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LONG STRAIGHT SECTIONS IN SEPARATED SPIRAL SECTOR ACCELERATORS

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This is a summary of a preliminary study of the behavior of spiral sector FFAG parameters when long, spiraled straight sections are included in the structure. The incentive has been to learn to what extent occasional long straight sections are feasible since they are clearly desirable for targeting, R.F. structures, etc. Three types of geometry for long straight sections are presented, and calculations for two of them made using hard edge matrix methods are summarized.

Geometry

We identify the parameters in the unperturbed (no long straight sections) machine as in Figure 1. Here ρ is the angle between a radius vector from the center of the machine and the sector boundary,

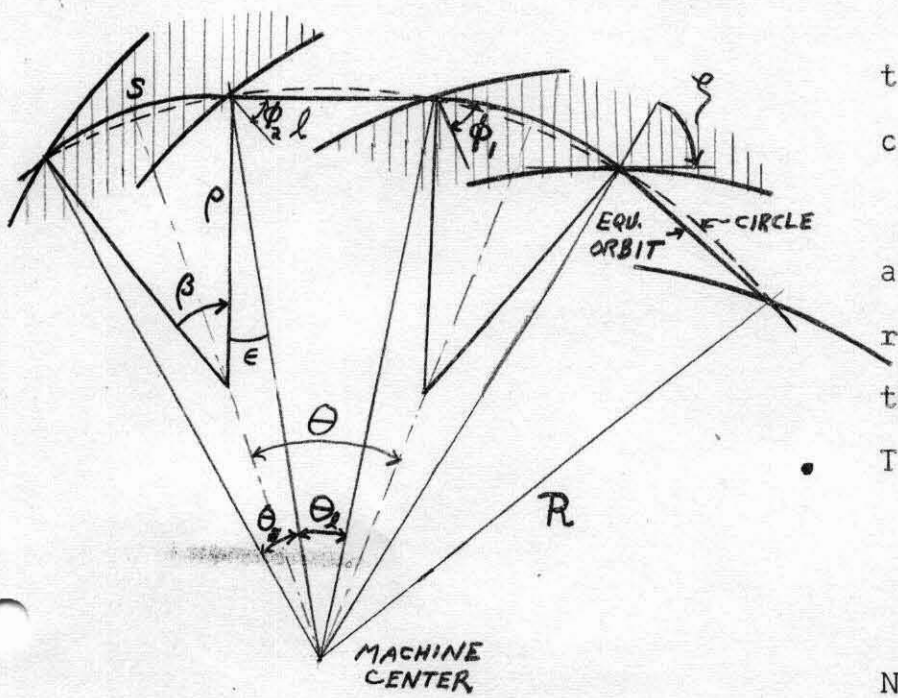

FIGURE 1.

ϵ is the angle between the equilibrium orbit and a circle at the magnet edge, ϕ_1 and ϕ_2 are the angles between the equilibrium orbit and the normal to the edge at magnet boundaries. Therefore,

$$\phi_1 = \rho + \epsilon$$
$$\phi_2 = \rho - \epsilon$$

Notice that $\epsilon = \frac{\theta_\ell}{2}$ so that if $S = \ell$,

$$\epsilon \cong \frac{\theta}{4} \cong \frac{\beta}{4}$$

Geometry 1.

If occasionally around the machine a straight section of length ℓ is extended to length L but no changes are made in ϕ_1 , ϕ_2 , or s in any magnets, we have a geometry from which σ_s may be readily calculated. However in this case ρ is not the name for different magnets and the structure does not scale. This may be seen by considering that (for example) four long straight sections are introduced when one divides the machine into four spiraled quadrants and moves them radially apart (Figure 2.). This lack of scaling, resulting in considerable change in ϕ and L/R with radius makes the σ_s calculated for this case valid for only one equilibrium orbit.

