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EFFECTS OF BUMPS WHEN MAGNET EDGES ARE IMPORTANT

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ABSTRACT: It is possible, using impulse approximations, to estimate quickly the effects of magnet misalignments in accelerators where magnet edge effects are important such as spiral sector and small radial sector designs. The results of this simple calculation agree with a more precise calculation for the parameters of the Michigan Radial Sector betatron. The method is also used to estimate misalignment effects in a 15 Bev spiral sector design.

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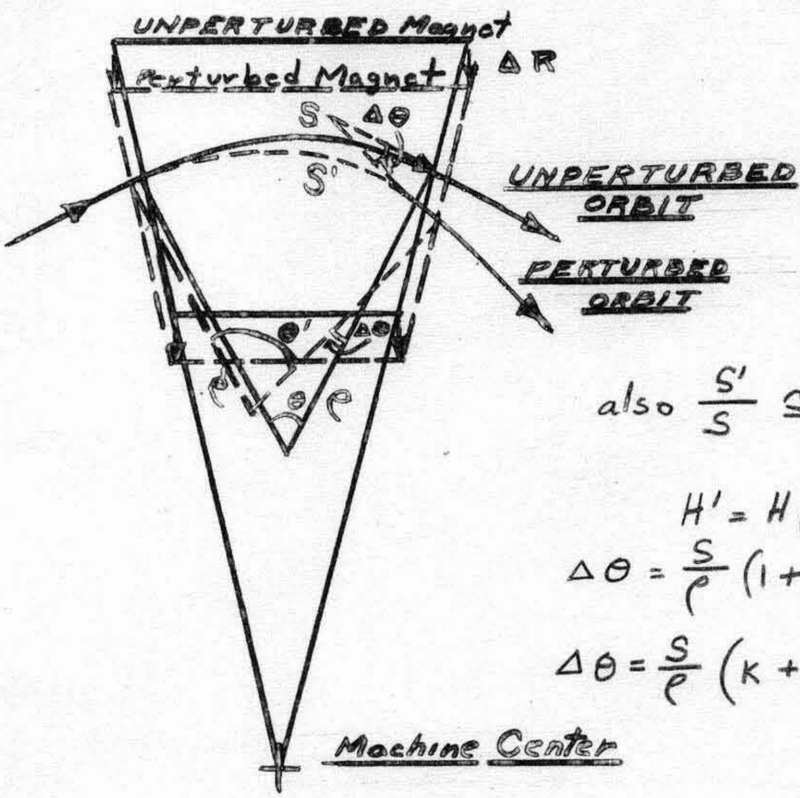
In small radial sector accelerators, such as the Michigan Model, or in any separated spiral sector accelerator the amplitude of the closed equilibrium orbit may be estimated using simple impulse methods including the effects of edges. Assuming the wavelength of betatron oscillation is long compared to the azimuthal extent of the bump (here assumed to be one magnet), the new closed orbit will be a curve which resembles a betatron oscillation (about the unperturbed equilibrium orbit) closing on itself at the bump with an angle .

Determination of $\Delta\theta$

We may find $\Delta\theta$ for the effect of a radially displaced sector on radial oscillations as follows. In the unperturbed machine the orbit is S with radius of curvature ρ in an average field H subtending an angle θ. In the perturbed machine the same notation with primesis used. Consider a scaling sector moved radially inward.



$$\theta = \frac{S}{\rho}$$

$$\theta' = \frac{S'}{\rho'}$$

$$\Delta\theta = \theta' - \theta = \frac{S'}{\rho'} - \frac{S}{\rho}$$

Since $H'\rho' = H\rho$,

$$\frac{1}{\rho'} = \frac{1}{\rho}\,\frac{H'}{H}$$

also $\frac{S'}{S} \cong \frac{R + \Delta R}{R}$ for scaling machines.

$$H' = H\left(1 + \frac{K\Delta R}{R}\right)$$

$$\Delta\theta = \frac{S}{\rho}\left(1 + \frac{\Delta R}{R}\right)\left(1 + \frac{K\Delta R}{R}\right) - \frac{S}{\rho}$$

$$\Delta\theta = \frac{S}{\rho}\left(K + 1\right)\frac{\Delta R}{R}$$

This expression is independent of magnet spiraling and of scalloping of the equilibrium orbit; i.e., as long as the sector scales, there are no explicit edge effects. The term + 1 in the bracket is due to the extra arc length in the magnet and is important in machines with model-like parameters.

