Fermilab Library
0 1160 0038486 1

MURA #114

16 May 1956

Memorandum to : Jim Snyder

From: A. M. Sessler

Topic: Proposed Digital Computer Program to Study the Coupling of Radial Betatron Oscillations; and Synchrotron Oscillations; in the Presence of Foils; and Non-slaunched, Non-Radially Terminating, Leakage Flux and Magnetic Effects Absent, R. F. Gaps.*

Motivation: 1) It appears important that we study the effects of coupling between orbital motion and synchrotron oscillations in order to be able to understand completely such things as R. F. knockout.

2) The Princeton people have made the important observation that it is possible to devise systems which are non-Liouvillian as far as the accelerated particles are concerned. This is readily reconciled with general theorems of dynamics by noting that the proposed schemes introduce other particles (electrons in foils) so that the total phase space is still conserved, or alternatively the accelerated particles are subject to dissipative forces. The possibilities opened up by the observation must be studied, since successful use of foils may allow a storage ring to be substituted for an accelerator--at a considerable saving in cost.

3) The separated sector accelerator has slaunched cavities, and some of the proposed R. F. schemes employ cavities which only extend over part of the radial aperture. It is important to study the effects these cavities may engender, but it was felt that the simpler problem in which these effects were ignored, should at least be formulated first. It may

*I am indebted to Dr. Laslett for constant encouragement and support during the writing of this title.

then be considered whether one wants two programs, or to include all effects in one program. In this regard the question of speed of computation should be given great consideration. The program outlined here, is I believe, quite rapid.

4) In the interest of simplicity, and hence speed of computation, we have confined our attention to orbital plane motion. Also the A. G. characteristics have been ignored, the guiding field assumed to vary as r^k as far as the synchrotron oscillations are concerned, but the betatron frequency may be inserted separately. (Thus allowing one to study some of the A. G. effects in-so-far-as they change the betatron frequency--scallop and ripple ignored.) Also the betatron oscillations are treated in linear approximation. One can include one of the effects of non-linearities by requiring that the amplitude of betatron oscillations remain below a limit which can arbitrarily be imposed--employing our knowledge of stability limits in A. G. accelerators, to determine this limit.

Program:

1) Number of oscillators = N_o (suggest $N_o \leq 4$)

2) Oscillator index = j (j = 1, 2, ... N_o)

3) Number of foils = N_f (suggest $N_f \leq 10$)

4) Number of oscillator transits of particle = n

($n = N N_o + j$)

5) Array: This specifies order of foils and oscillators.

For example: O_1 F_1 F_2 O_2 F_3 F_4 ... etc.

6) Auxiliary Functions:

A) Each foil has a function $U(R)$ which should specified on an appropriate master tape. For initial use of the program we should have the following functions:

a) $U(R) = U_o$, $R > R_{fm}$ $R < R_{fM}$

b) $U(R) = \begin{cases} U_o - U_1 R, & R < U_o/U_1 \\ 0, & R \geq U_o/U_1 \end{cases}$

c) $U(R) = \begin{cases} 0, & R < \frac{U_o}{U_1} \\ -U_o + U_1 R, & R \geq \frac{U_o}{U_1} \end{cases}$ where U_o, U_1 are constants.

B) Each oscillator has a voltage $V(t)$, and frequency $W(t)$, function which is given on an

appropriate master tape. Initially we might have available the following functions:

a) $V(t) = V_0$

b) $V(t) = \begin{cases} \dfrac{V_0 (t - t_1)}{(t_2 - t_1)} & t_1 \leq t \leq t_2 \\ V_0 & t_2 \leq t \leq t_3 \\ \dfrac{V_0 (t_4 - t)}{(t_4 - t_3)} & t_3 \leq t \leq t_4 \end{cases}$

c) $w(t) = W_0$

d) $w(t) = W_0 + W_1 t$ where:

$V_0, W_0, |W_1|$ are positive constants (W_1 may be negative), t_1, t_2, t_3, t_4, may be specified.