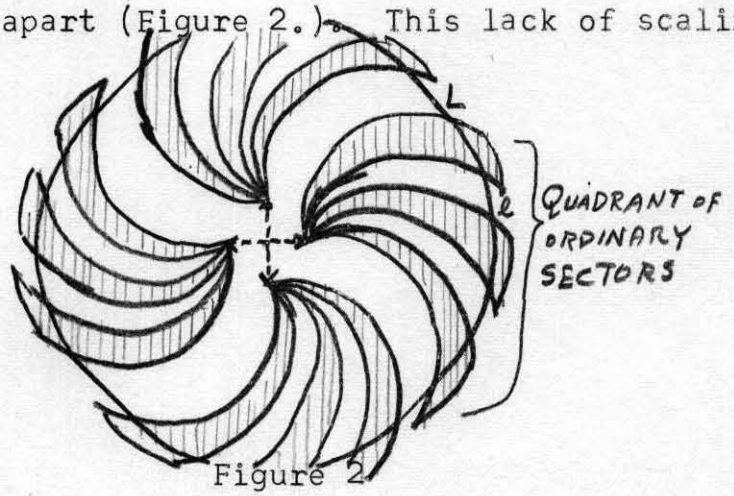

Figure 2

Geometry 2.

Periodic extra long straight sections may be introduced into a machine by using sectors which are too short or too few. For example, if a sector is simply removed from the accelerator, a straight section of about three times normal length results. Scaling of orbits is preserved since ρ remains independant of radius. However ϕ_1 and ϕ_2 are different for each magnet between successive long straight sections since ϵ varies and ρ is constant. A portion of the geometry is sketched in Figure 3 for the case of four extra long straight sections.

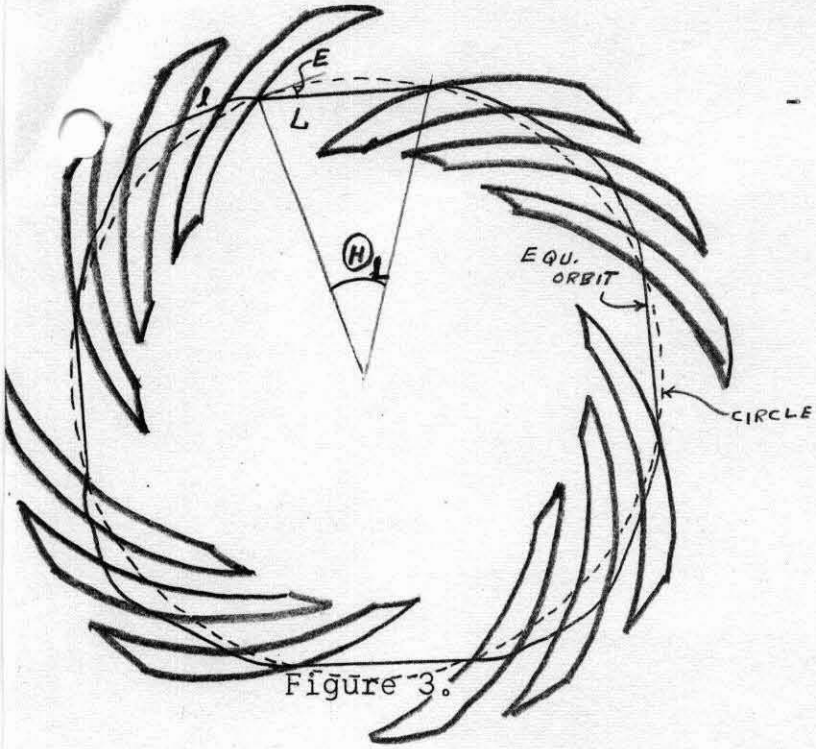

Figure 3.

Θ_L is the angle subtended by the long straight section. It can be shown that if β is the same in each magnet, ϕ_1 and ϕ_2 vary from $\rho - E$ to $\rho + E$ where $E = \frac{\Theta_L}{4} \cong \frac{L}{4R}$. If, for example, $L = 3\ell$, then $E = 3\epsilon_0$ where ϵ_0 is defined for the unperturbed machine. The behavior of ϕ with θ around the machine may be represented graphically as follows:

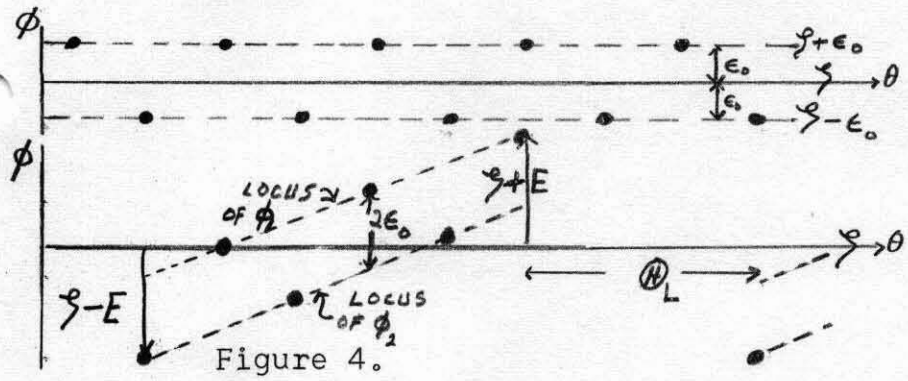

Figure 4.

