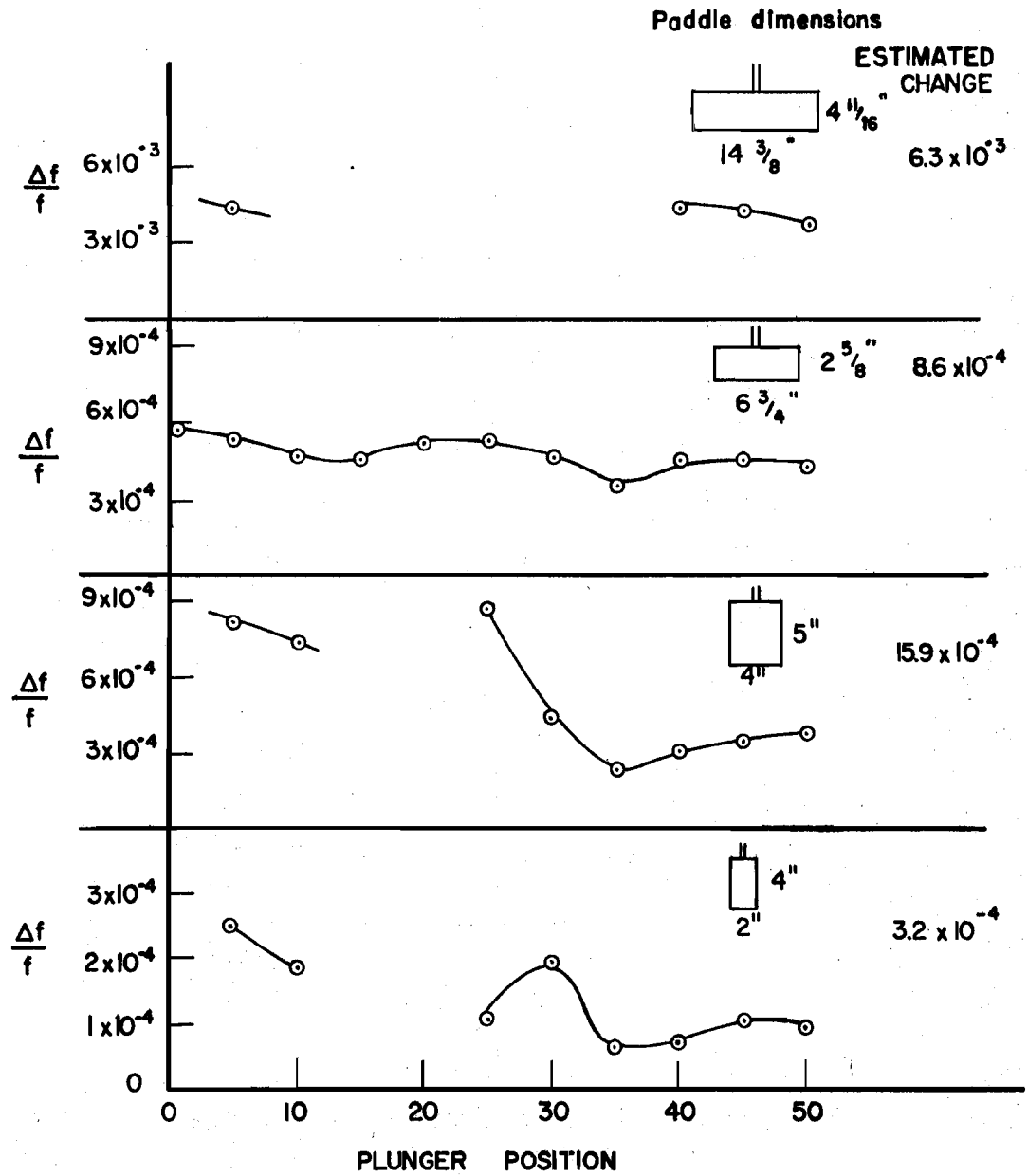


Figure 10 - Effect of Paddle Size on Cavity Frequency.

