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Mura Notes

MURA 122

APPROXIMATE CALCULATION OF THE POTENTIAL AT THE BOUNDARY OF A CURRENT CARRYING RECTANGULAR COIL, AT THE CORNER OF A POLE.

Mr. E. Weinberg

As a first approximate calculation it is assumed that the effect due to the corner is small, and hence that an infinitely long pole may be considered. The validity of this approximation has not been investigated. Under the conditions the potential will be due to the current flowing in the coil and an "image current" as indicated in Fig. 1. (current coming out from paper)

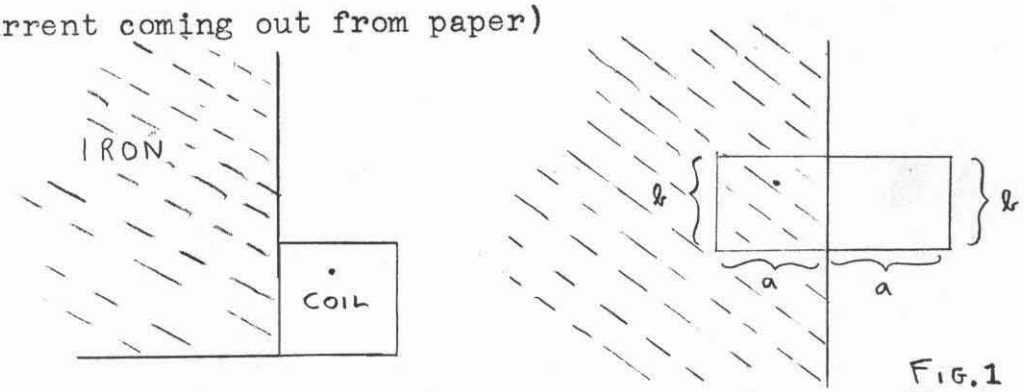
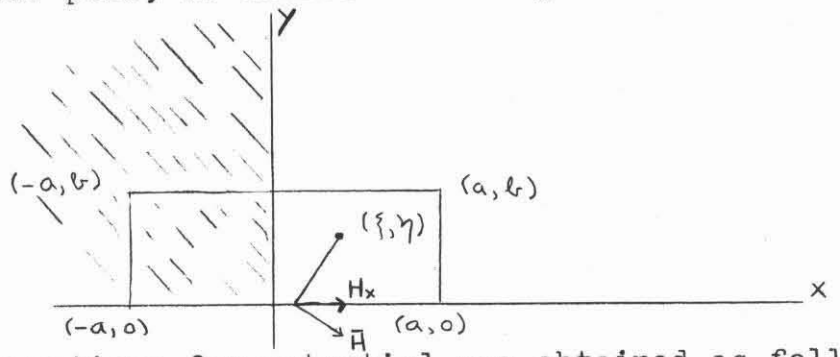


FIG. 1

For convenience consider a rectangular coordinate system with origin at corner of coil coincident with the corner of the pole, as indicated in Fig. 2.



The equations for potential are obtained as follows: Calculate the magnetic field at any point on the boundary

due to the current in the coil and its image. Calculate potential from relation $V = \int \bar{H} \cdot d\bar{s}$

e.g. Given a point (x,o) such that $0 \le x \le a$

$$H_x = \int_0^b \int_{-a}^{a} \frac{\cos\theta \, d\xi \, d\eta}{[(\xi - x)^2 + \eta^2]^{1/2}} = \int_0^b \int_{-a}^{a} \frac{\eta \, d\eta \, d\xi}{(\xi - x)^2 + \eta^2}$$

$$V_{x,0} = \int_0^x H_x \, dx$$

Potential at an arbitrary point on the boundary of the coil is then given by

$$V_{x,0} = \frac{1}{4}[(x-a)^2 - b^2] \ln[(x-a)^2 + b^2] - \frac{1}{4}[(x+a)^2 - b^2] \ln[(x+a)^2 + b^2] - \frac{(x-a)^2}{2} \ln|x-a|$$
$$+ \frac{(x+a)^2}{2} \ln(x+a) - b(x-a) \tan^{-1}\left(\frac{a-x}{b}\right) - b(x+a) \tan^{-1}\left(\frac{a+x}{b}\right)$$

