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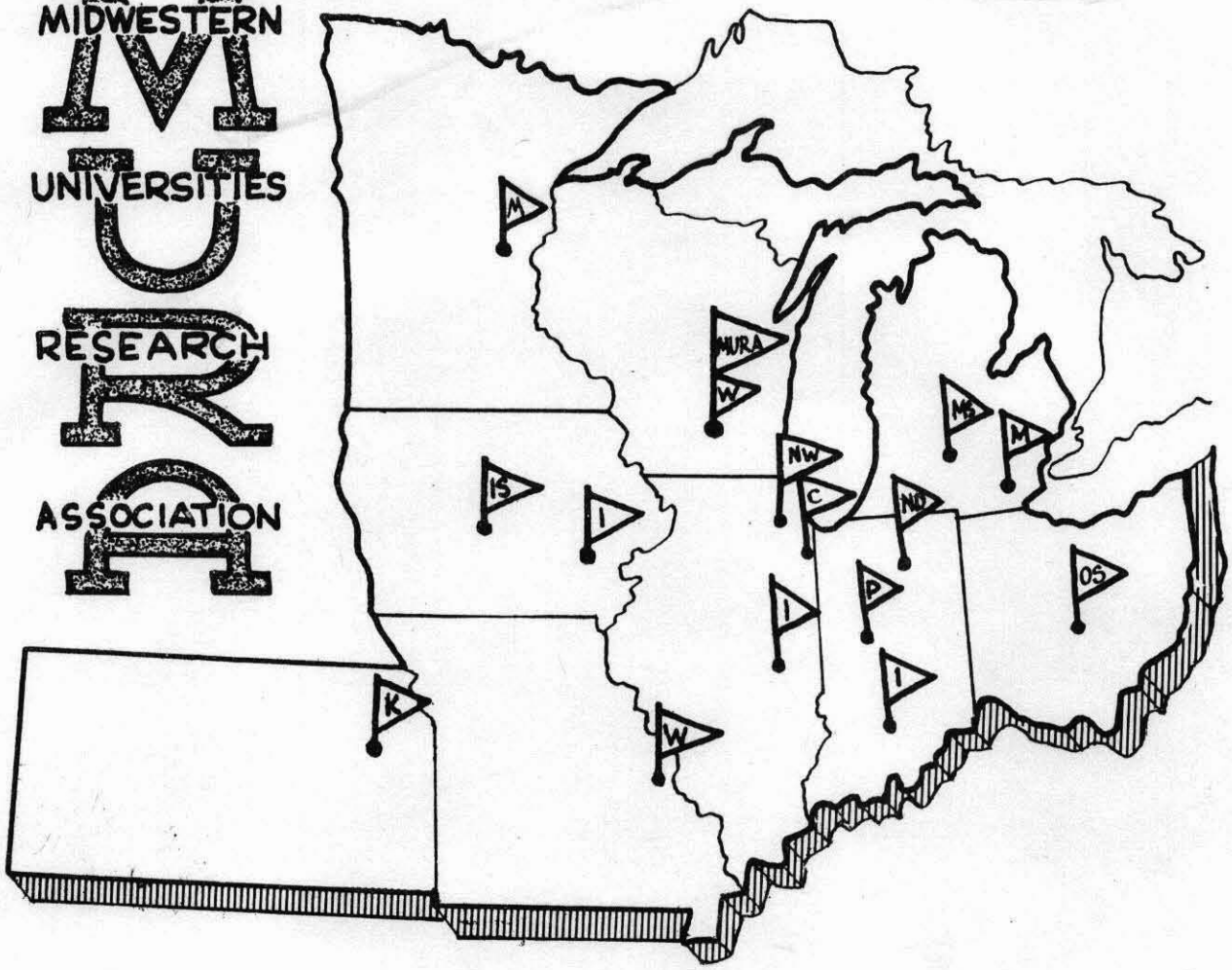

THE OHKAWA INTERSECTING BEAMS MACHINE

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THE OHKAWA INTERSECTING BEAMS MACHINE

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Ohkawa[1] has suggested a geometry for intersecting beams in which particles of the same charge travel in opposite directions in a single accelerator. This is achieved with fields which reverse periodically with azimuth around a circle and which increase radially. Since the amounts of iron, copper, power, R.F., etc., required in this case are characteristic of one machine only instead of two, and since stacked beams would intersect 2N (where N is the number of sectors) times around the machine instead of at only one point, it was thought worthwhile to look into some possible design parameters to determine to what extent such a machine would be feasible.

