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AXIAL-AMPLITUDE LIMITATIONS EFFECTED BY $\sigma_x + 2\sigma_y = 2\pi$

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ABSTRACT

Evidence, based on Feckless Five computations, is presented appearing to support Parzen's suggestion that the $\sigma_x + 2\sigma_y = 2\pi$ resonance, rather than $\sigma_y = 2\pi/3$, is responsible for the limit of stable y-amplitude in spirally-ridged accelerations free of imperfections. The computations covered a small number of structures with $k = 0.2$, $f = 1/4$, and $N = 5$, for which σ_x was in the neighborhood of $\pi/2$.

1. Introduction:

The question has been raised by Parzen (Madison summer session) whether the stable limit of y-amplitude observed[1] in Feckless Five[2] runs with σ_x near 0.6π is attributable to the $\sigma_x + 2\sigma_y = 2\pi$ resonance rather than to $\sigma_y = 2\pi/3$. Because of the importance of this question in connection with the design of spirally-ridged (or separated-sector) FFAG accelerators,[3] a quick computational examination was made to distinquish between the two possibilities. The computations were performed by aid of the Feckless Five ILLIAC Program. The results of this study are summarized below and, although unfortunately carried out with σ_x undesirably close to $\pi/2$, appear to substantiate Parzen's proposition.

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2. Results:

The parameters and characteristics of the structures studied are summarized in Table I. The results of 80-sector searches for the axial stability limit are also included in the table. In all cases the x-motion was started substantially on the fixed point. Figures 1 and 2 depict the y-stability limit, expressed in terms of the initial value y(0) with $y^1(0) = 0$, as a function of $1/w$ and of σ_{y_0}/π.

3. Conclusion:

The results of this brief survey appear to substantiate Parzen's suggestion that the $\sigma_x + 2\sigma_y = 2\pi$ resonance, rather than $\sigma_y = 2\pi/3$, is responsible for the limitation of stable axial motion in this region of the working diagram for a structure free of misalignments. It is expected that this matter will receive further study. It may be of interest to mention in closing that it has been conjectured that generally, in structures free os misalignments, resonances of the form

$$p\sigma_x + q\sigma_y = r(2\pi) \qquad (p,q,r = \text{integers})$$

are significant only if q is even.

4. References:

1. F. T. Cole, L. J. Laslett, and J. N. Snyder, Bull. Amer. Phys. Soc., Ser. II, #4, Paper G5 (April 26, 1956).

2. L. J. Laslett, MURA Report LJL(MURA)-5 (July 30, 1955), Appendix II.

3a. D. W. Kerst, et al., Bull. Amer. Phys. Soc. 30, #1, Paper D5 (January 27, 1955).

b. K. R. Symon, et al., "Fixed Field Alternating Gradient Particle Accelerators" (to be published in The Physical Review).

c. Pop. Mech. 106, #1, P. 94 (July 1956).

TABLE I

EXAMINATION OF 80-SECTOR AXIAL STABILITY LIMIT
IN THE NEIGHBORHOOD OF $\sigma_y = 2\pi/3$ AND $\sigma_x + 2\sigma_y = 2\pi$

k = 0.2 f = 1/4 N = 5

$\frac{1}{\omega}$	Small-Ampl. Freqs. σ_{x_0}/π	Small-Ampl. Freqs. σ_{y_0}/π	Small-Ampl. Freqs. $\sigma_{x_0}/\pi + 2\sigma_{y_0}/\pi$	Last Unstable Run Initial y_0	First Stable Run Initial y_0	First Stable Run Max. $\lvert y\rvert$ $N\theta = 0 \bmod 2\pi$	First Stable Run Max. $\lvert y\rvert$ $N\theta = \pi \bmod 2\pi$	π_f	p_{x_f}
29.69	0.4782	0.626	1.730	0.00530	0.00460	0.0055648	0.0126041	−.002400	−.054863
30.49	0.4809	0.650	1.781	0.00460	0.00400	0.0052565	0.0126207	−.002409	−.054863
31.00	0.4811	0.666	1.812	0.00348	0.00300	0.0035865	0.0089536	−.002415	−.054905
31.5243	0.4830	0.682	1.847	0.00264	0.00230	0.0028005	0.0073961	−.002421	−.054921
32.2167	0.4851	0.702	1.889	0.00174	0.00150	0.0018110	0.0052916	$-.002428_5$	−.054940
32.9091	0.4871	0.729	1.945	0.00087	0.00075	0.0009169	0.0029405	−.002432	−.054958
33.5555	0.4891	0.753	1.995	($\cong 0.00037_5$)		----	----	−.002443	−.054973
34.7676	0.4930	0.805	2.103	0.00150	0.00132	0.0016944	0.0077997	−.002560	$-.054998_5$

FIG. 1.