$$V_{a,y} = -\frac{1}{4}[4a^2 - (b-y)^2 \ln[(b-y)^2 + 4a^2] + \frac{1}{4}(4a^2 - y^2) \ln(y^2 + 4a^2) - \frac{(y-b)^2}{2} \ln|y-b|$$
$$+ \frac{y^2}{2} \ln y + 2a(b-y) \tan^{-1}\left(\frac{b-y}{2a}\right) - 2ay \tan^{-1} \frac{y}{2a} - \pi ab$$

$$V_{x,b} = -2\pi ab + \frac{1}{4}[b^2 - (x-a)^2] \ln[(x-a)^2 + b^2] + \frac{1}{4}[(x+a)^2 - b^2] \ln[(x+a)^2 + b^2]$$
$$+ \frac{(x-a)^2}{2} \ln|x-a| - \frac{(x+a)^2}{2} \ln(x+a) + b(x-a) \tan^{-1}\left(\frac{a-x}{b}\right) + b(x+a) \tan^{-1}\left(\frac{a+x}{b}\right)$$

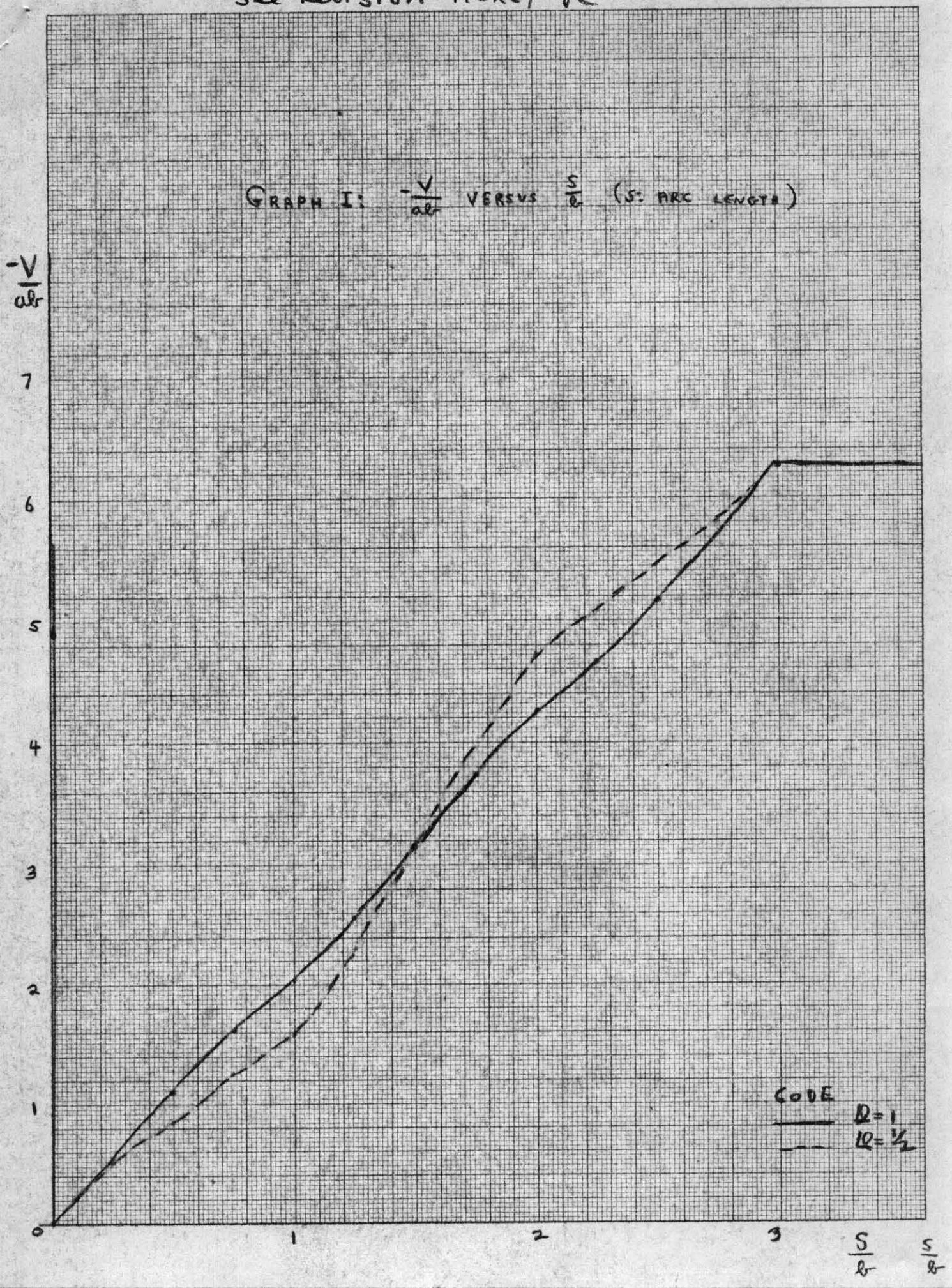
$$V_{0,y} = -2\pi ab$$

If $a = Lb$ Graph 1 indicated the relation of potential to arc length for the cases $L = 1$ and $L = \frac{1}{2}$

The author is indebted to Professor E. S. Akeley who suggested this problem and the method of solution.

