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BEAM LOADING ON AN R.F. CAVITY AND AN ACCELERATION SCHEME BY VELOCITY MODULATION

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When a beam current approaches the order of magnitude of the current in the cavity, the voltage and the phase of the cavity are affected by beam loading. The particles, governed by synchrotron oscillations, would fill "the bucket" uniformly and the density of particles in terms of the phase relative to the R.F. voltage can be expressed as

$$P(\varphi)\,d\varphi = \frac{2\sqrt{2(C+\alpha\cos\phi+\beta_1\phi)}}{A}\,d\phi \qquad (1)$$

Where

$$\begin{cases} A: & \text{Area of the bucket} \\ \alpha = & h\,\dfrac{d\omega}{dE}\,\dfrac{\omega}{2\pi}\,V \\ \beta_1 = & \dot{\nu} \\ C: & \text{constant which determines the bucket size.} \end{cases}$$

The effects of the beam current on the cavity mainly depend on the first Fourier components of the current.

$$I_{eff} \sim A N e v \frac{1}{\pi}\left[\int_{-\pi}^{\pi} P(\varphi')\sin\phi'\,d\phi'\sin\phi + \int_{-\pi}^{\pi} P(\phi')\cos\phi'\,d\phi'\cos\phi\right] \qquad (2)$$

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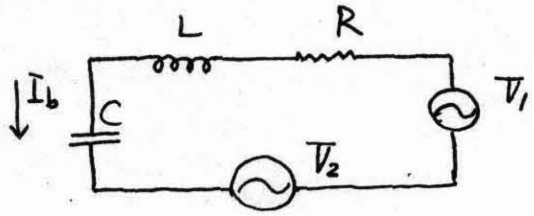
However, if the shape of the bucket remains unchanged, it might be plausible to say that the magnitude of the effective current is proportional to the area of the bucket and its phase is close to φ_s.

$$\begin{cases} |I_{eff}| \propto A \propto V^{1/2} \\ \text{Phase of } I_{eff} \sim \varphi_s \end{cases} \qquad (3)$$

The effects of beam loading on the cavity can be expressed as a voltage generator (or a current generator) connected in series (or in parallel) in the cavity circuit or an admittance parallel to the gap.

The figure shows an equivalent circuit where a series voltage generator is used.



C, L and R are the equivalent circuit of the cavity

V_1 : an R.F. oscillator

V_2 : the equivalent voltage generator of the beam loading of which the phase is delayed by $\pi/2$ relative to the beam phase

We put

$$\begin{cases} I_b = I_{eff} \sin(\omega t - \varphi_b) \\ V_1 = V_{10} \sin(\omega t - \varphi_0) \\ V_2 = V_{20} \sin(\omega t - \varphi_b - \pi/2) \\ V = V_0 \sin(\omega t - \varphi_3) \end{cases}$$

and V_{20} and φ_3 are given by

$$\begin{cases} V_{20} = \frac{\beta}{\omega C} I_{eff} \\ \varphi_3 - \varphi_b = \varphi_s - \pi/2 \end{cases} \tag{4}$$

where β : gap coefficient

φ_s : equilibrium phase of the synchrotron oscillations

We can get V from

$$\vec{V} = (\vec{V_1} + \vec{V_2}) \frac{\frac{1}{j\omega C}}{R + j(\omega L - \frac{1}{\omega C})} \tag{5}$$

1. Resonant Cavity

Since the cavity is working at the resonance frequency

$$L\omega = \frac{1}{C\omega}$$

Hence we get

$$\begin{cases} V = \frac{V_3}{\omega CR} \sin(\omega t - \varphi_3) \\ \varphi_3 = \varphi_0 + \pi/2 - \delta, \end{cases}$$

With

$$\begin{cases} V_3^2 = V_{10}^2 + V_{20}^2 + 2 V_{10} V_{20} \cos(\varphi_0 - \varphi_b - \pi/2) \\ \tan\delta = \frac{V_{20} \sin(\varphi_0 - \varphi_b - \pi/2)}{V_{10} + V_{20} \cos(\varphi_0 - \varphi_b - \pi/2)} \end{cases} \tag{6}$$

or using (4),

$$\tan\delta = \frac{-a\cos(\delta-\varphi_s)}{1+a\sin(\delta-\varphi_s)} \quad \text{or} \quad \sin\delta = -a\cos\varphi_s \tag{7}$$

and

$$V_3 = V_{10}[\,1+a^2+2a\sin(\delta-\varphi_s)\,] \tag{8}$$

$$\text{where} \qquad a = \frac{V_{20}}{V_{10}}$$

a is related to V through $I_{eff} = \lambda[V]^{1/2}$ and also $\sin\phi_s \propto V^{-1}$

$$a = \lambda' \frac{\sqrt{V_3}}{V_{10}}$$

We can get expressions for V and ϕ_s using the above relations.

