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ANALYSIS AND COMPUTATIONS OF MAGNETIC FIELDS ARISING FROM TWO-DIMENSIONAL POLE CONFIGURATIONS OF INTEREST IN SPIRAL-SECTOR ACCELERATORS

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ABSTRACT

Computations have been made, for the greater part by aid of the ILLIAC digital computer, of the median-plane fields which result magnetostatically from various two-dimensional pole configurations resembling those which would be employed in a spirally-ridged FFAG accelerator. The results which are presented are analized in terms of a parameter $\mathcal{Q}$, defined as

$$\mathcal{Q} \equiv \sqrt{\frac{2}{w^2}\left[\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1\right]},$$

which is believed to be the analogue of the quantity f/w which determines the frequency of betatron oscillations in a strictly-sinusoidal focusing field.

For the various structures studied the following quantities are tabulated:

g/λ, the ratio of semi-gap to the wave length of the structures;

$\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1$, being a measure of the flutter of the median-plane field;

g, the semi-gap, in units of q;

[CF], a "circumference factor" defined in terms of the median-plane field as $B_{max}/\langle B\rangle$; and

$g \cdot$ [CF].

The median-plane fields resulting from a number of structures considered of greatest interest are also presented graphically. The suggestion is advanced that optimization may consist in maximizing g for a constant value of the parameter $\mathcal{Q}$.

CONTENTS

I. INTRODUCTION

1. General:

The Mark V or spirally-ridged FFAG accelerator was originally discussed in terms of a structure for which the spatial variation of the median-plane field was strictly sinusoidal. A field of this character is not necessarily optimum from the standpoint of orbit characteristics and moreover can impose severe limitations of a magnetostatic character on the magnitude of the gap between the pole-pieces. To permit a study of the characteristics of spirally-ridged accelerators with a more general type of field, a computational program based on LJL(MURA)-8 is in preparation. In the belief, however, that useful orientation concerning the field themselves could be obtained by a survey based on the two-dimensional Laplace equation, the work summarized in the present report was undertaken. The report confines itself to solutions of the two-dimensional Laplace equation with periodic boundary conditions.

2. Basis of Comparison of Fields:

In the consideration of various field configurations it is believed desirable to compare structures which may be expected to yield the same advance of betatron phase per sector. It is recalled that, in the case of the purely sinusoidal median-plane field

$$B_0 = \bar{B}(1+x)^k \left[1 + f \sin\left(\frac{z}{w} - N\theta\right)\right],$$

the strength of the A-G focusing is substantially determined by $(f/w)^2$, k, and N. For fields with a harmonic content, and which hence show a more elaborate spatial variation, it seems appropriate to characterize the effect of the flutter by the quantity

$$q^2 \equiv \frac{2}{w^2}\left[\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1\right],$$

where, as before, $2\pi w$ is the wavelength (λ) of the radial variation of the field, measured in units of the radius. The supposition that the foregoing quantity correctly characterizes the influence of the field-flutter on the betatron oscillations is believed to be concordant, within the framework of the smooth approximation, with the unified theory of FFAG developed by Dr. Symon.

In what follows, we shall, accordingly, express and compare such features as the pole-gap for various configurations in terms of q. Specifically, if the median-plane field is determined for pole surfaces with a semi-gap g and spatial wavelength λ, one writes

$$g = \frac{g}{\lambda}\sqrt{\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1} \Bigg/ \left\{\frac{1}{2\sqrt{2}\,\pi}\sqrt{\frac{2}{w^2}\left[\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1\right]}\right\}$$

$$= \frac{\left(\frac{g}{\lambda}\right)\sqrt{\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1.}}{0.1125395\ldots\, q}$$

, measured in units of the radius, the coefficient of $\frac{1}{q}$ thus serving as a figure-of-merit for the structure under consideration. It is recognized that the character of <u>large-amplitude</u> betatron oscillations may differ, of course, for particle motion in various fields of the same mean-square flutter.

II. ANALYTICAL EXAMINATION OF SIMPLE FIELDS

1. General:

We examine here analytically the maximum aperture which may be achieved by purely magnetostatic means (avoiding the use of forward and back-wound current-carrying conductors) for a strictly sinusoidal median-plane field and for fields with a third-harmonic content of the type suggested by Powell (MURA-JLP-6). Considerations of this nature, given previously by the present writer in informal MURA Notes (October & December, 1955) have been presented originally by Terwilliger (MURA/KMT-2) for a step distribution of field and by Kerst (MURA/DWK-10) for the sinusoidal distribution.