Besides being tedious to calculate, the large extremes of ϕ may lead to smaller stability limits than in the unperturbed case. If the machine is to scale (constant ρ), ϕ at the ends of the straight section must be $\rho \pm \frac{L}{4R}$, so that long straight sections require large variations of ϕ. However it is possible to have the large values of ϕ only at the ends of the long straight sections and to have ϕ elsewhere only $\rho \pm \epsilon_0$ instead of varying smoothly between the extreme values as above. This is achieved in Geometry 3 described below.

Geometry 3.

Consider the sectors to be defined from magnet center to magnet center of an unperturbed machine. At sector boundaries the equilibrium orbit is tangent to a circle, and sectors of different lengths may be joined without disturbing the equilibrium orbit within any one kind of sector from what it would have been in a machine constructed only of that type of sector. If the magnetic fields are to be continuous across sector boundaries, in the simplest case the "circumference factor" must be the same for all sectors, so that the ratio of magnet length to straight section length is the same for every sector. Figure 5 illustrates a section of such a machine.

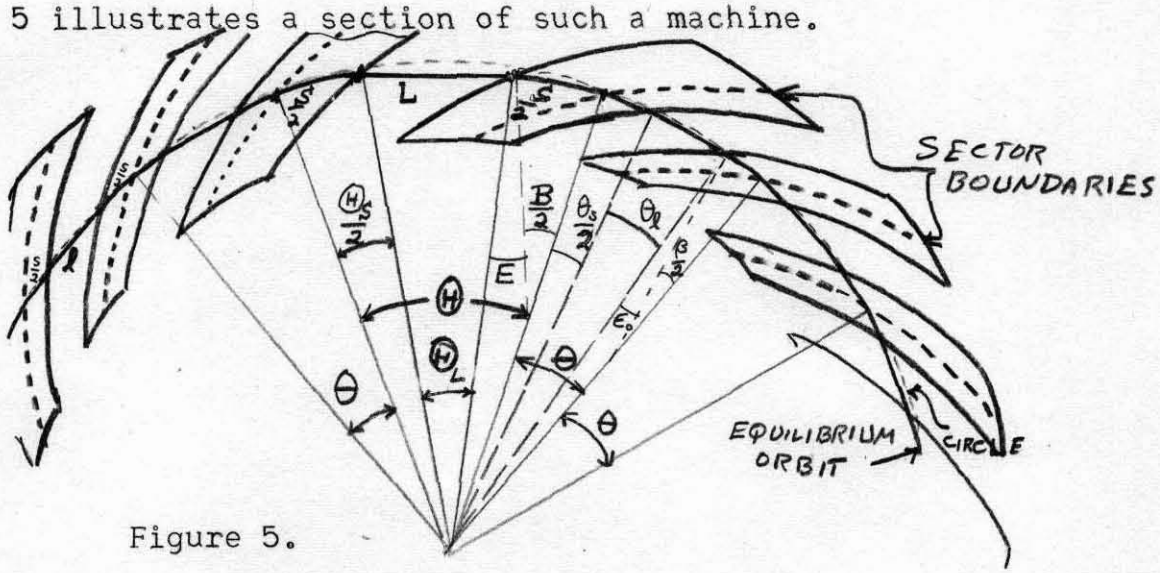

Figure 5.

Since such a machine is made up of parts of machines all identical in h, ξ, and s/ℓ but different in N, the smooth approximation would suggest that insofar as it is valid, ν_s would be unchanged by inserting straight sections in this manner. Since σ_x is quite large, the smooth approximation would be expected to be less valid for the radial motion, and consequently most of the computations have been made for radial oscillations.

Calculations

The introduction of longer straight sections periodically changes (usually increases) the amplitudes of oscillation for a given angle of crossing of the equilibrium orbit. A more important effect is the introduction of stopbands, so that if a long straight section is introduced every m sectors, the stability region is subdivided into m^2 smaller regions separated by bands of instability.

