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RESULTS OF MISCELLANEOUS TWO-DIMENSIONAL POTENTIAL COMPUTATIONS, INVOLVING POLE-FACE CURRENTS, OF POSSIBLE INTEREST IN FFAG MAGNET DESIGN

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ABSTRACT

For orientation purposes a few two-dimensional potential computations have been made with the ILLIAC to study fields of possible interest in FFAG structures. The results which are reported thus supplement those summarized in MURA-LJL-9. One case concerns a structure similar to the reversed-field FFAG model which is in operation at the University of Michigan.

I. INTRODUCTION

With the object of achieving median-plane fields which might be of interest in separated-sector FFAG accelerators, a few relatively-simple pole configurations with pole-face currents have been studied with the ILLIAC by means of the two-dimensional "FL" program. In the event that any of the resulting fields, or fields similar thereto, should be considered of sufficient interest, further studies could be undertaken by means of the "Scapocyl" and "Stormesh" programs. The general objective was to obtain conveniently (i) a substantially sinusoidal median-plane field and (ii) a flat-topped field containing a third-harmonic component of relative amplitude near -1/9 (cf. MURA-JLP-6).

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As a second part of the study, the median-plane field produced by a pole configuration similar to that of the Michigan reversed-field model was obtained. The harmonic content of this field may be informative in formulating future computational programs based on a Fourier representation of the field.

II. RESULTS

A.

The configurations employed in the first series of runs and the median-plane fields which were found are depicted in Figs. 1-8. A summary of the results in the manner employed in MURA-LJL-9 (7 June 1956) is given in Table I.

TABLE I

SUMMARY OF CHARACTERISTICS OF MEDIAN-PLANE FIELDS

Run No.	Figs.	g/λ	$\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1$	$f_{eff.}$	g	(C.F.)	g(C.F.)
FLSL02	1-2	4/36 = 1/9	0.197597	0.62864	0.43888/q	1.612	0.7077/q
FLSL03	3-4	do.	0.191447	0.61878	0.43199/q	1.634	0.7057/q
FLSL04	5-6	do.	0.208437	0.64566	0.45075/q	1.572	0.7085/q
FLSL05	7-8	do.	0.528443	1.02805	0.71771/q	1.884	1.3521/q

It appears that the configurations of Figs. 1 and 3 give a variation of the median-plane field which is quite close to a pure sinusoid. The configurations of Figs. 5 and 7 give fields (Figs. 6 and 8, respectively) which are materially flatter in the regions of maximum and minimum field, the last example also exhibiting a comparitively large effective flutter

$$f_{eff.} \equiv \sqrt{2\left[\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1\right]} \quad .$$

Mr. Leffler has evaluated the first Fourier components for these cases, writing $B = \langle B \rangle (1 + f_1 \cos N\theta + f_2 \cos 2N\theta + f_3 \cos 3N\theta)$, with the results displayed in Table II.

TABLE II

FOURIER COMPONENTS OF MEDIAN-PLANE FIELDS

Run No.	Figs.	f_1	f_2	f_3	f_2/f_1	f_3/f_1
FLSL02	1-2	0.6282	0.0153	-0.0062	0.024	-0.010
FLSL03	3-4	0.6184	0.0193	0.0084	0.031	0.014
FLSL04	5-6	0.6423	-0.0136	-0.0636	-0.021	-0.099
FLSL05	7-8	1.0218	-0.0580	-0.0974	-0.057	-0.095

B.

The configuration of Fig. 9 was intended to represent the Michigan reversed-field model and led to the median-plane field depicted in Fig. 10. In performing the computations, a constant "bias" was added to the values which appear on Fig. 9 (since the computational program did not admit negative values) and subsequently subtracted out in analysizing the results.

The ratio of half-gap to wavelength was 8/98. From the computated potentials, the vertical field strength at a height 7/8 of the distance from the median plane to the pole was determined (Fig. 11) by the algorism

$$12h\,B(x_i, y_m) = V(x_{i-1}, y_{m+1}) + 4V(x_i, y_{m+1}) + V(x_{i+1}, y_{m+1}) - V(x_{i-1}, y_{m-1}) - 4V(x_i, y_{m-1}) - V(x_{i+1}, y_{m-1}).$$

A Fourier analysis of this field has been undertaken by Mr. Condo and Mr. Carrier. The relative coefficients are listed below:

$f_o = 1$	$f_4 = 0.9957$	$f_7 = 0.2849$
$f_1 = 3.1824$	$f_5 = -0.8548$	$f_8 = -0.3478$
$f_2 = -1.7945$	$f_6 = 0.0307$	$f_9 = 0.1532$
$f_3 = 0.0699$		

It is thus seen that, at this distance from the median plane, the magnitude of the eighth Fourier coefficient of the axial Field-component is evidently some 11% of F_1 and over one-third of F_o. It may, therefore, be seriously doubted that an accurate representation of this field could be achieved without inclusion of many more Fourier components. (From an analysis of a square wave, with the positive portion twice as long as the negative portion, one expects the coefficients f_3, f_6, f_9, to be small in the present example and expects the signs of F_2 and F_1, F_5 and F_4, F_8 and F_7,... to be opposite.)

The Fourier coefficients of the field in the median-plane (Fig. 10) although obtainable independently from the data, may be expressed in terms of those which pertain to the height y_m through division of the latter of $\cosh\left(s\,\frac{2\pi\, y_m}{\lambda}\right)$ where s is the order of the harmonic. Introduction of this factor has the effect of reducing the magnitudes of the seventh and eighth Fourier coefficients to about 2% of F_o. The general magnitude of these median-plane Fourier coefficients is similar to those reported by Dr. Cole for the actual model (private communication, Madison, 7 August 1956), although the latter appear to be somewhat the larger.