2. Analytical Work:

(i) Sinusoidal Field:-

We consider first the character of the two-dimensional median-plane field

$$B_0 = -\bar{B}\left[1 + f \sin\frac{x}{w}\right].$$

This field may be represented by the complex potential function

$$U + iV = r_1 \bar{B}\left[(x+iy) - wf \cos\frac{x+iy}{w}\right],$$

from which

$$V = r_1 \bar{B}\left[y + wf \sin\frac{x}{w} \sinh\frac{y}{w}\right].$$

The limiting equipotential, V_0, is determined by setting $\sin\frac{x}{w} = -1$ and $\frac{\partial V}{\partial y} = 0$:

$$1 - f \cosh\frac{y}{w} = 0$$

$$y = w \cosh^{-1}\frac{1}{f}$$
$$= w \operatorname{sech}^{-1} f \, , \quad \text{and}$$

$$V_0 = r_1 \bar{B} w \left[\operatorname{sech}^{-1} f - \sqrt{1-f^2}\right].$$

The minimum gap presented by this equipotential is obtained when $\sin\frac{x}{w} = +1$ and is given by

$$y + wf \sinh\frac{y}{w} = w\left[\operatorname{sech}^{-1} f - \sqrt{1-f^2}\right]$$

or

$$\frac{y}{w} + f \sinh\frac{y}{w} = \operatorname{sech}^{-1} f - \sqrt{1-f^2}.$$

To maximize the semi-aperture, subject to the condition that f/w remain constant (§ I, 2), we rewrite this last equation in terms of $Y \equiv (f/w)\,y$:

$$\frac{Y}{f} + f \sinh\frac{Y}{f} = \operatorname{sech}^{-1} f - \sqrt{1-f^2}.$$

The value of Y obtained by solution of this equation is found to be greatest when

$$f = 0.236,$$

for which

$$Y = 0.214 \; .$$

Hence $\quad y = (w/f)\,Y$

$$= 0.907\, w \, ,$$

the semi-aperture is $\quad y = 0.144\,\lambda$,

and the full gap is $\quad \underline{G = 0.288\,\lambda}$.

$$\mathcal{g} = 0.127 / \mathcal{q}$$
$$= 0.5335\, w, \quad \text{and}$$
$$G = 0.170\, \lambda\ .$$

Due to the influence of the factor Sinh $3\frac{y}{w}$ in one term of the magnetostatic potential, the maximum permisssible aperture thus appears to be 59.2% of the corresponding value previously found for a pure sinusoidal field with the same q .

The character of the limiting equipotential for the approximately saw-tooth field is illustrated in Fig. 2.

(iii) Flat-Topped Field:-

A third-harmonic component with $f_2 = +\frac{1}{9} f_1$ will yield a flat-topped field. In this instance the "crisis" which determines the limiting equipotential surface develops at points other than those for which sin $\frac{x}{w} = -1$. We have not undertaken optimization in this case, but for

$$f_1 = 0.236$$

we find

$$g = 0.168/q$$
$$= 0.707\, w, \quad \text{and}$$
$$G = 0.225\, \lambda\ .$$

The limiting equipotential surface for this case is illustrated in Fig. 3.

3. Summary of Analytic Work:

In table I we summarize the results of the preceding analytic work. We include the "circumference factor", somewhat arbitrarily defined in terms of the median-plane field as $[B]_{max} / \langle B \rangle_{Av}$.

TABLE I

CHARACTERISTICS OF LIMITING MAGNETOSTATIC EQUIPOTENTIAL

Case	f_1	f_2	Semi-Aperture, g	[C.F.]
(i) Sinusoidal	0.236	0	$0.214/\ell = 0.907w = 0.144\lambda$	1.236
(ii) Saw-Tooth	0.236	$-\frac{0.236}{9}$	$0.127/\ell = 0.5335w = 0.085\lambda$	1.262
(iii) Flat-Top*	0.236	$+\frac{0.236}{9}$	$0.168/\ell = 0.707w = 0.11\dot{3}\lambda$	1.210

* Not optimized

It appears clear from the foregoing considerations that one is limited illadvisedly by strict conformance to some supposedly-simple median-plane field. The permissible aperture certainly can be increased substantially, with little change in the working field, by rounding off or filling in the crevices in the pole contour. Study of such modified profiles, including cases more easily fabricated, is thus appropriate and motivates the remainder of the work reported here. It is recognized that investigation of orbit characteristics will subsequently be necessary for any significantly-modified fields which may appear to be of particular interest.

