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A Simple Method for RF Acceleration in a Fixed Field Machine*

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In planning any method of radio frequency acceleration, a number of factors must be considered in addition to the usual phase space considerations. Among these factors are the problems of scattering by the residual gas, the limitations caused by space charge, resonances caused by coupling of the RF cavity to betatron oscillations, and the restrictions caused by the practical limitations on the oscillators. Thus, although it seems desirable at first glance to stack particles at low energy, space charge and gas scattering limitations rule out this procedure.

Gas Scattering

Betatron oscillations are induced in particles due to Coulomb scattering by the residual gas.[1] The importance of this phenomenon decreases strongly with energy, so that it becomes important to accelerate particles rapidly at the low energies. In particular, "bucket lift" arrangements, in which particles slip from harmonic to harmonic as they are accelerated, are deficient in this respect. Gas scattering also imposes a limitation on how long particles can be stacked at a given energy; a good criterion to use is that the amplitude of oscillations built up while particles are stacked should be smaller than the oscillations induced while the particles are being accelerated. Gas scattering questions become rapidly less important

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as the pressure in the vacuum chamber falls below 10^{-6} mm.

Space Charge

Carl Nielsen has calculated the effect of space charge in changing the frequency of betatron oscillations.[2] If we impose the condition that the change in betatron oscillation frequency due to beam space charge be no greater at any time than at injection, we get

$$\frac{N}{N_i} = \frac{P_i \, E_i}{P E} \qquad (1)$$

where N is the number of particles that can be stacked at any energy E (including the rest energy), and P is the corresponding momentum. The subscript i refers to the value at injection. Equation (1) includes the effect of adiabatic damping.

Electrons captured by the beam will also change the frequency of betatron oscillations. As long as particles are in buckets, electrons can escape from the beam, and their effect will be small. In stacked beams, however, electrons can become important, and it may be necessary to have additional oscillators running tuned to the stacking frequencies to prevent the collection of electrons.

Nielsen has pointed out that the longitudinal repulsion of the beam space charge may have a serious effect on synchroton oscillations. Since the effect is proportional to the harmonic number of the oscillator, it is desirable to operate the oscillator on the fundamental frequency. If longitudinal space charge repulsion is the factor that limits the number of particles that can be held in a bucket, we must satisfy the relation

$$\frac{N}{N_i} = \frac{E_i^2}{E^2} \qquad (2)$$

at all stacking energies.

RF Knockout[-3]

It is clear that particles in synchronism with an RF voltage cannot be given excessive betatron oscillations by this voltage. This is true because an RF bump will not be different from a magnetic bump in its effect upon betatron oscillations. For particles not in buckets, however, the effect of an oscillator is like that of a bump whose frequency can be different from the going around frequency of the particles. In fact, the particles will experience a whole series of frequencies $n\omega \pm \nu_0$ where ω is the particle frequency, ν_0 is the oscillator frequency, and n is any integer. If ν_x is the number of radial oscillations around the machine and ν_y the number of vertical oscillations, then there is a possibility for resonance whenever

$$\pm(n_x\omega\nu_x + n_y\omega\nu_y) = m(n\omega \pm \nu_0), \quad n_x + n_y \leq 4$$

where m, n_x and n_y are integers. This can be written

$$m\nu_0/\omega = n \pm (n_x\nu_x + n_y\nu_y) \tag{3}$$

as the condition for possible resonance. From this equation it can be seen that the fewest resonances will be crossed for any given percentage frequency modulation of ν_0 if $\nu_0/\omega \sim 1$. Thus again we see that it is desirable to operate on the fundamental. It will require additional study to see which of the resonances of Eq (3) are important. However, Jones and Terwilliger in experiments with the Michigan model, have found RF knockout when

$$\nu_0/\omega = n \pm \nu_x \quad \text{or} \quad n \pm 2\nu_x \tag{4}$$

so that these resonances, at least, are important.

Oscillator Limitations

The accelerating scheme which is chosen will depend on the range over which an oscillator can be frequency modulated and on the repetition rate of the oscillator. If an oscillator operates on the fundamental (50-500 kc), it is not unreasonable to expect to frequency modulate by a factor 2 or 3 with a repetion rate of more than 100 per second.* Provided such an oscillator is not objectionable for extraneous reasons (such as interfering with radio broadcasting), it should be possible to design a simple RF scheme to accelerate many particles (of the order of 100 buckets per second) to an energy of 15 Bev. Two such schemes are considered in the next section.

RF Schemes

We choose to design an acceleration scheme for the machine of the MURA proposal.[4] We express all energies in units of $E_0 = 938$ Mev and all frequencies in terms of the transition frequency $\omega_t = .635 \times 10^6$ sec^{-1}. We denote kinetic energies by T and total energies by E. The subscript i will indicate the value of any quantity at injection and the subscript $k (k = 1, 2 \cdots \mathcal{N})$ will indicate the value after being accelerated by the k-th oscillator (there being $\mathcal{N}$ oscillators in all).