Weinberg.

See Revision next page



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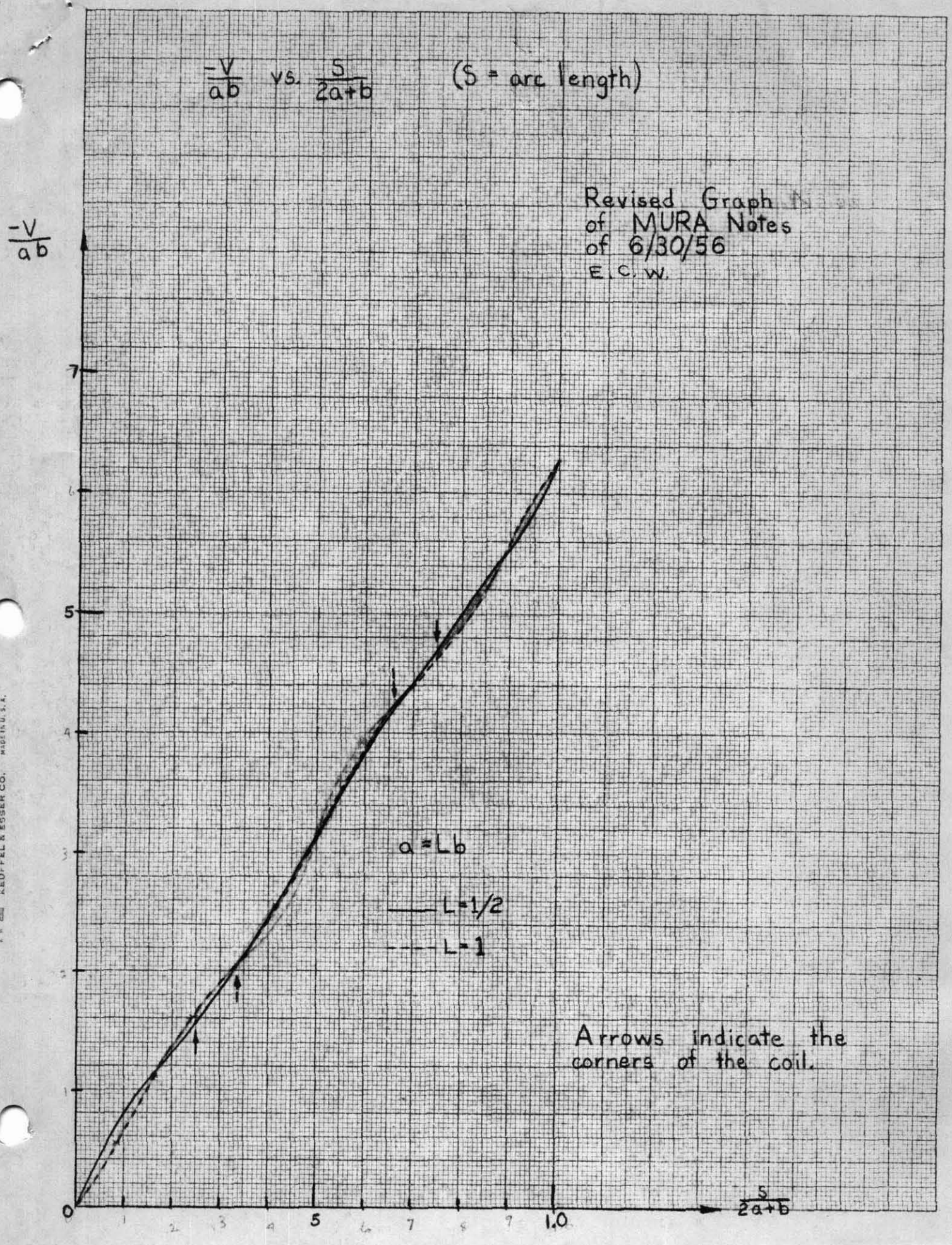
Elliot Weinberg



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