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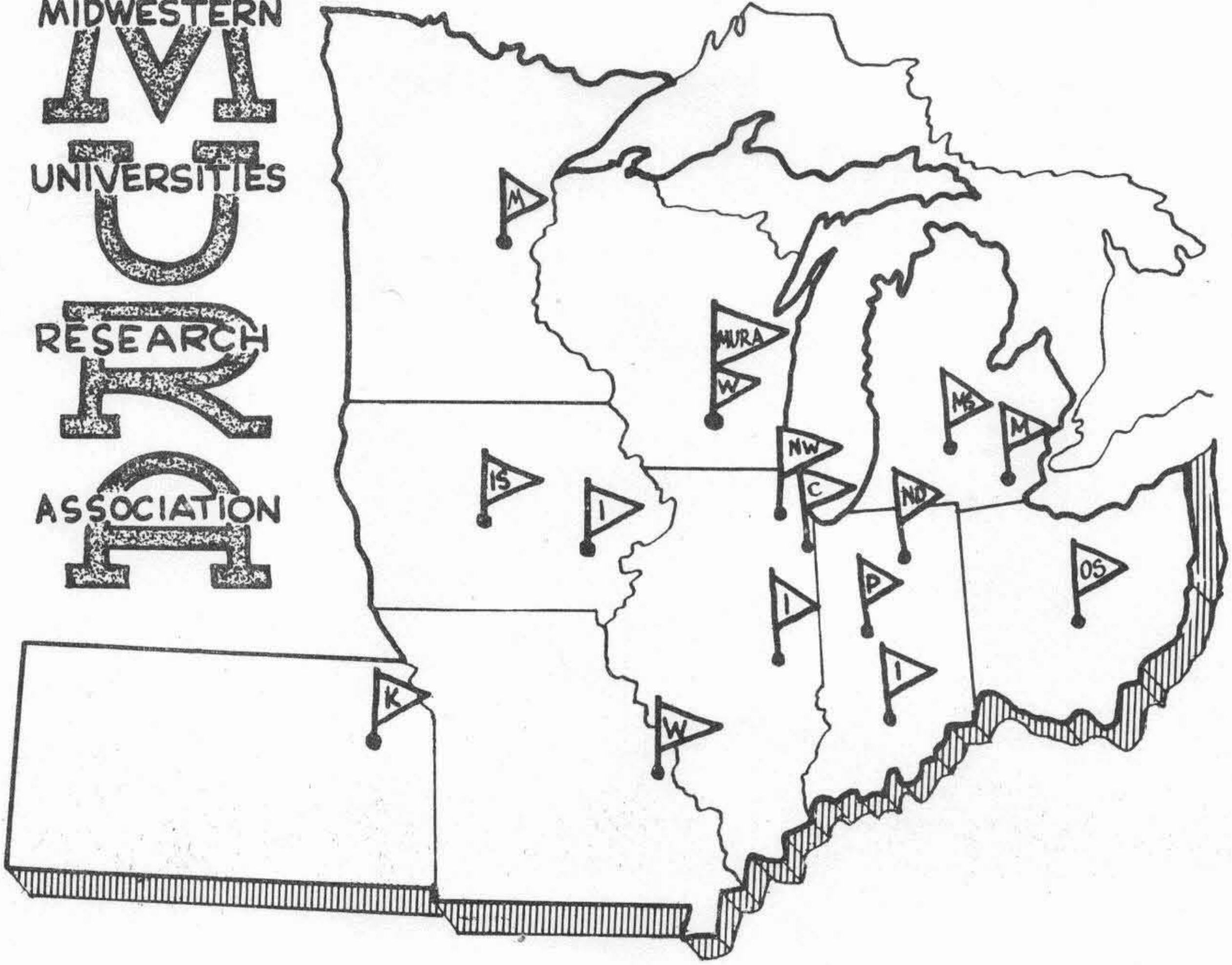