The basic idea will be to run the oscillators in relay, each oscillator handing a bucket of particles to the next one and returning to get another bucket from the previous oscillator (or from the injector). We arbitrarily choose an injection energy $T_i = .005$ (4.7 Mev) corresponding to a particle frequency $\omega_i = .11$, and let $V = 5 \times 10^{-5}$ (47Kev) be the maximum energy

*This statement is a result of conversations with several members of the MURA technical group.

per turn supplied by all oscillators.[5]

We now make use of a number of results obtained by Symon and Sessler.[6] Defining a quantity $W = \int d E/\omega$, these authors show that

$$\dot{W} = \gamma V \qquad (5)$$

where

$$\gamma = \dot{\omega}/V\, Y(E)$$

and the dot indicates the time derivative. The function $Y(E) = \omega \frac{d\omega}{dE}$ is plotted in Fig. 1, reference 6.[7] If we accelerate particles at constant γ, Eq. (5) becomes

$$W = \gamma V t \qquad (6)$$

so that we can give oscillators equal duty factors by letting them accelerate particles over equal increments of W. The function W is plotted against T in Fig. 1

We assume that we inject particles until the space charge limit is reached. In Scheme No. 1 we assume that the number of buckets that can be stacked at any given energy is limited by Eq. (1). In Scheme No. 2 we assume that the limitation is Eq. (2).

We try to avoid RF knockout by satisfying the requirement that an oscillator which stacks particles must not be frequency modulated by more than 10%. This requirement prevents crossing the obvious resonances of Eq. (4) and prevents crossing a number of additional resonances of Eq. (3). If further study should reveal that important resonances are still crossed, it will be necessary to add a "stacking oscillator" at each stacking position whose only function will be to remove particles from the influence of the preceding oscillator. These stacking oscillators need to be frequency modulated only a negligible amount.

Acceleration scheme No. 1 is shown in Table 1. Gas scattering is unimportant for this proposal, since particles are accelerated rapidly at low energies. The fact that 10 buckets are stacked by oscillator No. 7 and 7 by oscillator No. 9 comes from the space charge limitation of Eq. 1. Only two buckets are stacked by oscillator No. 10 because no more are needed. If more buckets were stacked it would only mean a lower duty cycle on the last oscillator. The γ's are chosen from the requirement that buckets be large enough to hold all the particles.[8] These calculations are based on the assumption that the area in phase space occupied by the particles at injection is $A_i = 2\pi A W_i = 3 \times 10^{-4}$. This is substantially the same number as the one given in the MURA proposal.

The time required for the first oscillator to accelerate particles is given by Eq. (6). It is t = .0037 sec. If we give the oscillators a duty factor of .4 so that no oscillator will interfere with particles being accelerated by neighboring oscillators, we get t = .0092 sec. for the effective time to accelerate a pulse. This means that 109 pulses can be accelerated per second. If the space charge limitation at injection is 5×10^{12} particles per pulse, this acceleration scheme will bring of the order of 5×10^{14} particles per second to 15 Bev final energy.

Acceleration scheme No. 2 is shown in Table 2. The limitation given by Eq. (2) is much more severe than that of Eq. (1), since the reduction of the longitudinal space charge effect is a purely relativistic phenomenon. We have assumed that the first oscillator can be frequency modulated by a factor 3. This scheme is a factor 2 slower than scheme No. 1 so that 54 pulses are accelerated per second. To avoid RF knockout in this scheme, we have imposed the restriction that a stacking oscillator can be

frequency modulated by no more than 2%. An attractive feature of the schemes proposed here is that buckets can be transferred adiabatically from oscillator to oscillator with very small loss in phase density. This means that a high current densities can be achieved in the final circulating beam, a condition which is important for colliding beams.

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FOOTNOTES

1. See D. B. Lichtenberg, MURA DBL-1 for a further discussion of gas scattering.
2. Carl Nielsen, private communication. Most of the information on space change in this section was pointed out to me by Professor Nielsen.
3. Much of the material in this section evolved from conversations with Professor Keith Symon.
4. Program and Proposal For MURA High Energy Research Facility (Apr. 5, 1956) Appendix E.
5. Since the proposals given here are for illustrative purposes only, the numbers which are quoted were computed only roughly.
6. K. R. Symon and A. M. Sessler, MURA-KRS/AMS-1.
7. The notation of Ref. 1 is $Y(E) = (\omega^2 K/E) E_0/\omega_t^2$.
8. See Ref. 6 for ~~an~~ information about bucket areas.