Although the amplitudes of oscillation must certainly be considered, the first problem is to find parameters for realistic machines which will preserve at least stability, and to learn in general how unstable stopbands are related to the long straight sections.

Calculations were made using parameters similar to those in the proposal ($N \simeq 40$, $k \simeq 80$—100, $\varphi \simeq 82^\circ$) and considering extra straight sections up to three times as long as in the unperturbed machines. Geometries 1 and 3 were considered as follows:

Let M_o be the transfer matrix for an ordinary sector and M_L the transfer matrix for the sector containing the extra long straight section. If the superperiodicity occurs every m sectors, then the transfer matrix for the m sectors, $M_m = (M_o)^{m-1} M_L$. If M_o is put in the standard form,

$$M = \begin{pmatrix} \cos\sigma + \alpha \sin\sigma & \beta \sin\sigma \\ -\frac{1+\alpha^2}{\beta} \sin\sigma & \cos\sigma - \alpha \sin\sigma \end{pmatrix},$$

$$\text{then } (M_o)^{m-1} = \begin{pmatrix} \cos(m-1)\sigma + \alpha \sin(m-1)\sigma & \beta \sin(m-1)\sigma \\ -\frac{1+\alpha^2}{\beta} \sin(m-1)\sigma & \cos(m-1)\sigma - \alpha \sin(m-1)\sigma \end{pmatrix}$$

so that M_m may be found as the product of two matrices only. Then

cos σ_m for the m sectors may be found as usual from

$$\cos \sigma_m = 1/2 \, Tr(M_m).$$

The transfer matrices through any one sector are found by the methods of MURA-LWJ-8. This method of determining cos σ_m applies to and was carried out for Geometries 1 and 3; no calculations were made using Geometry 2.

Results

With Geometry 1 and parameters typical of the proposal, cos σ_{mx} was found for various values of m and L/ℓ (these examples are not completely self consistant since N/m = P, the total number of long straight sections, is not integral.) The results are summarized in table 1.

m	$L/\ell = 1$		$L/\ell = 1.5$	$L/\ell = 2$	$L/\ell = 3$
	σ_{mx}	cos σ_{mx}	cos σ_{mx}	cos σ_{mx}	cos σ_{mx}
1	94.8°	-.08	-.505	-.93	-1.78
8	758.4°	.785	.529	.267	- .252
9	853.2°	-.688	-1.008	< -1	<-1
10	948°	-.670	-.361	-.045	.581
11	1042.8°	.797	1.061	> 1	> 1

Table 1

Although because of the non-scaling nature of this example it is of no practical interest, one may see from the figures how the stop-bands develop.

Calculations on self-consistant machines were made using Geometry 3 and m of 5,7,8, and 10. In this case transfer matrices were always

taken from the centers of sectors as described under Geometry 3. Results are summarized in Table II below.

N	m	P	k	ζ	$L/\ell = 1$		$L/\ell = 2$		$L/\ell = 3$
					$\cos m\sigma_x$	$m\,\sigma_x$	$\cos \sigma_{m\,x}$	$\sigma_{m\,x}$	$\cos \sigma_{m\,x}$
40	5	8	90	81.5°	-.42	475°	-4.5		
42	7	6	100	82°	.64	670°	1.69		
40	8	5	90	81.5°	.766	760°	1.78		6.24
40	10	4	90	81.5°	-.642	950°	1.419		
40	10	4	96	81.5°	-.174	980°	-.68	947°	
40	10	4	98	81.5°	.174	1000°	.065	994°	

Table II

From the last three sets of figures it is clear that stable regions may be found, centered in this case near where $\cos m\sigma \cong 0$ for the unperturbed machine. However it appears that the stopbands are very broad (perhaps equal in width to the stable regions) when $L/\ell = 2$ and $m = 10$. Calculations of vertical frequencies have not yet been made for any of the above cases. In addition to calculations of the z motion, k and ζ should be varied to find the stopband profiles and to determine to what extent the stable regions are compressed. This compression of course crowds all imperfection resonances closer together and increases the required alignment and tolerance precision, so that if the stopbands are indeed as wide as the stability regions, the tolerances are as bad as in an unperturbed machine of twice as high ν .