Tuning of Vertical Oscillations in
Separated Sector Mark V by
Twisting Magnets

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Tuning of Vertical Oscillations in Separated Sector Mark V by Twisting Magnets*

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I. Introduction

In a scaling Mark V accelerator, a spiral line crosses circles about the center of the machine at an angle which is independent of the radii of the circles. This follows from the equation of the spiral.

$$\begin{cases} \ln \frac{r}{r_0} - w N\theta = \text{Constant} \\ \text{or} \quad r = r_0 e^{wN(\theta - \theta_0)} \end{cases} \tag{1}$$

from which

$$\tan \alpha = \frac{1}{r} \frac{dr}{d\theta} = wN, \tag{2}$$

and from Figure I, α is the angle of the spiral with circles of radius. r_1 and r_2,

which from (1) are connected by $\frac{r_2}{r_1} = e^{wN\Delta\theta}$ (3)

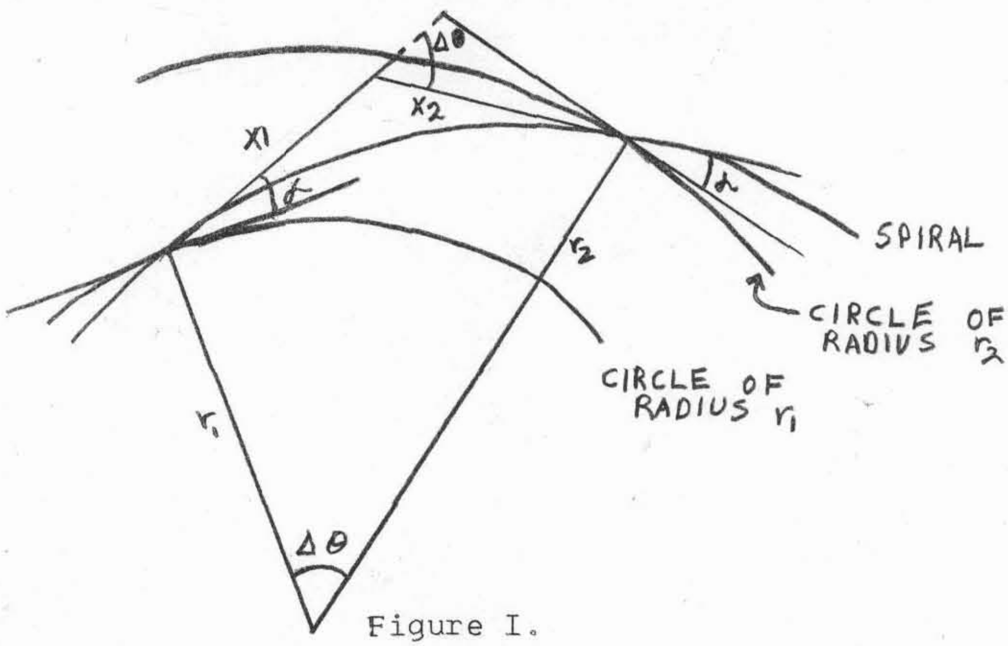

Figure I.

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From Figure I we note that

$$\begin{cases} x_1 = \dfrac{r_2 \cos \alpha - r_1 \cos(\alpha - \Delta\theta)}{\sin \Delta\theta} \\ x_2 = \dfrac{r_1 \cos\alpha - r_2 \cos(\alpha + \Delta\theta)}{\sin\Delta\theta} \end{cases} \tag{4}$$

Kerst has suggested that the vertical oscillations in a separated sector Mark V might be tuned by twisting the magnets, i.e., by changing α. Then α is no longer independent of radius and the machine no longer scales; that is, the frequency of vertical oscillation is now a function of radius. It is this lack of scaling which the present note considers.