R. F. Acceleration Scheme No. 1

$T_i = .005$ $\omega_i = .11$

Oscillator Number	Final Frequency ω/ω_t	Final Energy T/E_0	γ	No. of pulses stacked
1	.21	.020	.8	1
2	.31	.045	.8	1
3	.41	.081	.8	1
4	.48	.12	.8	1
5	.53	.16	.8	1
6	.60	.21	.8	1
7	.65	.27	.8	10
8	.87	1.0	.8	1
9	.94	1.9	.8	7
10	1.0	6.8	.6	2
11	1.0	15.0	.6	

R. F. Acceleration Scheme No. 2

$T_i = .005 \quad \omega_i = .11 \quad \gamma = .8$

Oscillator	Final Frequency	Final Energy	No. of pulses stacked
1	.32	.048	1
2	.49	.13	1
3	.62	.24	1
4	.72	.38	1
5	.73	.54	1
6	.82	.70	2
7	.88	1.0	1
8	.89	1.5	1
9	.94	1.8	2
10	.98	2.6	1
11	.99	3.4	4
12	1.0	6.6	3
13	1.0	15.	

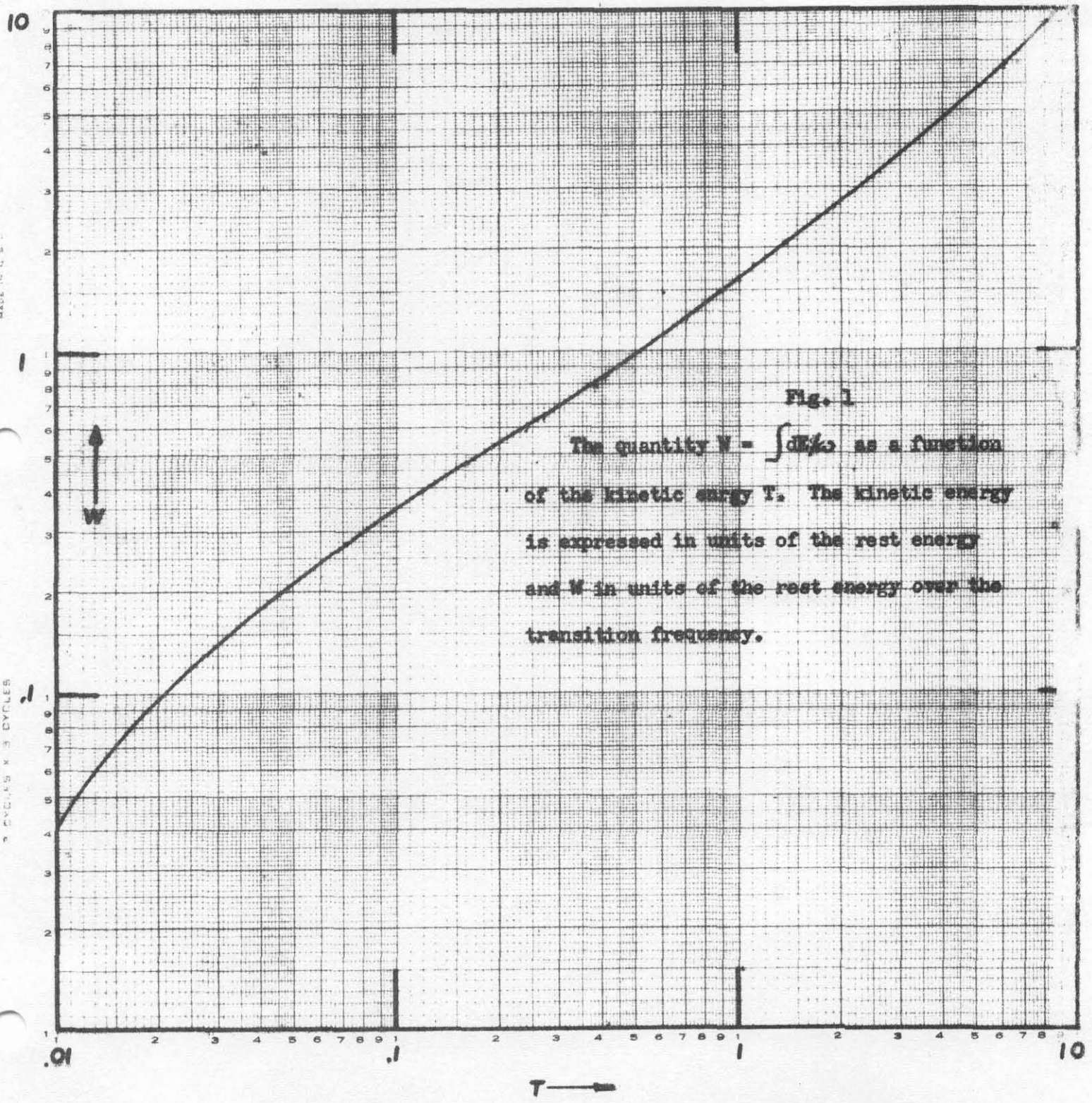

Fig. 1

The quantity $W = \int dE/\omega$ as a function of the kinetic energy T. The kinetic energy is expressed in units of the rest energy and W in units of the rest energy over the transition frequency.

FIG. 1.