For example in the case of a standing bucket ($\sin\phi_s = 0$)

$$\left\{ \begin{array}{l} V = \dfrac{1}{\omega CR}\left[\sqrt{V_{10}^2+\dfrac{\lambda'^4}{4}} - \dfrac{\lambda'^2}{2}\right] \\ \sin\delta = \dfrac{\lambda'}{V_{10}}\sqrt{V_3} \end{array} \right. \tag{9}$$

The equivalent admittance $\vec{Y}$ is given by

$$\vec{Y} = \frac{\vec{V}}{\vec{I_b}} = \frac{V_3}{\omega CR\,|I_{eff}|}(\sin\phi_s + j\cos\phi_s) \tag{10}$$

As seen easily, if the bucket is standing, the admittance is purely reactive. And since the magnitude depends on voltage, it is a non-linear element. Since the cavity is resonant at the frequency w, the voltage across the gap without beam is given by

$$V \sim Q V_{10}$$

and the a is given by

$$a = \frac{V_{20}}{V_{10}} = \frac{\beta}{V} I_{eff} \frac{Q}{\omega C} \qquad (11)$$

If we use a shunt impedance instead of a series R
(11) becomes

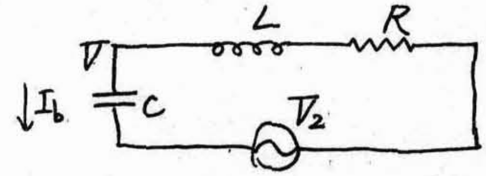
$$a = \frac{\beta I_{eff}}{V_0} R_{shunt} \qquad (12)$$

For example the CERN cavity has a shunt impedance of the order of $3\,k\Omega$ at a few mc/sec and the voltage is $\sim 3\,kV$. So the cavity voltage will be affected by a fairly large amount with a beam current of the order of one ampere.

If a single cavity with a high voltage is used, the beam loading effects are rather small because the oscillator voltage is large enough so that the effects of the induced voltage may be neglected. Using multiple cavities with similar impedances at lower voltage to save power consumption, the voltages induced by the beam become appreciable compared to the oscillator voltage. Since we are dealing with a high intensity beam and since to accelerate it a high power is needed anyway, it might be better to use a single low impedence cavity regardless of low power efficiency. When the cavity is a component of the self-excited oscillator, the situation is different, since it cannot be expressed as a linear voltage generator. Also, the frequency of the system is affected due to the reactance of the beam.

2. Off resonant cavity

If the cavity is not working at the resonant frequency or if it has a broad band characteristic, the phase of the induced voltage can be adjusted by changing the impedance of the cavity.



Let us consider a passive cavity, i.e., without oscillator.

V is given by

$$\left\{\begin{aligned} V &= \frac{V_{20}}{\omega C\sqrt{R^2+(L\omega-\frac{1}{\omega C})}}\sin(\omega t-\varphi_b+\tfrac{\pi}{2}+\delta_1) \\ \tan\delta_1 &= \frac{R}{L\omega-\frac{1}{\omega C}}, \end{aligned}\right. \tag{13}$$