II. Calculation of Change in α

We denote the value of α at radius r_i by α_i. Then we rotate about an axis perpendicular to the place of the paper passing through the point of radius r_1. We thus keep the length r_1 fixed. In addition, to keep the physical shape of the iron fixed, we keep x_1, x_2, and the angle between x_1 and x_2 $(\Delta\theta)$ fixed. Then at radius r_2,

$$\tan\alpha_2 = \frac{x_2 + x_1 \cos\Delta\theta + r_1 \sin(\alpha_1 - \Delta\theta)}{x_1 \sin\Delta\theta + r_1 \cos(\alpha_1 - \Delta\theta)}$$

$$= \frac{r_2 \sin\alpha + r_1 \left[\sin(\alpha_1 - \Delta\theta) - \sin(\alpha - \Delta\theta)\right]}{r_2 \cos\alpha + r_1 \left[\cos(\alpha_1 - \Delta\theta) - \cos(\alpha - \Delta\theta)\right]} \tag{5}$$

Where for $r_1 < r_2$, $\Delta\theta < 0$.

Calling $\delta\alpha_i = \alpha_i - \alpha$, to first order in the $\delta\alpha_i$ we have

$$\delta\alpha_2 = \frac{r_1}{r_2} \cos\Delta\theta \, \delta\alpha_1 \tag{6}$$

III. Effect on Frequency

We use the smooth approximation to estimate the change in frequency with radius. Calling $\Delta \nu_z(r_i)$ the change in frequency from the un-tuned value $(\alpha_i = \alpha)$ at radius r_i, we find from the smooth approximation result

$$\nu_z^2 = -k + \left(\frac{f}{wN}\right)^2$$

that

$$\frac{\Delta \nu_z(r_i)}{\nu_z} = -\frac{k+\nu_z^2}{\nu_z}\left[\frac{1+(wN)^2}{wN}\right]\delta\alpha(r_i) \qquad (7)$$

or

$$\frac{\Delta \nu_z(r_2)}{\Delta \nu_z(r_1)} = \frac{\delta\alpha(r_2)}{\delta\alpha(r_1)} = \frac{r_1}{r_2}\cos\Delta\theta, \qquad (8)$$

where r_1 is the fixed radius. $\Delta\theta$ and $\frac{r_1}{r_2}$ are related by (3), so that (8) is a function only of $\frac{r_1}{r_2}$, i.e.,