It is emphasised in closing that computational programs intended for the determination of particle orbits in FFAG accelerators will be required to employ fields at points remote from the median-plane and that careful consideration is required to avoid premature truncation of Fourier series which may be introduced to represent such fields.

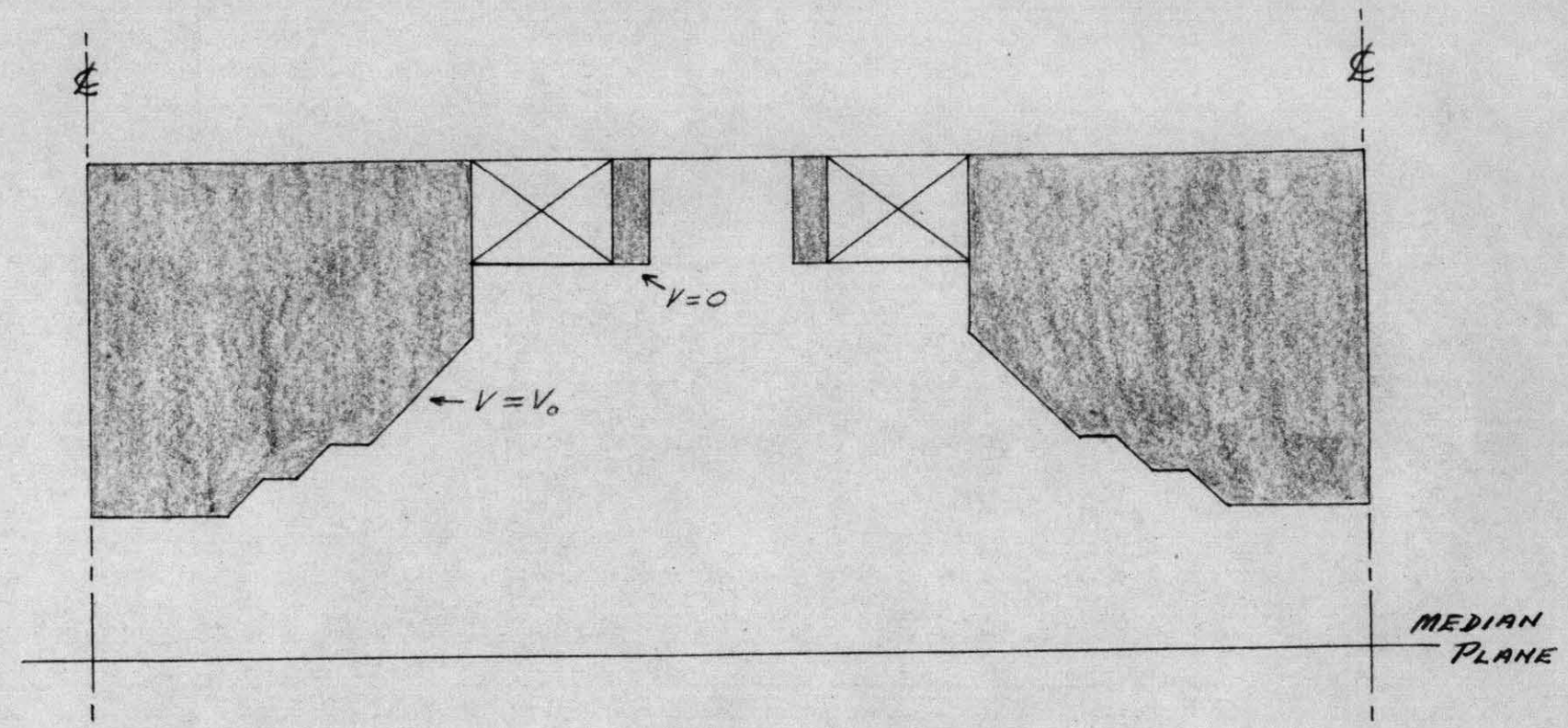
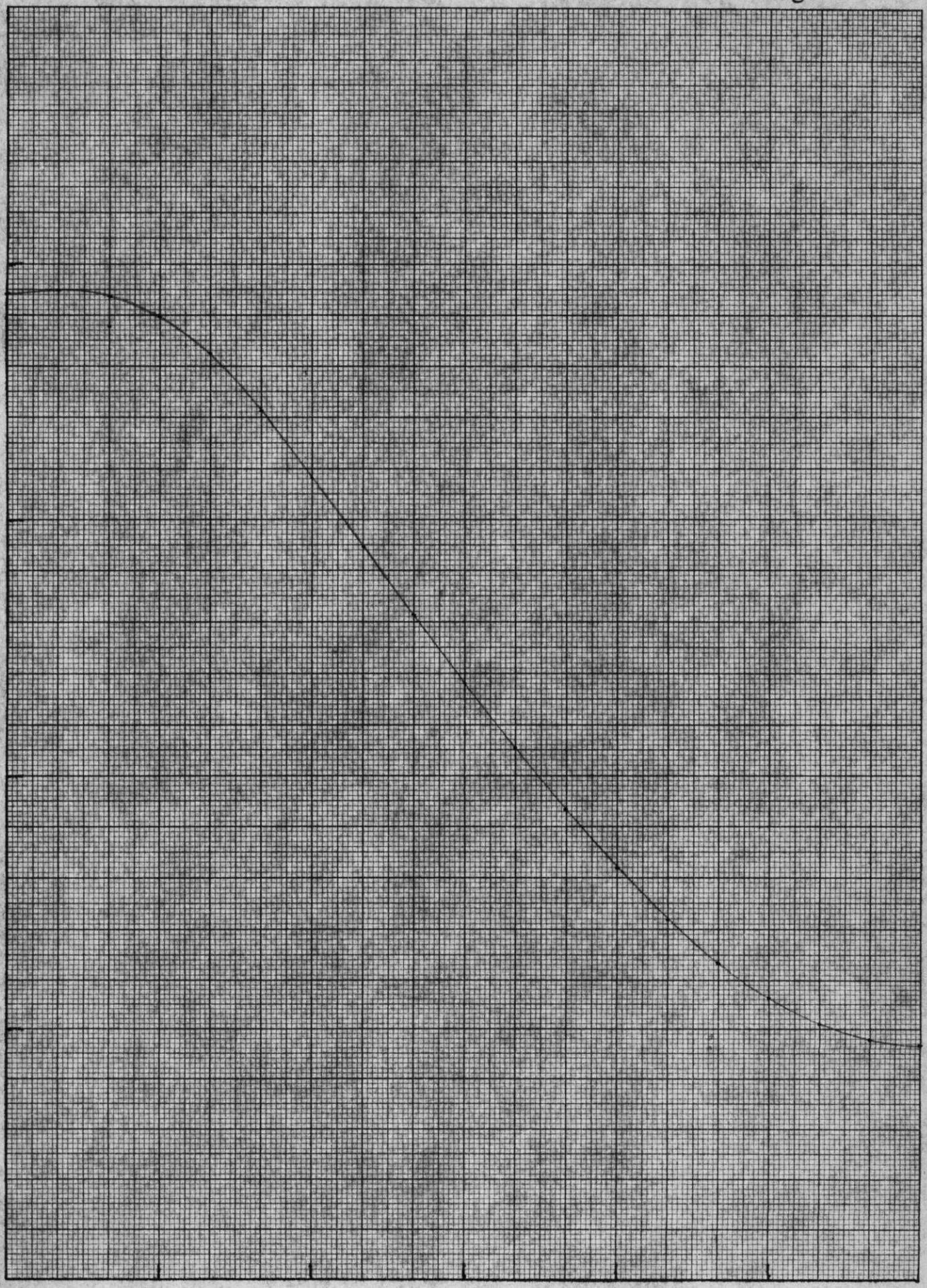