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1. MURA-TO-6

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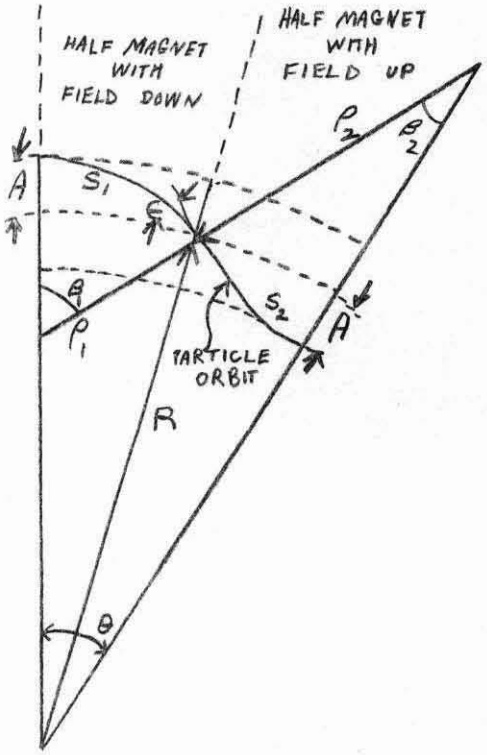
GEOMETRY

The first geometries considered were essentially of the Mark Ia type, where radial sector magnets of equal length and of equal but opposite field were used. With fields constant within magnets along arcs of circles from the machine center and varying as r^k with radius, particle equilibrium orbits within magnets are to a close approximation sections of sine curves. For ease of calculation the orbits were approximated by arcs of circles where the radius of curvature was taken for the average magnetic field along the sinusoidal orbit. Referring to figure 1, for the approximate half-sector geometry without straight sections,



$$A \cong R\frac{\epsilon}{N}$$

$$\epsilon = \frac{\beta_1 + \beta_2}{2}.$$

Therefore

$$A = \frac{R}{2N}(\beta_1 + \beta_2)$$

Fig. 1

Where N is the number of sectors, ϵ is the angle between the equilibrium orbit and the normal to the magnet edge, β_1 and β_2 are the half angles subtended by the orbits in each half magnet. The average over half a sine curve is $\frac{2}{\pi} = 0.643$. Here we consider the field along orbits in each magnet to have the value it has at $R = R \pm 0.65A$.

Let

$$R_1 = R + 0.65A$$
$$R_2 = R - 0.65A$$

Then
$$\left|\frac{H_1}{H_2}\right| = \left(\frac{R_1}{R_2}\right)^k = \left[\frac{2N + (.65)(\beta_1 + \beta_2)}{2N - (.65)(\beta_1 + \beta_2)}\right]^k .$$

An error in chosing the average Rs affects k only linearly, and any reasonable way of averaging results in the same general family of parameters.

Since $S_1 \cong S_2$ for large N,

$$\left|\frac{H_1}{H_2}\right| = \frac{\beta_1}{\beta_2} .$$

We also have

$$\beta_1 - \beta_2 = \theta = \frac{\pi}{N} .$$

Therefore, if N, β_1, and β_2 are chosen arbitrarily, k is determined. This is one more restriction than one normally has in F.F.A.G. designs; it arises since we demand complete symmetry between positive and negative fields, so that protons may travel in either direction around the machine. For the matrix solutions for cosine σ, we define as usual:

$$n_1 = \frac{\rho_1}{H_1}\frac{dH_1}{dr} \qquad n_2 = \frac{\rho_2}{H_2}\frac{dH_2}{dr}$$

$$\psi_1 = \sqrt{n_1}\,\beta_1 \qquad \psi_2 = \sqrt{n_2}\,\beta_2$$

$$\phi_1 = \sqrt{n_1+1}\,\beta_1 \qquad \phi_2 = \sqrt{n_2-1}\,\beta_2 .$$

If edge effects are neglected and there are no straight sections

$$\cos\sigma_x = \cos 2\phi_1 \cosh 2\phi_2 + \frac{1}{2}\left[\frac{\rho_1}{\rho_2}\frac{\sqrt{n_2-1}}{\sqrt{n_1+1}} - \frac{\rho_2}{\rho_1}\frac{\sqrt{n_1+1}}{\sqrt{n_2-1}}\right]\sin 2\phi_1 \sinh 2\phi_2 ,$$

$$\cos\sigma_z = \cos 2\psi_2 \cosh 2\psi_1 + \frac{1}{2}\left[\frac{\rho_2}{\rho_1}\sqrt{\frac{n_1}{n_2}} - \frac{\rho_1}{\rho_2}\sqrt{\frac{n_2}{n_1}}\right]\sin 2\psi_2 \sinh 2\psi_1 .$$