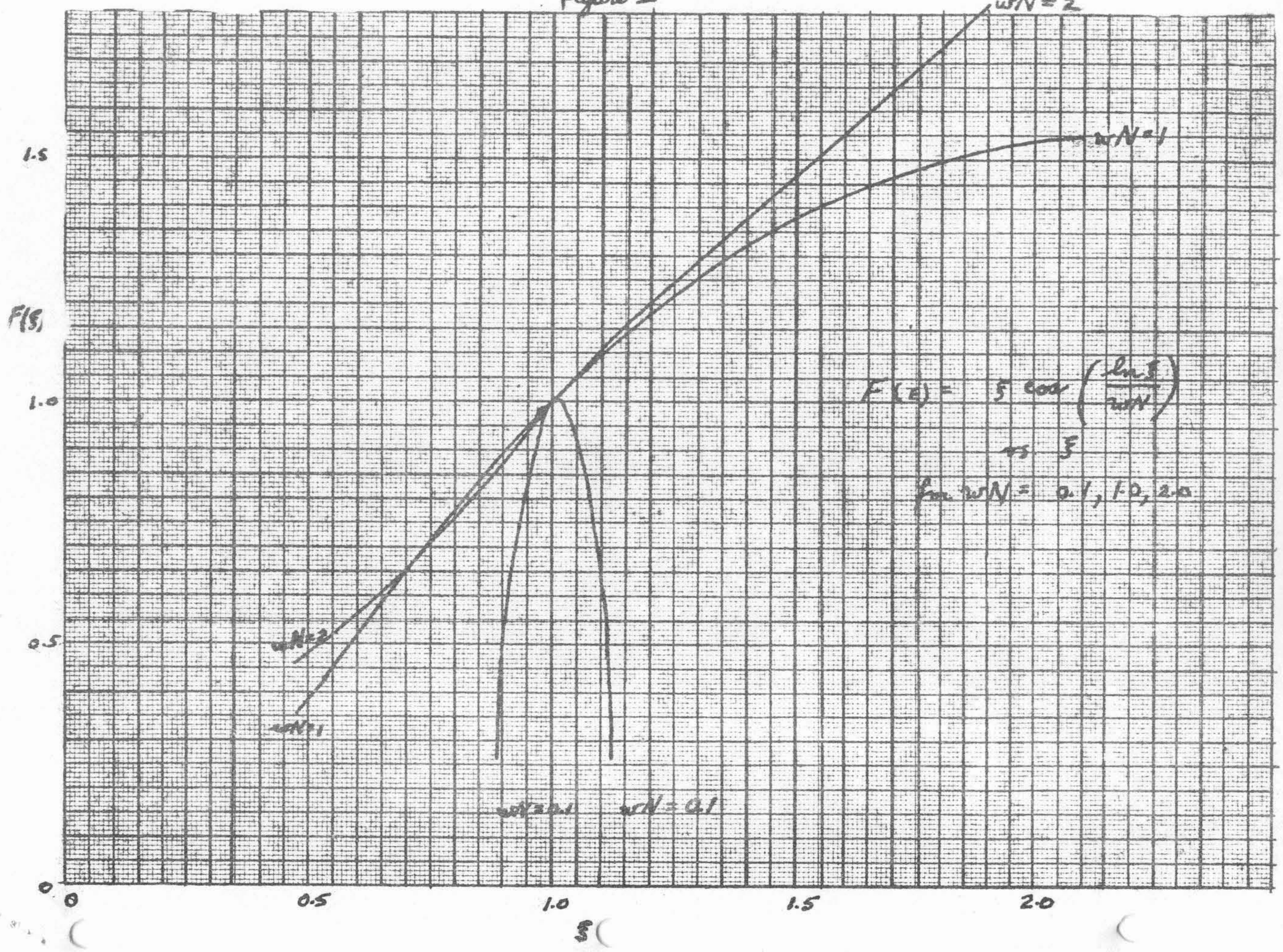
$$\frac{\Delta \nu_z(r_2)}{\Delta \nu_z(r_1)} = \frac{r_1}{r_2}\cos\left(\frac{\ln\frac{r_1}{r_2}}{wN}\right) \qquad (9)$$

In Figure II we graph this function for three values of wN, where wN = 1 and wN = 2 are in the range appropriate for small models and wN = 0.1 is in the range appropriate for a large machine. Because of the much tighter spiral in large machines, (9) is much more rapidly varying in this case, but the range of $\frac{r_1}{r_2}$ is much smaller, as is indicated on the graph.

In a small model where the radius change is of the order of 2, then tuning changes the frequency only 50% as much as the outer edge as at the inner, which makes tuning across resonances difficult. In a large machine, where the radius change is less than 10%, the frequency at the outer edge is changed about 80% as much as the frequency at the inner edge. These numbers cannot be improved materially by choosing an axis at any point between the inner and outer edges.

An interesting point about misalignments may be noted from (6). If a point on the inner edge of the radial aperture is held fixed and the magnet twisted about it, then the misalignment (the angle of twist) at larger radii is always smaller. If a point on the outer edge is fixed, then the misalignments at smaller radii are larger for some values of $\Delta\theta$. It appears wise, therefore, to align carefully the inner edges of separated sector magnets.

Figure II



$wN = 2$
$wN = 1$
$F(\xi)$
1.5
1.0
0.5
0
$F(\xi) = \xi \cos\left(\frac{\ln \xi}{wN}\right)$
vs ξ
for $wN = 0.1, 1.0, 2.0$
$wN=2$
$wN=1$
$wN=0.1$
$wN = 0.1$
0
0.5
1.0
1.5
2.0
ξ