Fig. 1

Fig. 2



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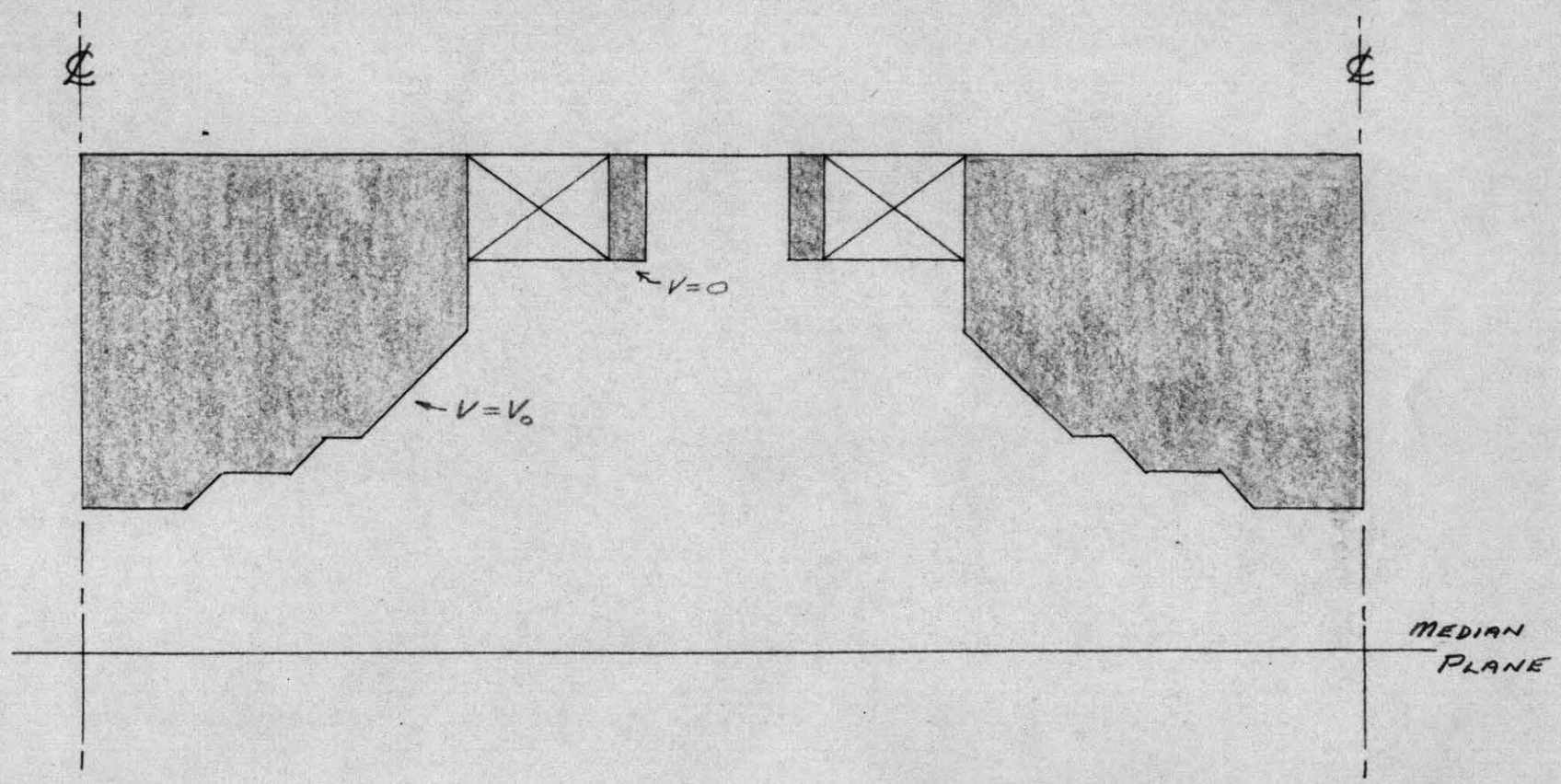
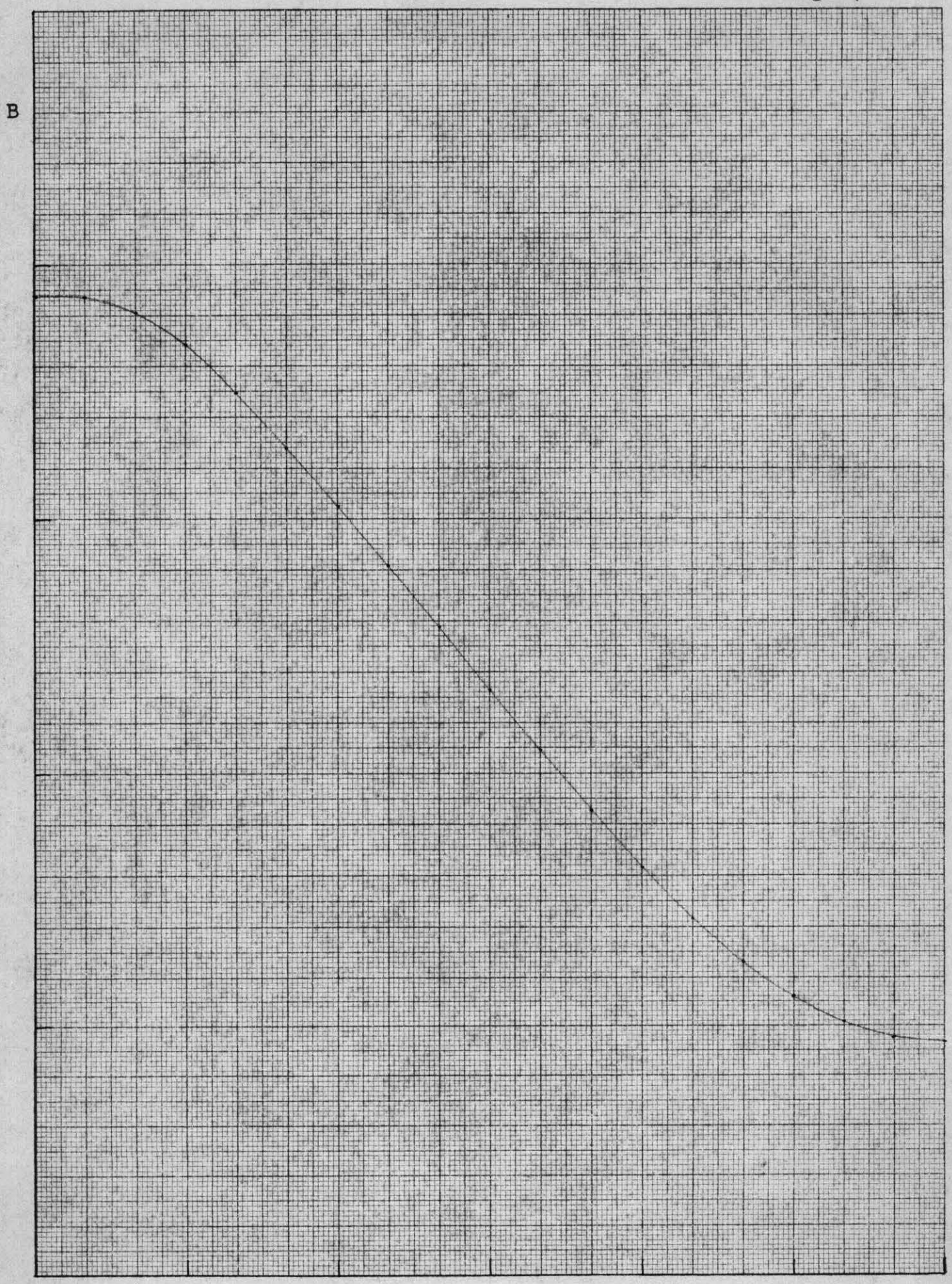