III. DIGITAL COMPUTATIONS

1. General:

The "FL..." program was devised for the ILLIAC by Dr. J. N. Snyder, to solve Laplace's equation in two-dimensional Cartesian coordinates on a net of dimensions 49 x 14. Positive boundary values, less than 0.25, were entered at mesh points corresponding to the pole-surface and median-plane ($V = 2^{-39}$ on the median-plane). The remaining starting-potentials, at various points in the gap remote from the region of interest, were estimated and entered with the input data to complete specification of the problem. The solution of the potential problem was considered complete when, after a couple of hundred iterations, the residuals at a specified check-point changed by less than 10^{-6} in a single iteration. From the potentials so obtained, the median-plane fields were computed by means of the algorism

$$-B_0 = \frac{V_{-11} + 4V_{01} + V_{11}}{6},$$

in which use is made of the symmetry of the complete solution (V odd, B_y even) about the median plane. In addition, averages involving the median-plane fields were computed, between specified points, to give

$$\langle B_0 \rangle_{Av} \qquad \text{and} \qquad \frac{\langle B_0^2 \rangle_{Av}}{\langle B_0 \rangle_{Av}^2} - 1 .$$

Several series of problems were run involving rectangular equi-potential pole surfaces. In any one series the ratio of gap (2g) to wavelength (λ) was varied. The first three series involved poles for which the pole-width was equal to the full gap, with the pole root respectively 3, 4, and 14/3 times as remote from the median plane as the pole surface. Two subsequent series were concerned with poles of other widths, in each case with the pole root located (14/3)g from the median plane. A later series was concerned with 45-degree poles and another with rectangular poles modified by a 45-degree bevel at the pole-tip and a 45-degree fillet at the pole-root.

To obtain approximate information concerning the effect of coils, additional runs were made for rectangular poles (of width 2g and pole-root (14/3)g from the median plane) on which the potential was zero at all points save on the pole-face itself. Additional, more extreme, runs were also made in which the potential was specified as alternately 0 and V_o along a surface of constant height above the median plane.

Finally some miscellaneous runs involving equi-potential pole-surfaces of various configurations were made, including profiles intended to give approximately sinusoidal, saw-tooth, and flat-top fields.

The results for these various series of runs are summarized in the following sections. In addition to giving the figure-of-merit, relating the semi-gap g to the parameter ℓ, the circumference factor $[CF] = [B_o]_{max} / \langle B_o \rangle_{AV}$ and $g \cdot [CF]$ are also computed, the latter quantity presumably being indicative of the number of ampere-turn centimeters necessary to derive the required flux across the gap between equipotential pole-surfaces if the full magnetostatically-possible gap is used. For comparison it may be recalled that these quantities were found analytically to have the following values for a strictly sinusoidal field with f = 0.236 ($\frac{\langle B^2 \rangle}{\langle B \rangle^2}$ - 1 = 0.02785) and g/λ = 0.144:

$$g = 0.2140/q$$
$$[CF] = 1.236$$
$$g \cdot [CF] = 0.2645/q \, .$$

2. <u>Rectangular Equipotential Poles</u>:

The results of several series of runs involving rectangular equipotential poles are summarized in Table II. An arrow is used to denote that run in each series for which g is a maximum, although a somewhat larger g/λ (reduced λ) might be preferable in an overall sense because of the resulting decrease of circumference.