The effect of a vertical magnet displacement on the vertical oscillations may be found by adding three contributions to $\Delta\theta$; the contributions due to the two edges and to the gradient. If the magnet is displaced by z vertically and the orbit enters the magnet at an angle ϕ from the normal to the magnet edge, $\Delta\theta$ due to that edge is given by

$$\Delta\theta = \frac{z \tan\phi}{\rho}$$

since $\Delta\theta = \frac{z}{f}$

and $\frac{1}{f} = \frac{\tan\phi}{\rho}$.

$\Delta\theta$ is negative if the edge is vertically focusing and positive if it is defocusing. Due to the gradient in the magnet, there will be the usual contribution to $\Delta\theta$ from the radial component of field

$$\Delta\theta = \frac{ksz}{\rho R}$$

Since $k = \frac{R}{H_o}\frac{\partial H_z}{\partial x} = \frac{R}{H_o}\frac{H_x}{z}$

when $H_x = 0$ in the median plane,

and $\Delta\theta = \frac{H_x S}{H_o \rho}$.

The same sign assignment holds for this $\Delta\theta$. Therefore in traversing a complete vertically displaced sector a particle experiences $\Delta\theta = \frac{z}{\rho}\left[\pm \tan\phi_1 \pm \frac{ks}{R} \pm \tan\phi_2\right]$.

For a radial sector magnet $\tan \phi_1 = \tan \phi_2$ and both are negative quantities. For a magnet of a separated sector spiral machine the smaller tangent term and the k term are positive while the larger tangent term is negative.

Amplitude due to $\Delta\theta$

If there is only one bump present, the equilibrium orbit will be symmetric about it and the amplitude, A, of the equilibrium orbit will be given, assuming sinusoidal oscillations, by

$$A_s = \frac{R\,\Delta\theta}{2\nu \sin\pi\nu}$$

If there are points of symmetry in sectors as in radial sector machines, the maximum amplitude measured at points of symmetry when the bump is also at a point of symmetry is

$$A = \frac{\nu\rho}{R}\beta A_s = \frac{\nu\rho}{R}\beta\frac{R\Delta\theta}{2\nu\sin\pi\nu}$$

where β is the coefficient in the one sector transfer matrix M;

$$M = \begin{pmatrix} \cos\sigma + \delta\sin\sigma & \beta\sin\sigma \\ \frac{-1+\delta^2}{\beta}\sin\sigma & \cos\sigma - \delta\sin\sigma \end{pmatrix}$$

since at points of symmetry δ - 0[1]. The independant variable in this formulation is length in units of the radius of curvature, and β is dimensionless.

In the ordinary A.G. or the radial sector case, the two points of symmetry correspond to maximum and minimum values of β, hence there are four special cases of interest:

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1. This is a particular case of the more general treatment given by Laslett (Brookhaven report LJL 7).

$A = \beta_{max} \frac{\nu\rho}{R} A_s$, bump and detector at point of β_{max},

$A = \sqrt{\beta_{max}\beta_{min}} \frac{\nu\rho}{R} A_s$, bump at point of β_{max} and detector at point of β_{min}, bump at point of β_{min} and detector at point of β_{max},

$A = \beta_{min} \frac{\nu\rho}{R} A_s$, bump and detector at point of β_{min}.

In spiral sector accelerators there are no points of symmetry.