Fig. 3

Fig. 4



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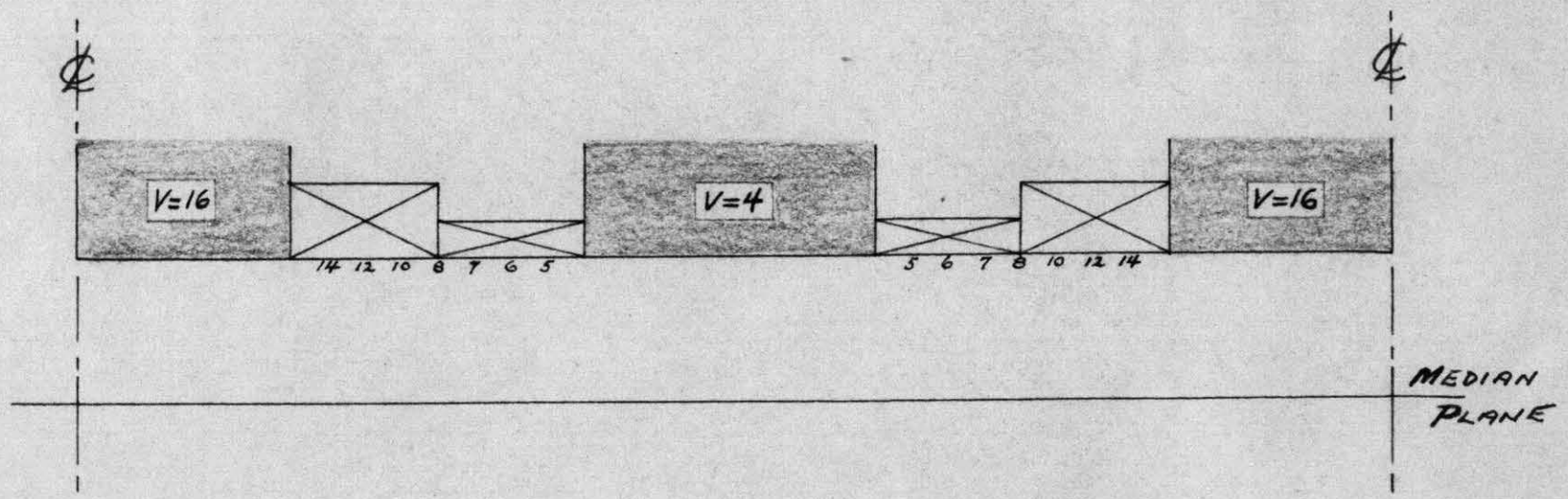
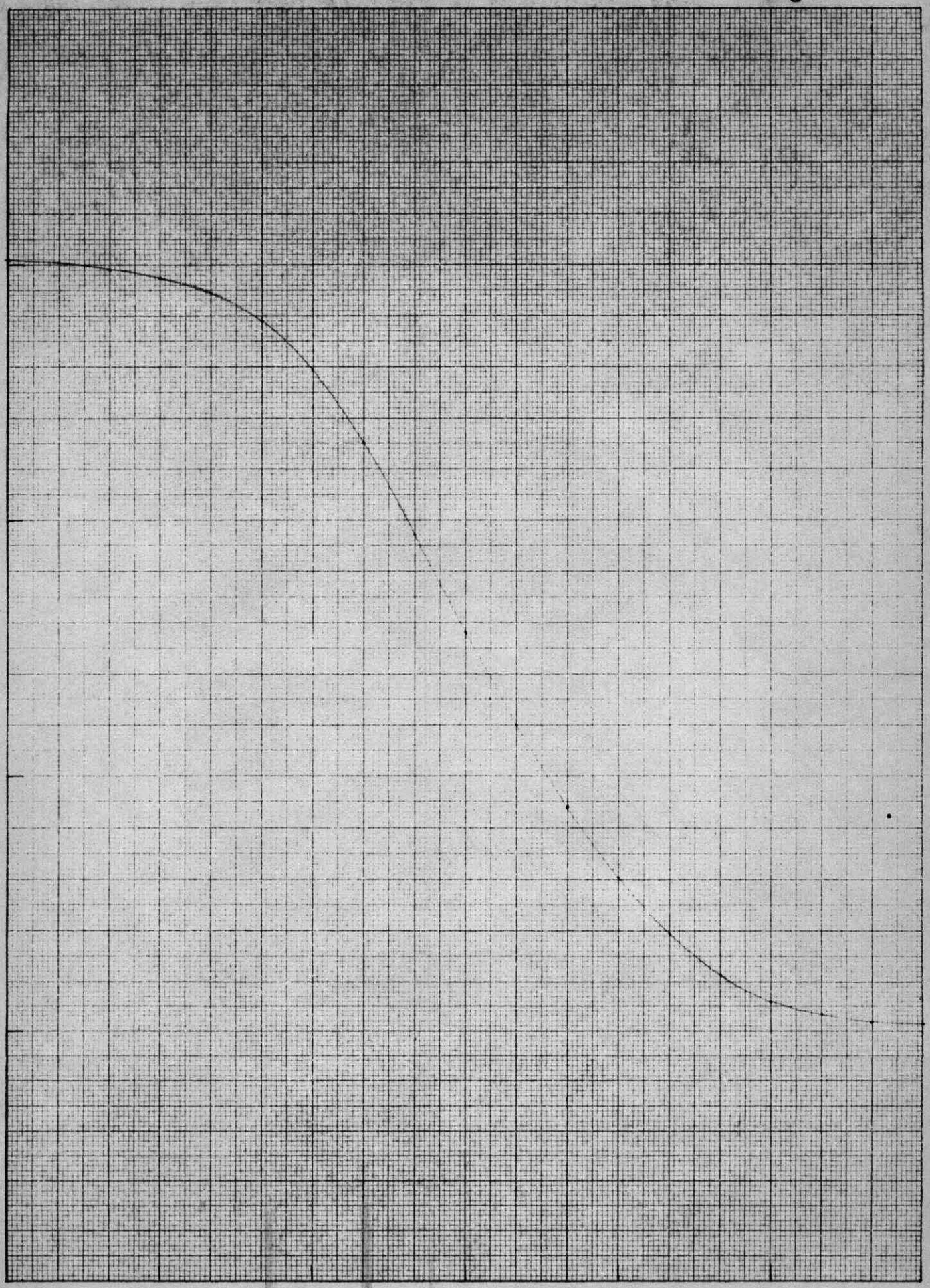