7) In the course of calculations there will appear certain other functions which you must build (permanently) into the program:

$$F(E, x, P, \alpha) = \frac{E \; R_e(E)}{c\left[E^2 - E_0^2\right]^{1/2}} \quad S(E, x, P, \alpha)$$

$$S(E, x, p, \alpha) = 2\pi\alpha + \frac{1}{4\nu}\left\{x^2\nu^2\left(2\pi\alpha - \frac{1}{2}\sin 4\pi\nu\alpha\right) + \right.$$

$$\left. + p^2\left(2\pi\alpha + \tfrac{1}{2}\sin 4\pi\nu\alpha\right) + xp\nu\left(\cos 4\pi\nu\alpha - 1\right)\right\}$$

$$R_e(E) = R_0\left[\frac{E^2 - E_0^2}{1 - E_0^2}\right]^{\frac{1}{2(k+1)}}$$

8) These require more data tape info beyond that specified in (1) - (5), namely:

9) U_0, R_{f_m}, R_{f_M}, or U_0, U_1

10) V_0 or V_0, t_1, t_2, t_3, t_4

11) W_0 or $W_0 + W_1$

12) k or $\frac{1}{2(k+1)}$

13) R_0 in meters

14) $c = 3 \times 10^8 \ \frac{\text{meters}}{\text{sec.}}$

15) $\nu =$ no. of β tron waves about the machine, a pure number between 0 and 100, usually ≈ 10.

16) Array: α_j This is a series of numbers such that $\sum \alpha_j = 1$, specifying the angles between foils and gaps.

(For example, the α between

$$OF_1 = 2\pi\alpha_1$$
$$F_1F_2 = 2\pi\alpha_2$$
$$F_2O_2 = 2\pi\alpha_3$$

etc...)

17) The calculation consists of taking a "particle", namely the vector $\begin{pmatrix} x \\ p \\ E \\ t \end{pmatrix}$ through the oscillators and foils in the prescribed manner. In this connection we need to specify what happens to a particle under the following possible situations:

A.) At a foil:

$$\begin{pmatrix} x \\ p \\ E \\ t \end{pmatrix} \longrightarrow \begin{pmatrix} [Re(E)(1+x) - Re(E-\nu)]\, Re^{-1}(E-\nu) \\ p\, Re(E)\, Re^{-1}(E-\nu) \\ E-U \\ t \end{pmatrix}$$

B.) Between foils and oscillators separated by angles $2\pi\alpha$:

$$\begin{pmatrix} x \\ p \\ E \\ t \end{pmatrix} \longrightarrow \begin{pmatrix} x \cos 2\pi\nu\alpha + \frac{p}{\nu} \sin 2\pi\nu\alpha \\ -\nu x \sin 2\pi\nu\alpha + p \cos 2\pi\nu\alpha \\ E \\ t + F(E, x, p, \alpha) \end{pmatrix}$$

C.) At an oscillator:

$$\varphi_{j,N} = \varphi_{j,N-1} + W_j\left(\frac{t_{n-N_0}+t_n}{2}\right)(t_n - t_{n-N_0})$$

$$E_{n+1} = E_n + \Delta E_n$$

$$\Delta E_n = V_n(t_n)\sin\varphi_{j,N}$$

$$t_{n+1} = t_n + F_j(E_{n+1}, \alpha_j, x, p)$$

So that:

$$\begin{pmatrix} x \\ p \\ E \\ t \end{pmatrix} \longrightarrow \begin{pmatrix} [Re(E)(1+x) - Re(E+\Delta E)]\,Re^{-1}(E+\Delta E) \\ p\,Re(E)\,Re^{-1}(E+\Delta E) \\ E+\Delta E \\ t \end{pmatrix}$$

where ΔE is given above.

18) I think in the interest of speed you will want to evaluate and store: $\sin 2\pi\nu\alpha_j$, $\sin 4\pi\nu\alpha_j$, $\cos 4\pi\nu\alpha_j$.

19) The program should have the following parameters to be specified on the data tape:

20) N_{p_0} - the step on which printing starts

21) N_p - number of steps printing continues for

22) N_e - end constant - number of steps at which problem ends (I don't think we need E_F.)

23) t_f - time at which problem ends.

24) $t_{j,-1}$ - transit time oscillator j on -1 step.

25) $\varphi_{j,-1}$ - phase of oscillator j on -1 step.

26) E_i - initial energy

27) x_i - initial x

28) p_i - initial p (I don't think we need t_i)

29) In order to approximate certain physical situations such as finite size of aperture, and non-linear stability limits we need cut-off factors:

30) Problem stops if: $|x| > x_c$

$$R_e(E) + x > R_{M_c}$$

$$R_e(E) + x < R_{m_c}$$

thus requiring on the data tape:

31) x_c

32) R_{M_c}

33) R_{m_c}

34) Print routine: Should print (x, p, E, t, φ). (if at an oscillator)

Print E and t to all figures, rest to 5 or 6.

34) Scaling:

Energies in units of 100 Bev

Radii in units of R_o, the radius at 100 Bev.

Time in units of 1 sec

Frequency in units of 2^{-30} sec^{-1}

Angles in units of φ/π (mod 2)

Then all quantities are <1.