TABLE II

CHARACTERISTICS OF RECTANGULAR EQUIPOTENTIAL POLES

Pole Width	Dist. of Pole Root from Med. Pl.	$\frac{g}{\lambda}$	$\frac{\langle B^2 \rangle}{\langle B \rangle^2}-1$	g	[CF]	g·[CF]	
2g	3g	3/28	0.15521	0.37507/q	1.670	0.6264/q	
		4/34	0.13911	0.38990/q	1.610	0.6278/q	
		4/30	0.11544	0.40255/q	1.520	0.6117/q	
		4/27	0.09409	0.40380/q	1.444	0.5832/q	←
		4/24	0.07030	0.39267/q	1.363	0.5350/q	
		4/21	0.04574	0.36199/q	1.275	0.4617/q	
	4g	3/43	0.29966	0.33936/q	2.150	0.7297/q	
		3/32	0.23871	0.40701/q	1.883	0.7666/q	
		3/28	0.20123	0.42708/q	1.755	0.7494/q	
		3/26	0.17926	0.43409/q	1.685	0.7313/q	
		3/24	0.15526	0.43765/q	1.611	0.7049/q	←

(cont'd)

Pole Width	Dist. of Pole Root from Med. Pl.	$\frac{g}{\lambda}$	$\frac{\langle B^2\rangle}{\langle B\rangle^2}-1$	g	[CF]	g·[CF]
		3/22	0.12935	$0.43579/\ell$	1.533	$0.6681/\ell$
		3/20	0.10244	$0.42660/\ell$	1.452	$0.6194/\ell$
		3/18	0.07534	$0.40649/\ell$	1.369	$0.5563/\ell$
		3/16	0.04976	$0.37166/\ell$	1.284	$0.4772/\ell$
	(14/3)g	3/32	0.26439	$0.42834/\ell$	1.926	$0.8249/\ell$
		3/30	0.24187	$0.43701/\ell$	1.856	$0.8109/\ell$
		3/26	0.19103	$0.44812/\ell$	1.705	$0.7640/\ell$
		3/24	0.16314	$0.44863/\ell$	1.625	$0.7290/\ell$
		3/22	0.13416	$0.44382/\ell$	1.542	$0.6844/\ell$
		3/20	0.10486	$0.43162/\ell$	1.457	$0.6289/\ell$
		3/18	0.07635	$0.40923/\ell$	1.371	$0.5610/\ell$
(8/3)g	(14/3)g	3/34	0.26866	$0.40638/\ell$	1.844	$0.7492/\ell$
		3/30	0.21959	$0.41639/\ell$	1.709	$0.7118/\ell$
		3/26	0.16399	$0.41519/\ell$	1.565	$0.6496/\ell$
		3/24	0.13461	$0.40752/\ell$	1.489	$0.6067/\ell$
		3/22	0.10518	$0.39297/\ell$	1.411	$0.5546/\ell$
(4/3)g	(14/3)g	3/30	0.24840	$0.44286/\ell$	1.999	$0.8853/\ell$
		3/26	0.20474	$0.46392/\ell$	1.847	$0.8568/\ell$
		3/24	0.18022	$0.47153/\ell$	1.765	$0.8322/\ell$
		3/22	0.15400	$0.47550/\ell$	1.679	$0.7984/\ell$
		3/20	0.12658	$0.47421/\ell$	1.590	$0.7539/\ell$
		3/18	0.09873	$0.46533/\ell$	1.497	$0.6968/\ell$

The results of these five series of runs are depicted in Figs. 4-8, while Fig. 9 is a composite graph of g for all the cases involving rectangular equipotential poles. The field-distributions in the median plane for representative cases in each series are shown in Figs. 10-14. It appears that, among the cases considered, optimum configurations are encountered for g/λ in the neighborhood of 1/7 or 1/8 and lead to fields which do not depart markedly from simple sinusoids. A deep gap between the poles is clearly desirable and thin poles appear to present an advantage.

3. <u>45-Degree and Beveled Equipotential Poles</u>:

The results obtained for a series of runs involving equipotential poles with some 45-degree surfaces, as sketched in Figs. 15 and 16, are listed in Table III.