Fig. 5

Fig. 6



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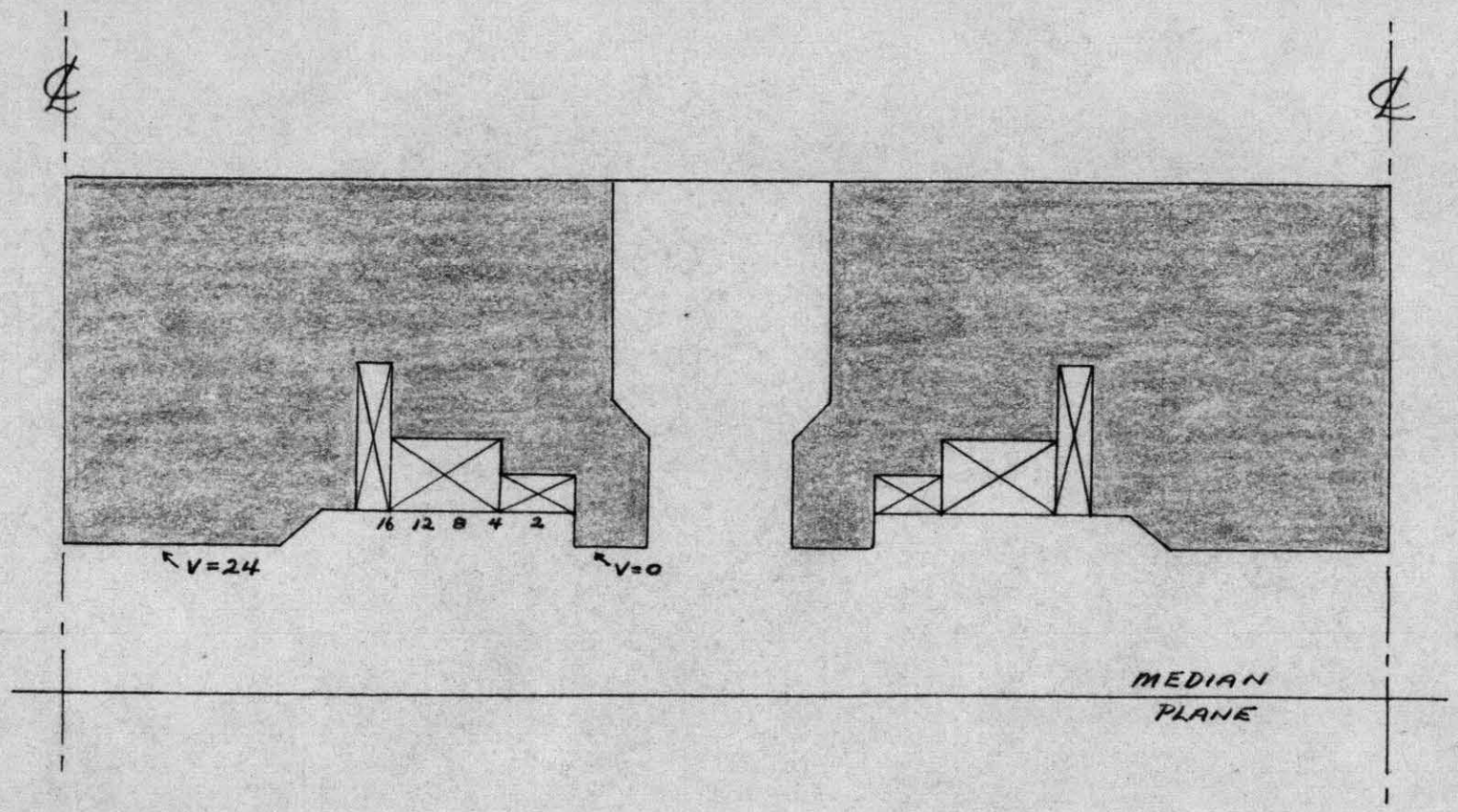
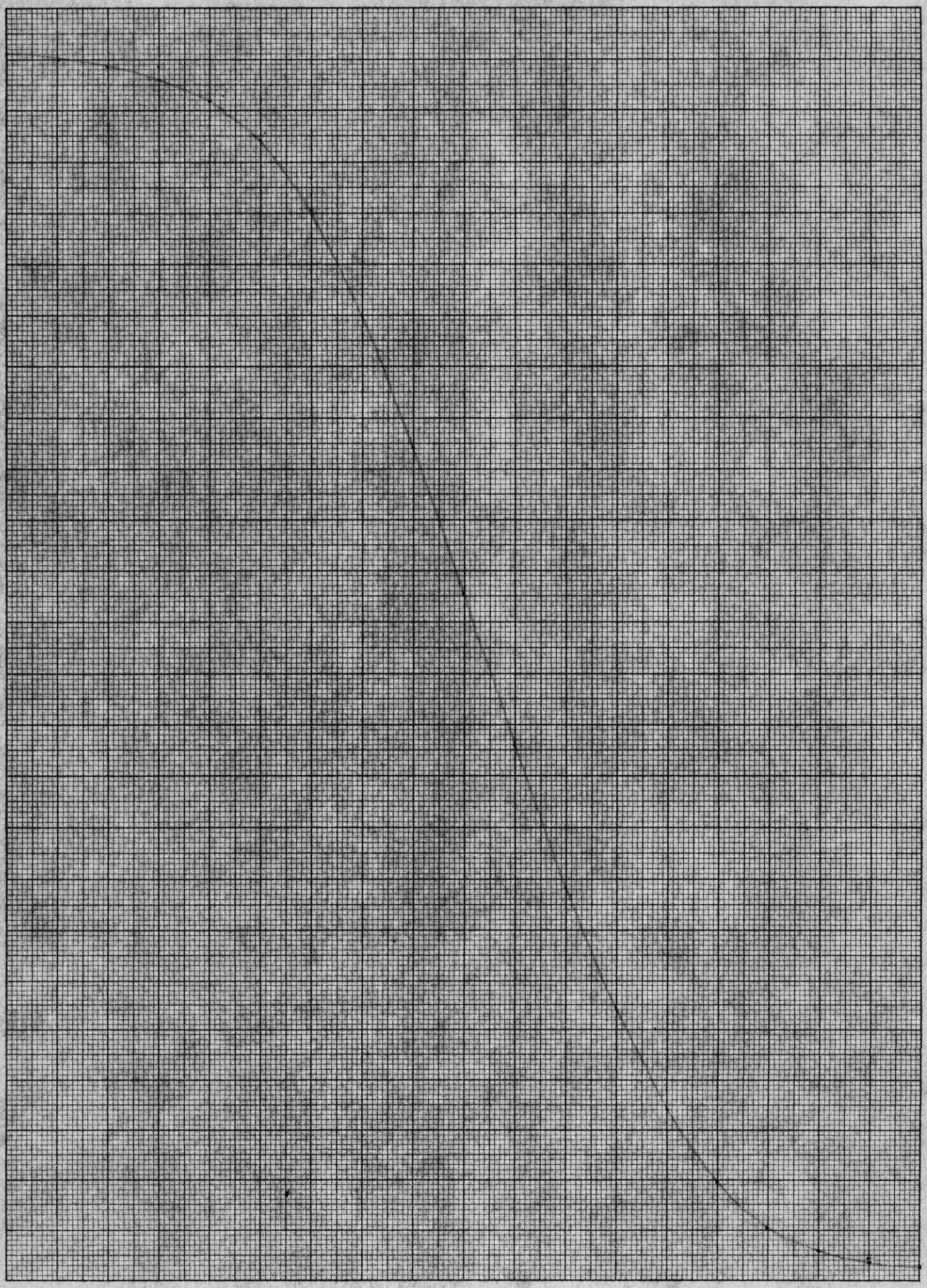