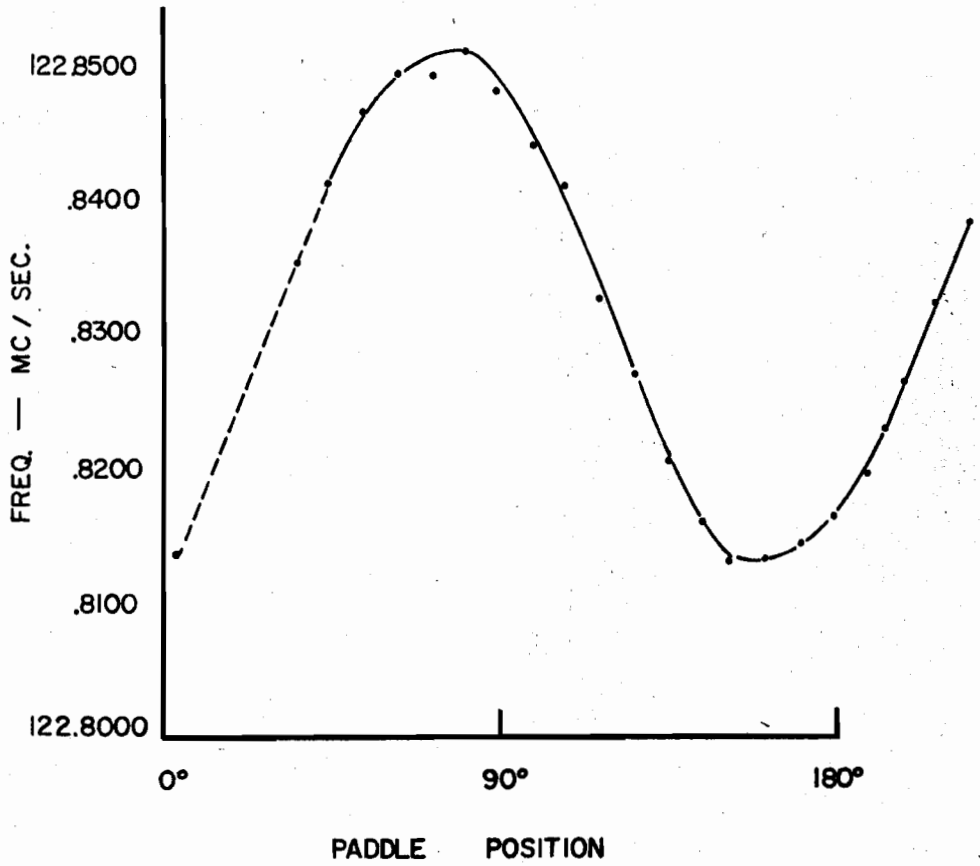
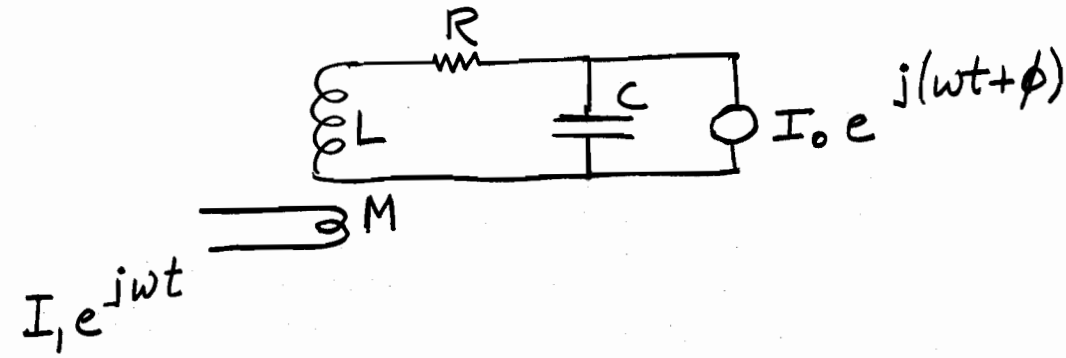


Figure 11 - Effect of Paddle Angular Position on Cavity Frequency.



Only the lowest frequency Fourier component of the beam is considered. Here the current generator represents the beam current (the phase is shown explicitly). Solving this circuit for the power taken from the current generator gives:

$$P = \frac{I_o^2 R}{2\,(L\,C\omega^2 - 1)^2 + \omega^2 R^2 C^2} +$$

$$\mathrm{Re}\left\{ \frac{CM\omega^2 R I_1 I_o e^{j\phi} - jM\omega\left[\,LC\omega^2 - 1\,\right] I_1 I_o e^{-j\phi}}{2\left[\,(LC\omega^2-1)^2 + \omega^2 R^2 C^2\right.} \right\}.$$

The first term represents loss of energy to the cavity from the beam, the second either gainor loss depending on the phase angle ϕ.

If the cavity is run as a self excited oscillator, the current I_1 comes from a vacuum tube, which has a grid voltage driven from a coil also coupled to L. The grid voltage varies as the rate of change of current in L, and, therefore, the current I_1 is just 90° out of phase with the current in L. Solving for the current in L and requiring a 90° phase shift between the current in L and I_1 gives the relation between the oscillation frequency and the phase of the beam current.

$$(LC\omega^2-1)\left[\,MC\omega^2 + \frac{I_o}{I_1}\cos\phi\,\right] - \frac{I_o}{I_1}\,\omega RC\sin\phi = 0.$$

So far it is assumed that the beam frequency and the oscillation frequency are exactly the same. If they are not, ϕ is no longer constant but is a function of time. This leads to an instability of the following type--if the equation above is satisfied for a particular frequency and phase, and then the phase increases slightly, the oscillation frequency will also change. This change in frequency will cause a changing phase between beam and cavity. If

cos Ø is positive, a stable situation exists since the frequency change tends to remove the phase shift. If cos Ø is negative, instability occurs. Energy is added to the beam when cos Ø is negative.

It is clear that when the beam current (or the Fourier component with frequency ω) is small with respect to the driving current, the change in frequency due to the beam is of little consequence, and ordinary phase stability will more than compensate for any instability. The only place where the instability noted here would be serious would be where phase stability is nearly zero. Here any perturbation of the beam will cause a phase shift to the region where the beam gives energy to the cavity.

A simple way to consider the instability is as follows: The beam is considered as an RF current, which may lead the voltage in the cavity. Thus the beam has the same effect as a condenser across the cavity, that is, it will lower the cavity frequency. If the phase between the beam current and the cavity increases, the effective capacity is increased, and the cavity frequency is lowered. This causes further increase in the phase difference, and so instability. Likewise, if the beam current lags behind the voltage, the beam has the effect of an inductance, which raises the cavity frequency. Again, if the phase increases the inductance increases causing a frequency increase in the cavity, which brings about still further phase shift.

If the power loss to the beam is high, then there is no reason to use a high Q cavity. Efficient operation only requires that the losses in the cavity be much less than the loss in energy to the beam.

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