Here n_1 and n_2 are related through the geometry:

$$n_1 = \frac{\rho_1}{R_1} k$$

$$n_2 = \frac{\rho_2}{R_2} k = \frac{\beta_1}{\beta_2} n_1 .$$

We define the circumference factor as:

$$C = \frac{R}{\rho_1} .$$

Through the above relationships, C and N determine k, σ_x, and σ_z. When edges are neglected, one may show that N and k are related in such a way that independent of N and k, C determines σ_x and σ_z. Using the method of adding edge effects in quadrature[2] the parameters for a 64 sector machine were found and are given in table 1.

There are two additional parameters which may be introduced to adjust σ_s for a given C: straight sections and spiraled sectors.

STRAIGHT SECTIONS

By introducing straight sections and holding the circumference factor constant, the ratio β_1/β_2 increases, and, since the scalloping is reduced, k must increase to produce the ratio of average fields in magnets equal to β_1/β_2. This increases the strength of radial focusing and slightly decreases vertical focusing, so that in general a slightly larger circumference factor is required for stability with straight sections than without. Figure 2 gives the geometry with straight sections.

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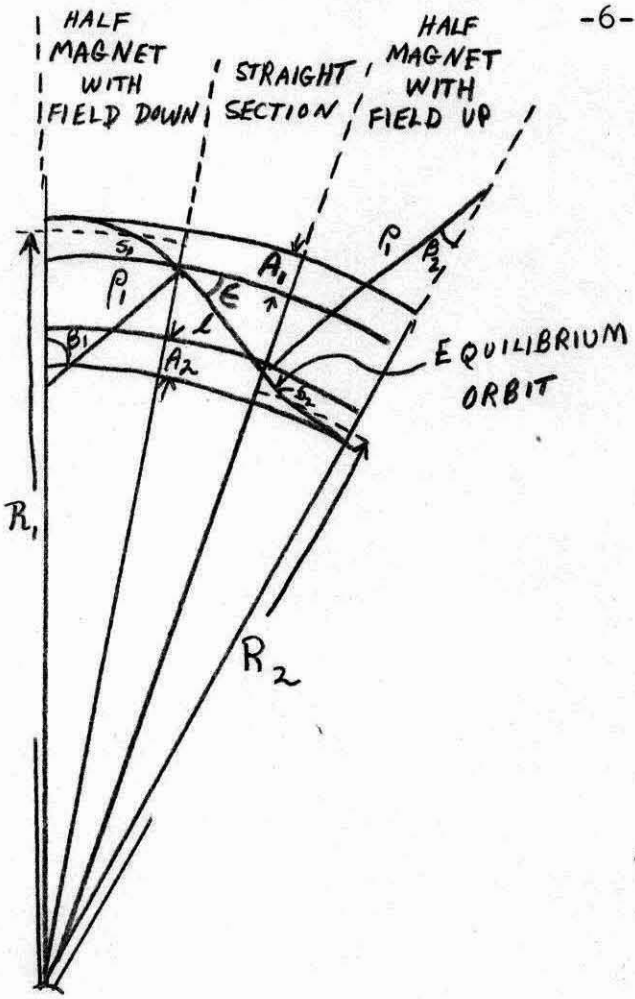
2. MURA-LWJ-KMT-2.



If we define P as the path length in magnets divided by path length in straight sections,

$$P = \frac{s_1 + s_2}{l}$$

$$A_1 \cong A_2 \cong \frac{\epsilon R}{N}\left(\frac{P}{P+1}\right)$$

$$R_1 - R_2 = 2(.65A) + l\epsilon$$

$$l = \frac{2\pi R}{2N(P+1)}$$

$$\frac{R_1 - R_2}{R} = (1.3P + \pi)\frac{\beta_1 + \beta_2}{2N(P+1)}.$$

Fig. 2

The cosine σ expressions with straight sections become

$$\cos\sigma_x = \cos 2\phi_1 \cosh 2\phi_2 + \frac{l\sqrt{n_2-1}}{\rho_2}\cos 2\phi_1 \sinh 2\phi_2 - \frac{l\sqrt{n_1+1}}{\rho_1}\sin 2\phi_1 \cosh 2\phi_2$$