Examples

In the Michigan radial sector model the detector is in a straight section. Since negative magnets are quite narrow we will assume the oscillation amplitude is the same as would be observed by a detector in a negative magnet. We also assume the bumps as impulsive and at the centers of magnets. The working expressions are summarized below:

Vertical: wide magnet displaced: $\frac{A}{Z} = \sqrt{\beta_{1v}\beta_{2v}} \frac{\rho\left(\frac{k s_1}{2\rho} - \frac{R}{\rho}\tan\phi\right)}{R \sin\pi\nu_z}$

narrow magnet displaced: $\frac{A}{Z} = \beta_{2v} \frac{\rho\left(\frac{k s_2}{2\rho} + \frac{R}{\rho}\tan\phi\right)}{R \sin\pi\nu_z}$

Radial: wide magnet displaced: $\frac{A}{\Delta r} = \sqrt{\beta_{1r}\beta_{2r}} \cdot \frac{\rho(k+1)s_1}{2R\rho\sin\pi\nu_r}$

narrow magnet displaced $\frac{A}{\Delta r} = \beta_{2r} \frac{\rho(k+1)S_2}{2R\rho\sin\pi\nu_r}$,

where subscripts 1 and 2 refer to wide and narrow magnets. Cole has found "hard-edge" parameters which closely represent the characteristics of the Michigan radial sector FFAG model and from which matrix calculations of the effects of misalignments were made.[2] These results agree well when plotted against the experimental points. The maximum amplitudes computed from this matrix calculation may be compared with the values from the above expressions, as a test of this approximate theory.

The equivalent hard edge parameters[2] are given below.

$$k = 3.36 \qquad \beta_{1r} = 2.40^{3}$$

$$\frac{s_1}{\rho} = 1.20 \qquad \beta_{2r} = 0.45$$

$$\frac{s_2}{\rho} = .415 \qquad \beta_{1v} = 0.95$$

$$\frac{R}{\rho} \cong 2.45 \text{ (average)} \qquad \beta_{2v} = 2.75$$

$$\phi \cong 19^{\circ} \text{ (average)} \qquad \nu_r = 2.712$$

$$\nu_z = 1.733$$

The results are compared with the matrix calculations in the following table.

		Present Approximation	Reference 2
Vertical:	wide magnet displaced	1.05	0.81
	narrow magnet displaced	2.33	1.94
Radial:	wide magnet displaced	1.41	1.65
	narrow magnet displaced	0.21	0.31

2. MURA-203

3. The value of β were found by Cole for these hard edge parameter, although they do not appear in MURA 203 or earlier references.

Considering the nature of the approximations and the fact that this calculation is for the center of negative magnets while that in reference 2 is for centers of straight sections, the agreement seems good.

Applying this method to the parameters for a 38 sector 15 Bev separated sector machine (considering only centers of magnets),

$$\frac{A}{z} \cong \frac{\nu_z \rho \beta_z}{R} \cdot \frac{[ks + R(\tan \phi_2 - \tan \phi_1)]}{2\rho \nu_z \sin \pi \nu_z}$$

$$\frac{A}{\Delta r} \cong \frac{\nu_r \rho \beta_r}{R} \cdot \frac{(k+1)s}{2\rho \nu_r \sin \pi \nu_r}$$

where

$k = 82.5$ $\qquad \nu_r = 10.8$

$S = .078R$ $\qquad \nu_z = 4.85$

$\rho = 0.5R$

$\phi_1 = 83.75^\circ$

$\phi_2 = 79.25^\circ$

assuming $\dfrac{\nu_z \rho \beta_z}{R} = \dfrac{\nu_r \rho \beta_r}{R} \cong 2$,

$$\frac{A}{z} \cong 1.5 \;, \quad \frac{A}{\Delta r} \cong 1.7.$$

It is worth noting that the net effect of a vertical displacement is smaller than any of its three components, and this seems to be a definite asset to the separated sector structure. This is analogous to the reduction in sensitivity to perturbations in a conventional A. G. machine when focusing and defocusing magnets are combined into single units.