Hence $\qquad \varphi_s = \pi+\delta_1.$

Since

$$\sin\delta_1 = \frac{R}{\sqrt{R^2+(L\omega-\frac{1}{\omega C})^2}} > 0 \quad \text{and} \quad \sin\phi_s < 0$$

$$\cos\delta_1 = \frac{L\omega-\frac{1}{\omega C}}{\sqrt{R^2+(L\omega-\frac{1}{\omega C})^2}} \gtrless 0 \quad \text{and} \quad \cos\phi_s \gtrless 0,$$

we can choose the sign of $\cos\phi_s$ to make ϕ_s a stable equilibrium phase depending on which side of the transition energy the particles are on. Then particles will be decelerated stably around ϕ_s and R can be changed slowly to keep ϕ_s constant (φ_s varies with R, ω and Ieff.) In order to make R zero or even positive, a wide band amplifier can be used. Free oscillations of the circuit are prevented by choosing the free oscillation frequency of the circuit well outside of the band width of the amplifier. Then R in the equivalent circuit can be zero or negative. With negative R the particles will be accelerated around the stable equilibrium phase ϕ_s and ϕ_s can be kept constant by changing the magnitude of the negative resistance or the gain of the amplifier and by changing the reactance of the cavity, if necessary.

The above acceleration method, however, might not work, because of the instability of the system as a whole. If the beam current tends to decrease due to any disturbances, the induced voltage and the bucket size will decrease and consequently tend to decrease the beam current. So the stability of the whole system must be considered. The equivalent admittance is given by

$$\vec{Y} = \frac{\vec{I_b}}{\vec{V}} = \frac{(\omega c)^2\sqrt{R^2+(L\omega-\frac{1}{\omega c})^2}}{\beta}\left[\cos(\pi/2+\delta_1) - j\sin(\pi/2-\delta_1)\right]$$

and is a linear element.

As well known, if all elements are linear, there is no stable amplitude of the oscillation and any kind of oscillator needs some non-linear elements, such as a vacuum tube to operate at a certain amplitude level. In the above equivalent circuit, the negative R corresponds to a vacuum tube in a simple oscillator and we must make this resistance non-linear to make the system stable.

$$R = R(V) \quad \text{or} \quad (R(I))$$

The stability depends on $\frac{dR}{dV}$ at the working voltage $\mathbf{V}$.

3. Velocity modulation

In a linear Klystron electrons receive velocity modulation at the first cavity and this appears as a density modulation at the second cavity. The phase of the induced voltage at the second cavity is such that the induced voltage decelerates electrons on the average. The kinetic energy of the electrons goes into R.F. power in the cavity. We may consider the reverse process, i.e., we energize the second cavity from an outside oscillator and adjust its phase in such a way that the particles receive an acceleration on the average. The applied voltage of the second cavity should be large enough to neglect the effects of beam loading.

To apply this scheme to a circular accelerator, two cavities are placed in a straight section. The frequency of the R.F. is chosen so high that the particles will forget their previous phases relative to the cavities after one turn, as a result of the changes of orbit length due to betatron oscillations

Then the particles come back to cavity No. 1 at random phases and the process will be repeated.

The R.F. component of the beam current I at cavity #2, after receiving velocity modulation at the cavity #1, is given by

$$I = 2 I_0 J_1(a) \sin(\omega t - \varphi_1 + \pi/2 - \theta)$$

ℓ : distance between cavities

ω : angular frequency of the cavities

$\left.\begin{matrix} V_1, \varphi_1 \\ V_2, \varphi_2 \end{matrix}\right\}$: voltage and phases of the cavities

a : bunching parameter $\qquad a = \dfrac{\omega \ell k \beta}{v_0}$

$\theta = \dfrac{\omega \ell}{v_0}$, $\qquad v_0$: initial velocity

$k = \dfrac{\Delta v}{v_0}$: depth of the velocity modulation

β : gap coefficient, $\quad 0 < \beta < 1$

If the phase difference between the cavities satisfies

$$\varphi_2 - \varphi_1 = \theta - \pi/2$$

the current and the voltage V_2 are in phase and the particles are accelerated on the average.

To follow the change of energy and velocity of the particles, one may vary the frequency or the drift space length or phase modulate cavity #2.

It must be noted that the frequency of the cavities can be chosen independent of the particle frequency and the frequency is so high that R.F. knockout might not hurt the beam.

As the second cavity, a non-resonant cavity as described in the previous section can be used, if the beam intensity is high, all parameters in the cavity remain constant since the frequency is not necessarily changed. But the depth of the modulation decreases with increase of the particle velocity.

In the above method of acceleration, the energy of the particles is much higher than the R. F. voltage and the depth of the modulation is quite low, while in the Klystron the energy of the electrons and the R. F. voltage are of the same order. Consequently, the efficiency of the acceleration method is extremely low at higher energies, because of the small depth of the modulation.