$$+\frac{1}{2}\left[\frac{\rho_1}{\rho_2}\frac{\sqrt{n_2-1}}{\sqrt{n_1+1}} - \frac{\rho_2}{\rho_1}\frac{\sqrt{n_1+1}}{\sqrt{n_2-1}} - \frac{l^2\sqrt{n_1+1}\sqrt{n_2-1}}{\rho_1\rho_2}\right]\sin 2\phi_1 \sinh 2\phi_2 ;$$

$$\cos\sigma_z = \cos 2\psi_2 \cosh 2\psi_1 + \frac{l\sqrt{n_1}}{\rho_1}\cos 2\psi_2 \sinh 2\psi_1 - l\frac{\sqrt{n_2}}{\rho_2}\sin 2\psi_2 \cosh 2\psi_1$$

$$+\frac{1}{2}\left[\frac{\rho_2}{\rho_1}\frac{\sqrt{n_1}}{\sqrt{n_2}} - \frac{\rho_1}{\rho_2}\frac{\sqrt{n_2}}{\sqrt{n_1}} - \frac{l^2\sqrt{n_1 n_2}}{\rho_1\rho_2}\right]\sin 2\psi_2 \sinh 2\psi_1 .$$

Table I summarizes results for machines of 64 sectors considering circumference factors of 6 and 8 with no straight sections ($P = \infty$) and with straight sections one third the magnet length ($P = 3$). Edge σ_s were calculated from the following:

$$\sigma_e^2 = \pm \frac{4\pi}{N}\left(\frac{R}{\rho_1} + \frac{R}{\rho_2}\right)\tan\epsilon = \pm \frac{4\pi C}{N}\left(1 + \frac{B_2}{B_1}\right)\tan\epsilon.$$

TABLE I

				No Edges			Corrected for Edge Effects			
C	P	h	ϵ	$\cos\sigma_x$	$\cos\sigma_y$	$\lvert\sigma_e^2\rvert$	σ_x	σ_y	$\cos\sigma_x$	$\cos\sigma_y$
6	∞	162	7.05°	-.312	1.185	.476	1.76	.34	-.19	.943
	∞	82	9.87°	-.076	1.115	.905	1.35	.825	.22	.68
6	3	234	4.94°	-.862	1.29	.314	2.72	.488i	-.91	1.12
8	3	120	7.05°	+.114	1.079	.647	2.13	.70	-.53	.765

It is apparent that without edge effects none of the parameter choices gives vertical stability, and that edge effects are important even with 64 sectors.

SPIRAL SECTORS

By spiralling the sectors but maintaining the condition that magnets are completely symmetric, we have an example of a "transition" machine between a spiral sector and a radial sector machine. The geometry and notation are given on the next page for the simple case with no straight sections. Note that here we let β refer to the whole magnet rather than the half magnet.