TABLE III

CHARACTERISTICS OF 45-DEGREE & BEVELED EQUIPOTENTIAL POLES

Type of Pole	g/λ	$\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1$	g	[CF]	$g \cdot$[CF]
45°, Min.Width=(4/3)g	3/26	0.13242	0.37309/g	1.643	0.6129/g
	3/25	0.12261	0.37336/g	1.609	0.6006/g
	3/24	0.11297	0.37332/g	1.575	0.5879/g
	3/20	0.07560	0.36648/g	1.437	0.5268/g
	3/18	0.05797	0.35656/g	1.368	0.4878/g
Beveled Pole, with Fillet	3/32	0.21894	0.38978/g	1.927	0.7513/g
	3/29	0.19506	0.40598/g	1.834	0.7445/g
	3/24	0.14586	0.42420/g	1.659	0.7039/g
	3/22	0.12336	0.42557/g	1.583	0.6738/g
	3/20	0.09997	0.42142/g	1.504	0.6337/g
	3/18	0.07655	0.40974/g	1.422	0.5827/g

The results for these respective series are depicted in Figs. 17 and 18.

The median-plane fields for two configurations of the second series are shown in Fig. 19. The fields produced by such beveled poles may be considered somewhat freer from rapid changes near the pole-corners, although the structure does not appear to offer quite as high a figure-of-merit as is possible with rectangular poles. The 45-degree poles appear to be less promising with respect to opening up the gap.

4. Rectangular Poles with Pole-Tip only at Potential Different from Zero:

In this series of runs the pole tip was considered to assume a constant specified potential, while the remainder of the rectangular pole and the median-plane were held at zero potential. In all cases the pole width was taken as 2g and the pole root was (14/3)g from the median plane. The results are listed in Table IV.

TABLE IV

CHARACTERISTICS OF RECTANGULAR POLES WITH POLE-TIP ONLY AT V 0

Pole Width	Dist. of Pole Root from Med. Pl.	$\frac{g}{\lambda}$	$\frac{\langle B^2\rangle}{\langle B\rangle^2} - 1$	g	[CF]	$g \cdot$[CF]
2g	14/3 g	3/28	1.13826	1.01573/2	2.936	2.9817/2
		3/26	0.98950	1.01989/2	2.728	2.7821/2
		3/24	0.84240	1.01945/2	2.521	2.5698/2
		3/22	0.69736	1.01186/2	2.315	2.3422/2
		3/18	0.41833	0.95786/2	1.907	1.8266/2
		3/14	0.17387	0.79395/2	1.512	1.2008/2
		3/12	0.08035	0.62971/2	1.326	0.8352/2
		3/10	0.02091	0.38545/2	1.158	0.4463/2

The values of the gap and circumference-factor for all but the last run in this series are represented graphically in Fig. 20*. The character of the median-plane field in some representative cases, near the optimum, is illustrated in Fig. 21. It is noted that a very substantial increase of gap and of flutter-factor, on the basis of the criterion adopted in this report, is attainable with the present type of configuration, this possible advantage being at the expense of increased total circumference-factor and of any associated elaboration of the copper required to accommodate the magnetizing current. The maximum of the curve representing g vs. g/λ (Fig. 20) occurs at a value of g/λ not greatly different from that found previously, the optimum value of this parameter being near 1/8 or 1/9. With this configuration the field drops to quite low values between the poles (leading to a large total circumference-factor), and the flat-bottomed aspect of the field-plots shown in Fig. 21 suggests the presence of significant harmonic content in those cases.

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* Note change of scale from previous graphs of this type.

5. Plane Pole-Face, with Potential Alternately V_0 and Zero:

A number of runs related to the series described in Section 4 were made with a plane pole-face, parallel to the median-plane, along which the potential alternated between the values V_0 and zero. The results of these runs, which by no means represent a complete survey of such a configuration, may to some extent illustrate the general character of the fields obtained with this arrangement and are summarized in Table V. The significance of the various dimensions is shown by Fig. 22.