Fig. 7

Fig. 8



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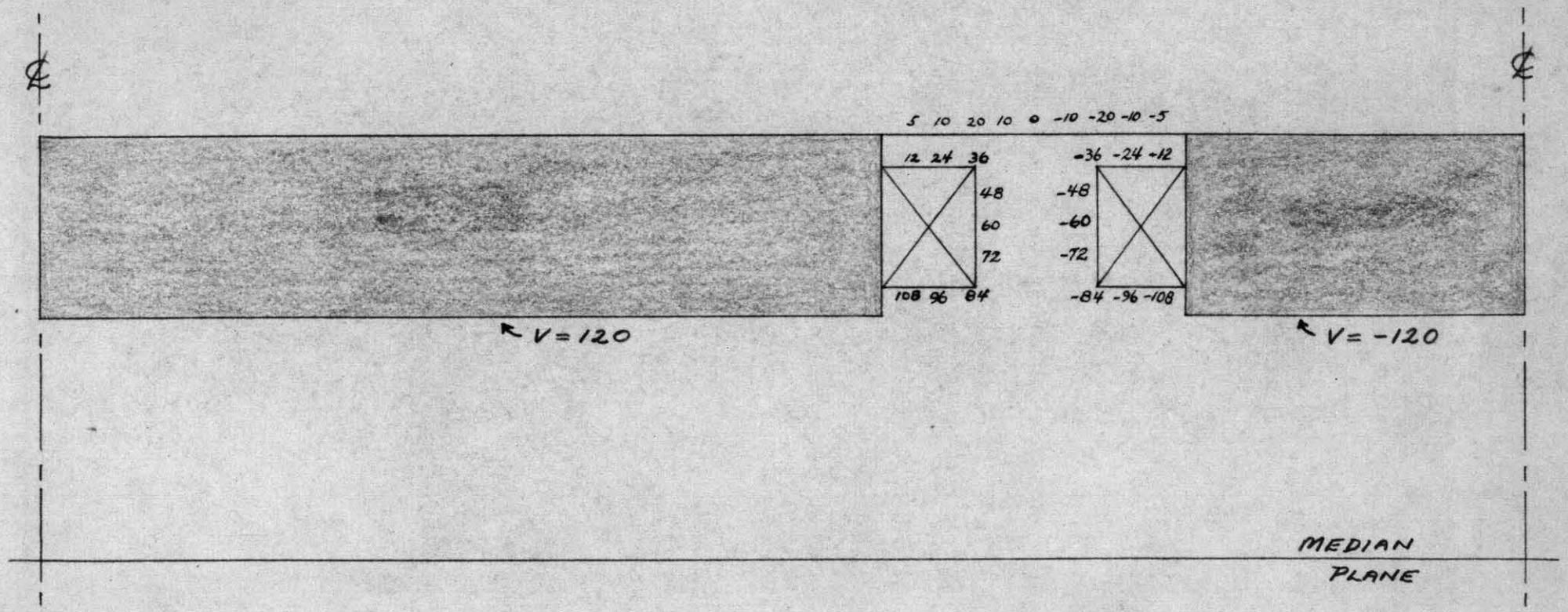
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12 24 36
48
60
72
108 96 84
-36 -24 -12
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-84 -96 -108
V=120
V= -120
MEDIAN
PLANE

Fig. 9

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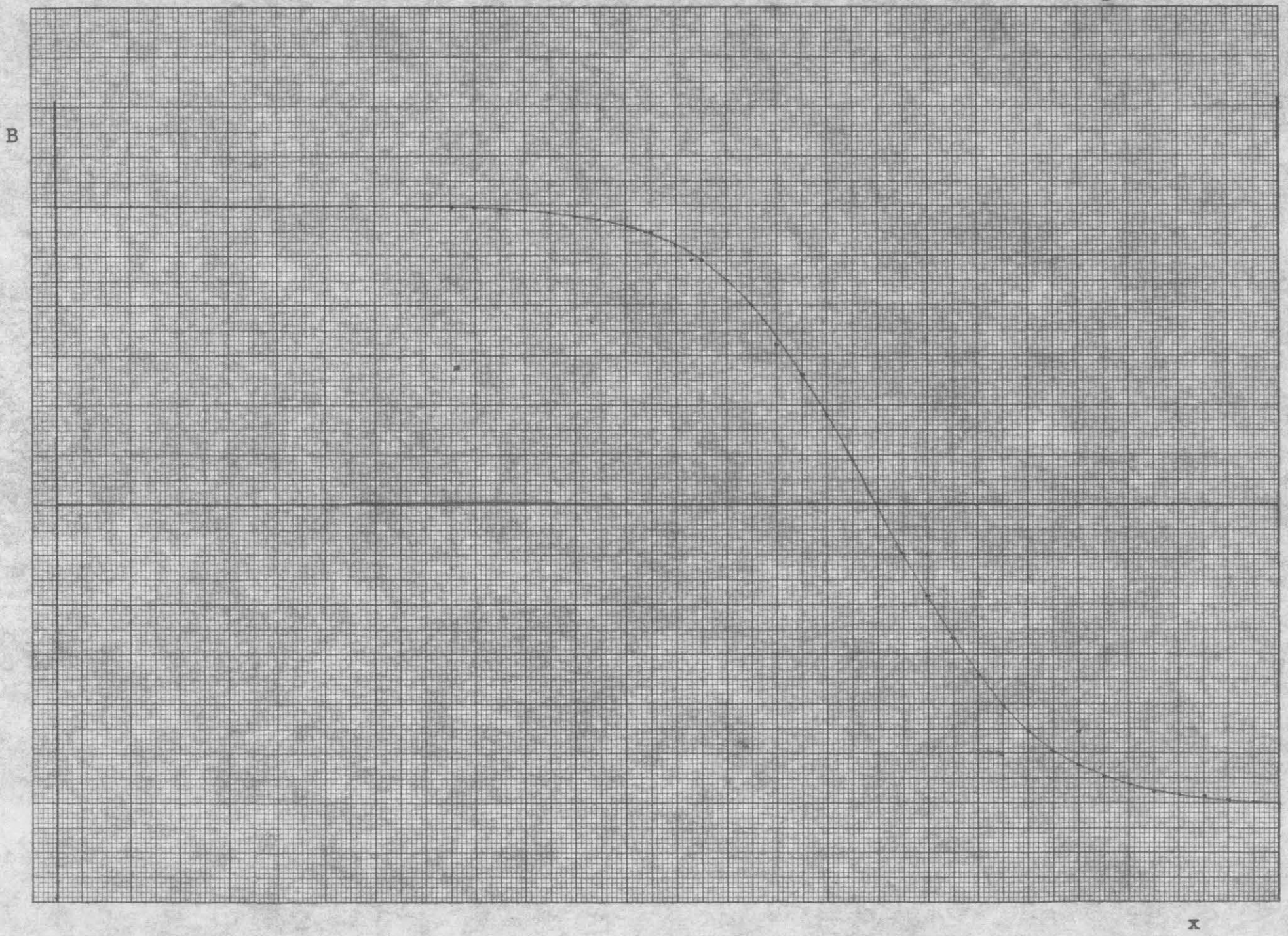
Fig. 10