AXIAL STABILITY LIMIT

IN NEIGHBORHOOD OF $\sigma_x + 2\sigma_y = 2\pi$ & $\sigma_y = 2\pi/3$

Results of 80-sector FF Search

× Unstable ⊙ Holds on

$k = 0.2$ $f = 1/4$ $N = 5$

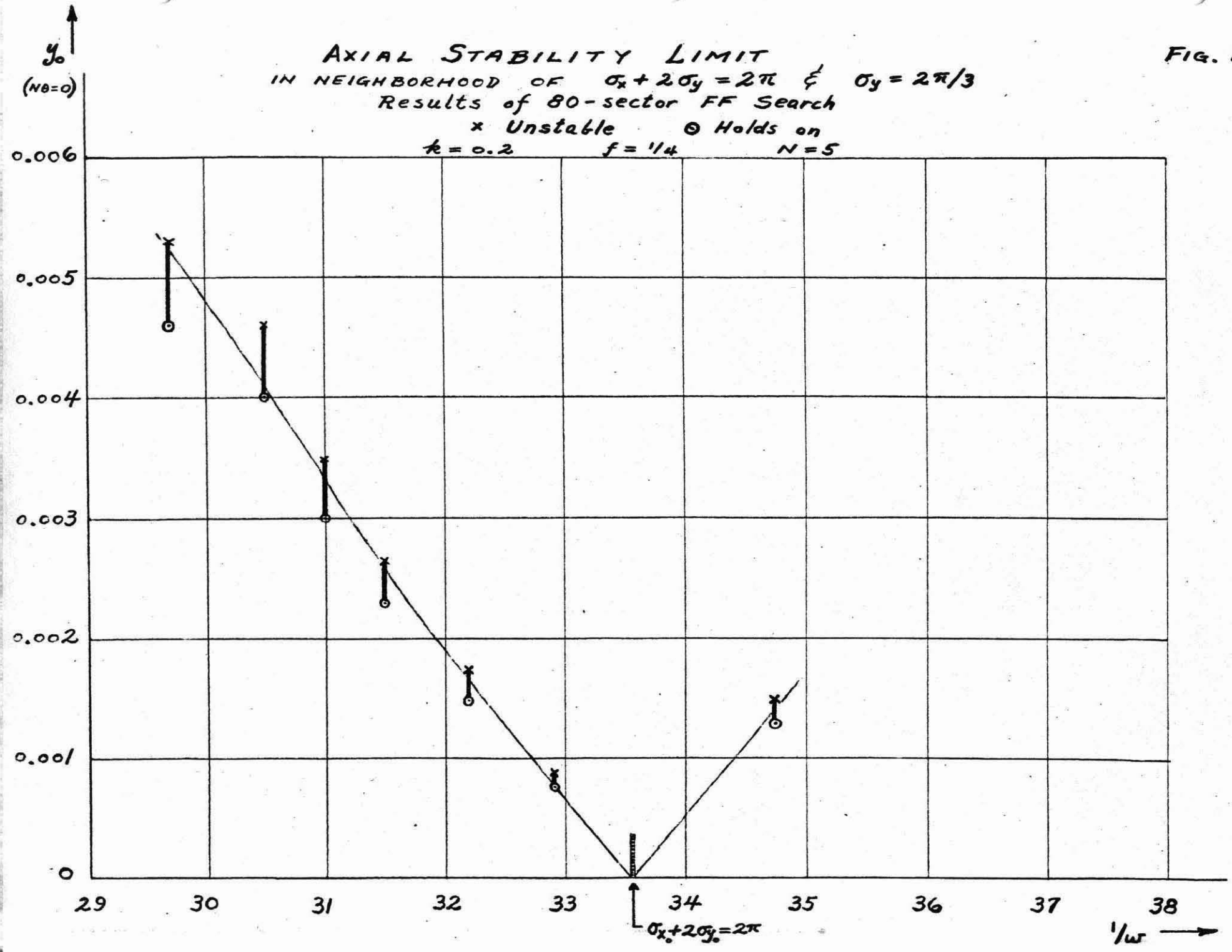

FIG. 2.

AXIAL STABILITY LIMIT

IN NEIGHBORHOOD OF $\sigma_x + 2\sigma_y = 2\pi$ & $\sigma_y = 2\pi/3$

Results of 80-sector FF Search

× Unstable ⊙ Holds on

$k = 0.2$ $f = 1/4$ $N = 5$