TABLE V

CHARACTERISTICS OF PLANE POLE-FACE, WITH POTENTIAL ALTERNATELY V_0 AND ZERO

Dimensions g	λ	p	s	g/λ	$\frac{\langle B^2\rangle}{\langle B\rangle^2}-1$	f	[CF]	g·[CF]
3	24	13	11	3/24	0.58291	0.84802/ℓ	1.841	1.5610/ℓ
do.	do.	7	17	do.	1.52582	1.37201/ℓ	3.2518	4.4615/ℓ
4	do.	15	9	4/24	0.33481	0.85693/ℓ	1.590	1.3624/ℓ
do.	do.	13	11	do.	0.49144	1.03820/ℓ	1.822	1.8911/ℓ
do.	do.	9	15	do.	0.93005	1.42822/ℓ	2.513	3.5887/ℓ
do.	do.	7	17	do.	1.23218	1.64392/ℓ	3.024	4.9717/ℓ
6	do.	17	7	6/24	0.12795	0.79460/ℓ	1.379	1.0958/ℓ
do.	do.	13	11	do.	0.32227	1.26108/ℓ	1.727	2.1778/ℓ
do.	do.	7	17	do.	0.75370	1.92856/ℓ	2.505	4.8302/ℓ
8	do.	15	9	8/24	0.12481	1.04641/ℓ	1.445	1.5124/ℓ
do.	do.	13	11	do.	0.18953	1.28948/ℓ	1.585	2.0435/ℓ
do.	do.	9	15	do.	0.34444	1.73833/ℓ	1.899	3.3014/ℓ
10	do.	9	15	10/24	0.17992	1.57043/ℓ	1.6352	2.5680/ℓ
12	do.	9	15	12/24	0.08320	1.28151/ℓ	1.4272	1.8290/ℓ

It would appear from the results of Table V that quite large gaps and high flutter factors may be attained with such plane pole-faces, with, for example, g/λ as large as 1/4 or more and with the larger portion of the pole-face at zero potential. It may be expected that large flutter-factors will increase the limits of stable particle motion, but only if the harmonic content of the field is not concurrently increased by a large amount.

6. Miscellaneous, Shaped Equipotential Poles:

A number of miscellaneous cases involving contoured equipotential poles were thought to be of special interest at the start of this work. The configurations investigated, and the resultant median-plane fields, are illustrated in Figs. 23-30, inclusive. The results for these cases are summarized in Table VI.

TABLE VI

CHARACTERISTICS OF MISCELLANEOUS, SHAPED EQUIPOTENTIAL POLES

Fig.	Character	$\frac{g}{\lambda}$	$\frac{\langle B^2 \rangle}{\langle B \rangle^2} - 1$	g	[CF]	$g \cdot$[CF]
23	Beveled & Shaped; Attempt to synthesize sinusoidal field	4/32	0.05156	0.25221/q	1.324	0.3339/q
24	Attempt to synthesize approximately-sinusoidal field	6/32	0.02958	0.28654/q	1.260	0.3610/q
25	Attempt to synthesize saw-tooth field	4/32	0.02933	0.19022/q	1.290	0.2454/q
26	Attempt to synthesize flat-top, flat-bottom field	4/32	0.04338	0.23135/q	1.299	0.3006/q
27	Sloping pole; attempt to synthesize flat-bottom field	3/32	0.12342	0.29266/q	1.619	0.4739/q
28	Sloping pole with boss	3/32	0.19820	0.37086/q	1.891	0.7011/q
29	Beveled, necked, and flared pole	3/32	0.22868	0.39836/q	1.954	0.7784/q
30	Under-cut Pole	3/32	0.24247	0.41020/q	1.893	0.7764/q

For comparison it may once again be mentioned that the corresponding characteristics of a magnetostatic structure giving a strictly sinusoidal field are (with $f = 0.236$):

$$\begin{aligned} [g/\lambda]_{opt.} &= 0.144 \\ \frac{\langle B^2\rangle}{\langle B\rangle^2} - 1 &= 0.02785 \\ g &= 0.2140/g \\ [CF] &= 1.236 \\ g\cdot[CF] &= 0.2645/g \end{aligned}$$

7. <u>Summary</u>:

The structure of Fig. 24, intended to give an approximately sinusoidal field, may be illustrative of pole-contours which have some merit if it is desired to enlarge the gap somewhat without introducing to any significant degree the complications of harmonic content. Aside from the question of harmonic content, however, the structures considered earlier in this report have the advantage of very materially increasing the effective flutter-factor and of permitting a very great enlargement of the magnet gap. Among the various types of equipotential poles, rectangular (Sect. 2) or beveled (Sect. 3) poles may attract the greatest interest, while the presence of current-carrying coils in a separated-sector accelerator directs attention to configurations similar to those considered in Section 4 of this report. A final, rational selection between competitive structures must, of course, await the outcome of computational tests concerning particle dynamics in the magnetic fields which these structures provide.