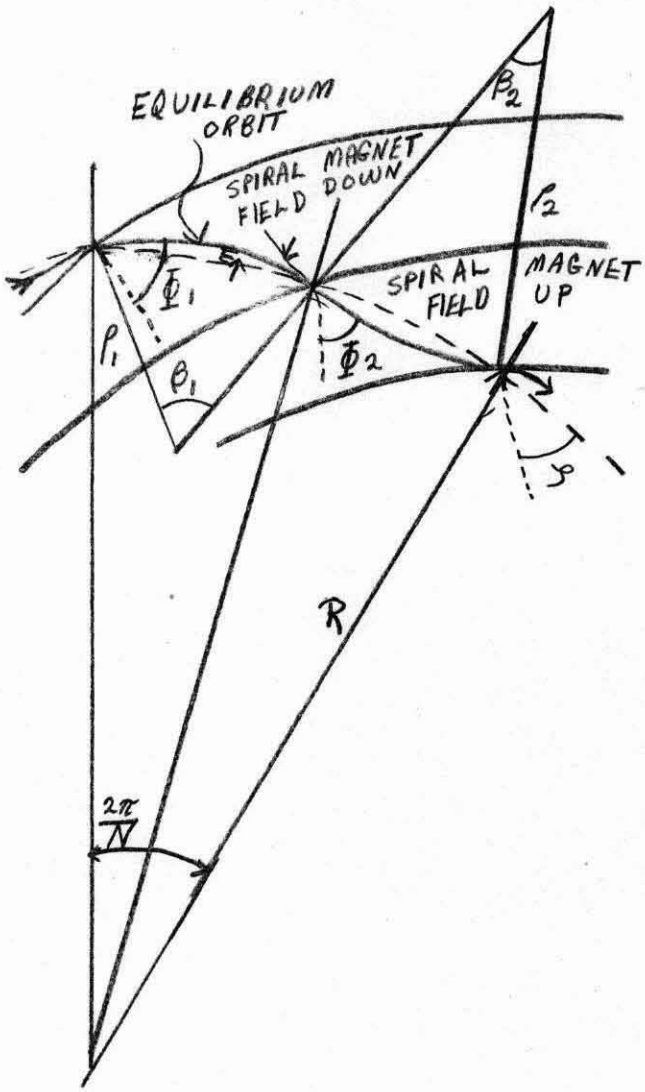


Fig. 3

$$\Phi_1 = \rho + \epsilon \qquad t_1 = \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)\tan\Phi_1$$

$$\Phi_2 = \rho - \epsilon \qquad t_2\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)\tan\Phi_2$$

let

$$n_1 \cong n_1 + 1 \therefore \psi_1 \cong \phi_1 = \sqrt{n_1}\,\beta_1$$

$$n_2 \cong n_2 - 1 \qquad \psi_2 \cong \phi_2 = \sqrt{n_2}\,\beta_2$$

$$\frac{\sqrt{n}}{\rho} = \eta$$

$$\frac{\beta_1}{\beta_2} = \gamma$$

$$C = \frac{2\gamma}{\gamma - 1}$$

$$\epsilon = \frac{\pi}{2N}(C-1)$$

$$\beta_1 = \frac{\pi}{N} C$$

$$\beta_2 = \frac{\pi}{N}(C-2)$$

$$\frac{R_1}{R_2} = \frac{2N + .65(\beta_1 + \beta_2)}{2N - .65(\beta_1 + \beta_2)}$$

$$n_1 = \frac{k}{C}$$

$$n_2 = \gamma n_1 .$$

$$\cos\sigma_x = \cos\psi_1 \cosh\psi_2 + \frac{t_1 - t_2}{2\eta_1}\sin\psi_1 \cosh\psi_2 + \frac{t_1 - t_2}{2\eta_2}\cos\psi_1 \sinh\psi_2$$
$$+ \frac{1}{2}\left[\frac{\eta_2}{\eta_1} - \frac{\eta_1}{\eta_2} - \frac{t_1 t_2}{\eta_1 \eta_2}\right]\sin\psi_1 \sinh\psi_2 ,$$

$$\cos\sigma_y = \cos\psi_2 \cosh\psi_1 + \frac{t_2 - t_1}{2\eta_2}\sin\psi_2 \cosh\psi_1 + \frac{t_2 - t_1}{2\eta_1}\cos\psi_2 \sinh\psi_1$$
$$+ \frac{1}{2}\left[\frac{\eta_1}{\eta_2} - \frac{\eta_2}{\eta_1} - \frac{t_1 t_2}{\eta_1 \eta_2}\right]\sin\psi_2 \sinh\psi_1 .$$

Parameters studied for a 50 sector machine are summarized below

C	k	ϵ	ξ	$\cos\sigma_x$	$\cos\sigma_z$
5	156	7.2°	60°	-.885	.221
			45°	-.819	.872
			30°	-.795	1.083
			0°	-.783	1.189 (radial sector)
6	99	9°	60°	-.229	-.493
			45°	-.152	.502
			0°	-.126	.960 (radial sector)

It is apparent that by spiraling the magnets any desired amount of vertical focusing may be achieved; however, the radial focusing is hardly affected by spiraling.

CONCLUSION

It appears that a reasonable machine might have a circumference factor of 5.5 or 6, sectors spiraling at about 45°, and straight sections of the order of one third as long as magnets. The total perimeter of this machine would be equal to that of two machines each of circumference factor 2.75 or 3; however, the weight of magnets, etc., would be much greater for the single machine. Although there would be 100 or more points of beam intersection, due to the angle of beam intersection (2ϵ) the intersecting region would be only a few times as long as the beam diameter. This may be an advantage or a disadvantage, depending on the details of a particular experiment. Each region of interaction would be accessible on three sides instead of just top and bottom (as in tangent intersecting beam designs). In this machine the energy

of the intersecting beams could be readily varied by changing the R.F. stacking point, and by changing the fields of one set magnets relative to the other, beams could be made to interact tangentially and at different energies (providing a net center of mass energy). Thus in spite of the greater circumference of such a machine, it appears that with its many target areas, greater flexibility, and design simplicity characteristic of almost radial sector geometries, this type of accelerator design merits very serious consideration as a means of obtaining center of mass energies of 6 to 40 Bev.