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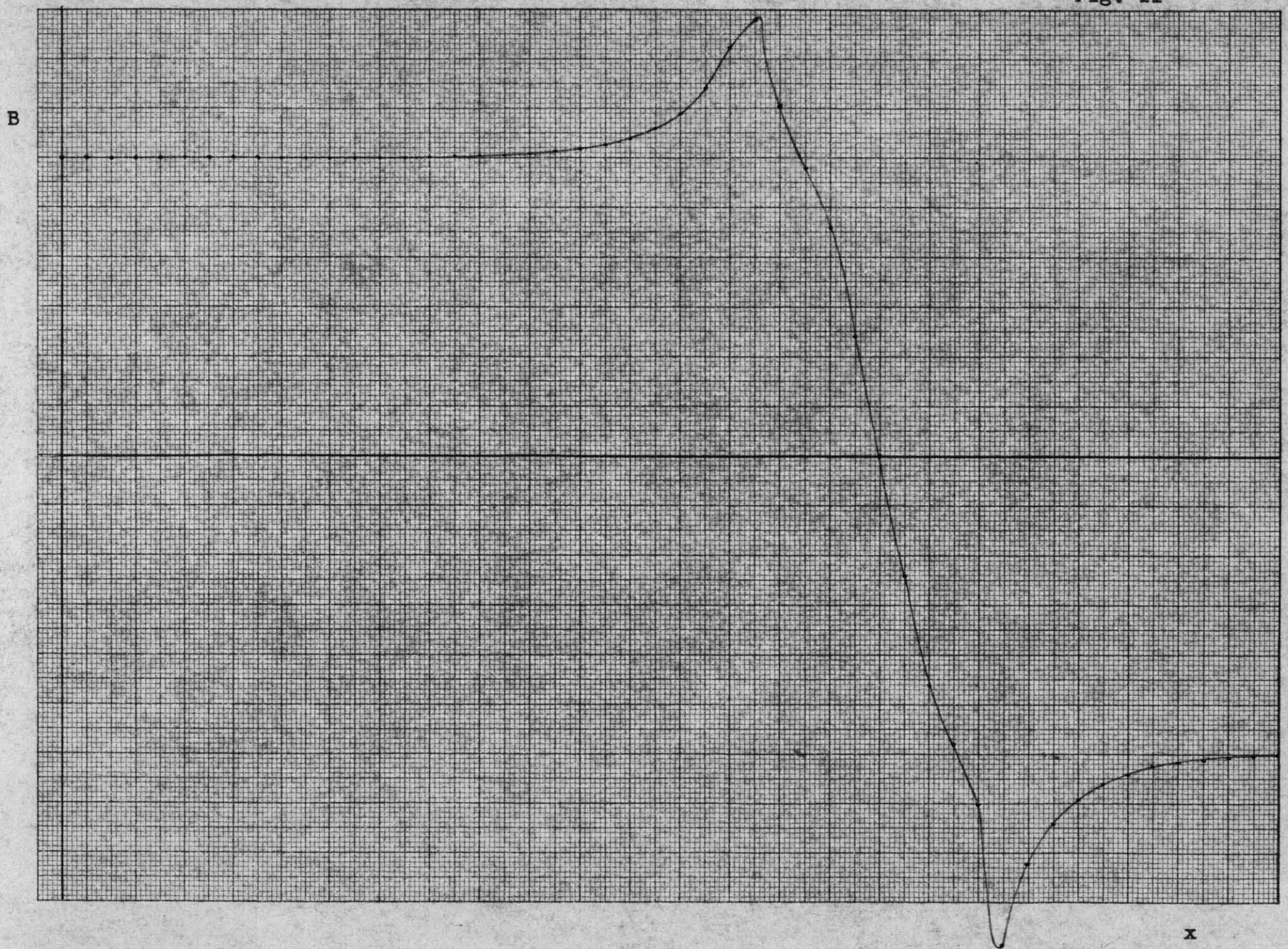
Fig. 11