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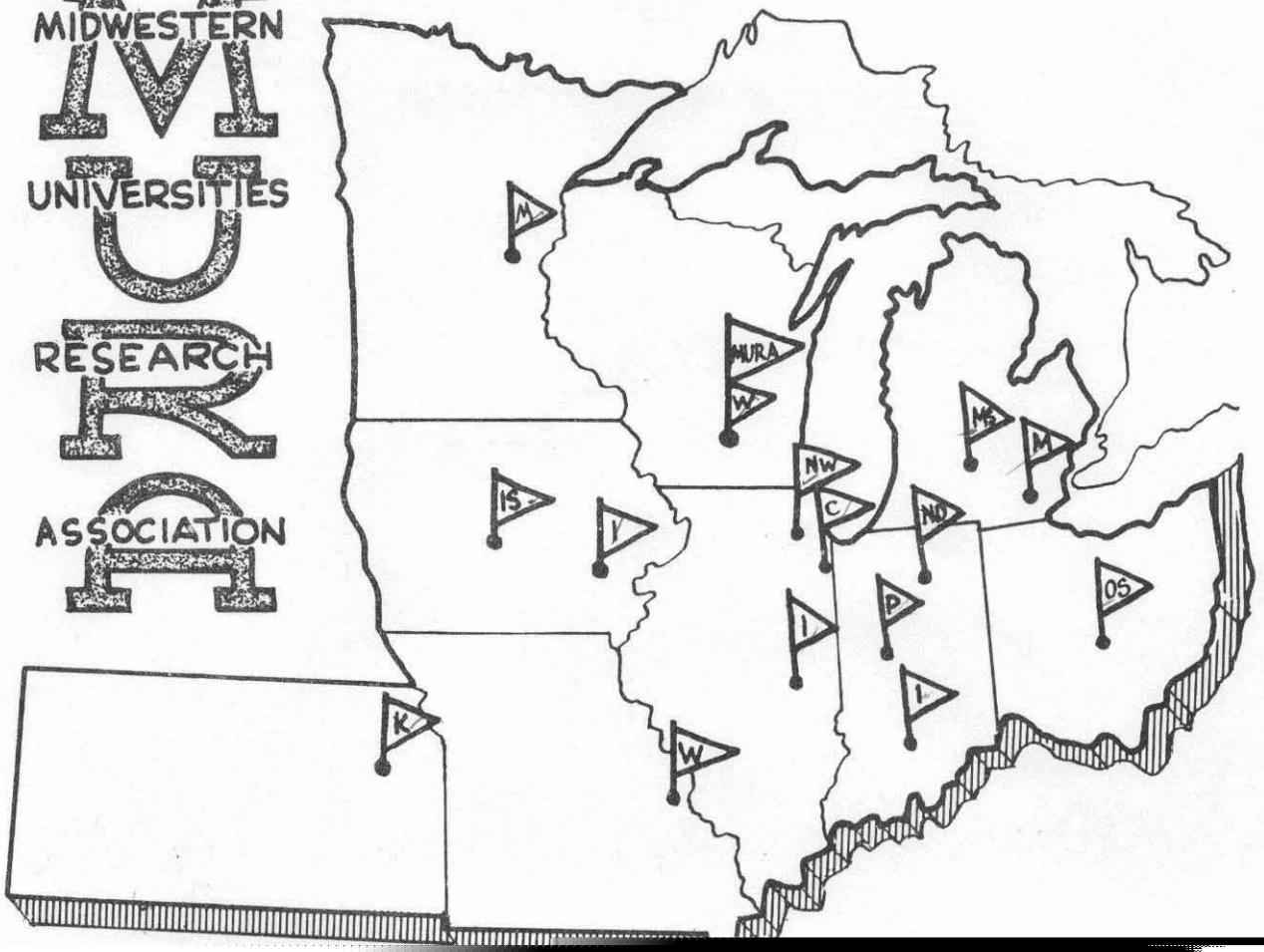

NUMBER _____ REPORT

THE y-STABILITY LIMIT IN SPIRAL SECTOR ACCELERATORS

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It is the purpose of this note to point out that a rather simple approximate expression may be obtained for the y stability amplitude in spiral sector accelerators. This "handy formula" seems capable of indicating the correct order of magnitude of the stability limit, while it has the distinct advantage of being applicable to separated sector machines where there are as yet no digital computer results.

Consider a medial plane magnetic field which is given by:

$$B_z = B_0 \left[1 + f_1 \cos \frac{x}{w} + f_2 \cos \frac{2x}{w} + \ldots \right] \tag{1}$$

This can be obtained from a scalor potential:

$$V(x,y) = B_0 \left[y + \sum_n f_n \left(\frac{w}{n} \right) \cos \frac{nx}{w} \sinh \frac{ny}{w} \right] \tag{2}$$

According to the smooth approximation

$$\nu_y^2 = \frac{f^2}{N^2 w^2} - k \tag{3}$$

where:

$$f = \sqrt{\sum_n f_n^2} \tag{4}$$

If a particle is not on the median plane then it will be subject to a magnetic field whose effective flutter is given by

$$f_{eff} = \sqrt{\sum_n \left(f_n \cosh \frac{ny}{w} \right)^2} \tag{5}$$

Let,

$$f_0 = \sqrt{\sum_n f_n^2} \tag{6}$$

then for $\frac{ny}{w} < 1$ we have:

$$f_{eff} \approx \sqrt{\sum_n \left[f_n\left(1 + \frac{n^2 y^2}{2w^2}\right)\right]^2} \tag{7}$$

$$f_{eff} \approx f_0 + \frac{1}{2} \frac{\left[\sum_n f_n^2 n^2\right]}{f_0} \frac{y^2}{w^2}$$

A variation in F_{eff} is immediately convertable into a change in the y-tune, since from Eq. 3:

$$\Delta \nu_y = \frac{f_0 \, \Delta f_0}{N^2 w^2 \nu_{y0}} \tag{8}$$

We know(experimentally) that the y stability amplitude near $\sigma_y = 2/3\,\pi$ is markedly decreased, and so it appears reasonable to evaluate the stability limit by requiring that the tune changes until σ y reaches $\frac{2\pi}{3}$. This results in:

$$y^2_{limit} = 2w^2\left(\frac{\sigma_{y0}}{\pi}\right) \frac{\left[2/3 - \frac{J_{y0}}{\pi}\right]}{\left[\left(\frac{\sigma_{y0}}{\pi}\right)^2 + \frac{4k}{N^2}\right]} \left[\frac{f_0^2}{\sum_n f_n^2 n^2}\right] \tag{9}$$

For the case of a Mark V with no additional harmonics the last bracket equals one. In this case we may compare the predicted formula with numerous values obtained from the Feckless Five. Table 1 and Graph 1 indicate numerical values for y limit, and compares them with ILLIAC results.

Separated sector accelerators have magnetic fields which involve a large number of harmonics, and hence effective flutters which vary strongly with y. One might thus fear that the y-stability limit will be markedly decreased in such a machine. Examination of Eq. 9 allows one to make quantitative statements about the stability limit in a separated sector accelerator in comparison with a Mark V sinusoidal field accelerator of the same parameters, namely:

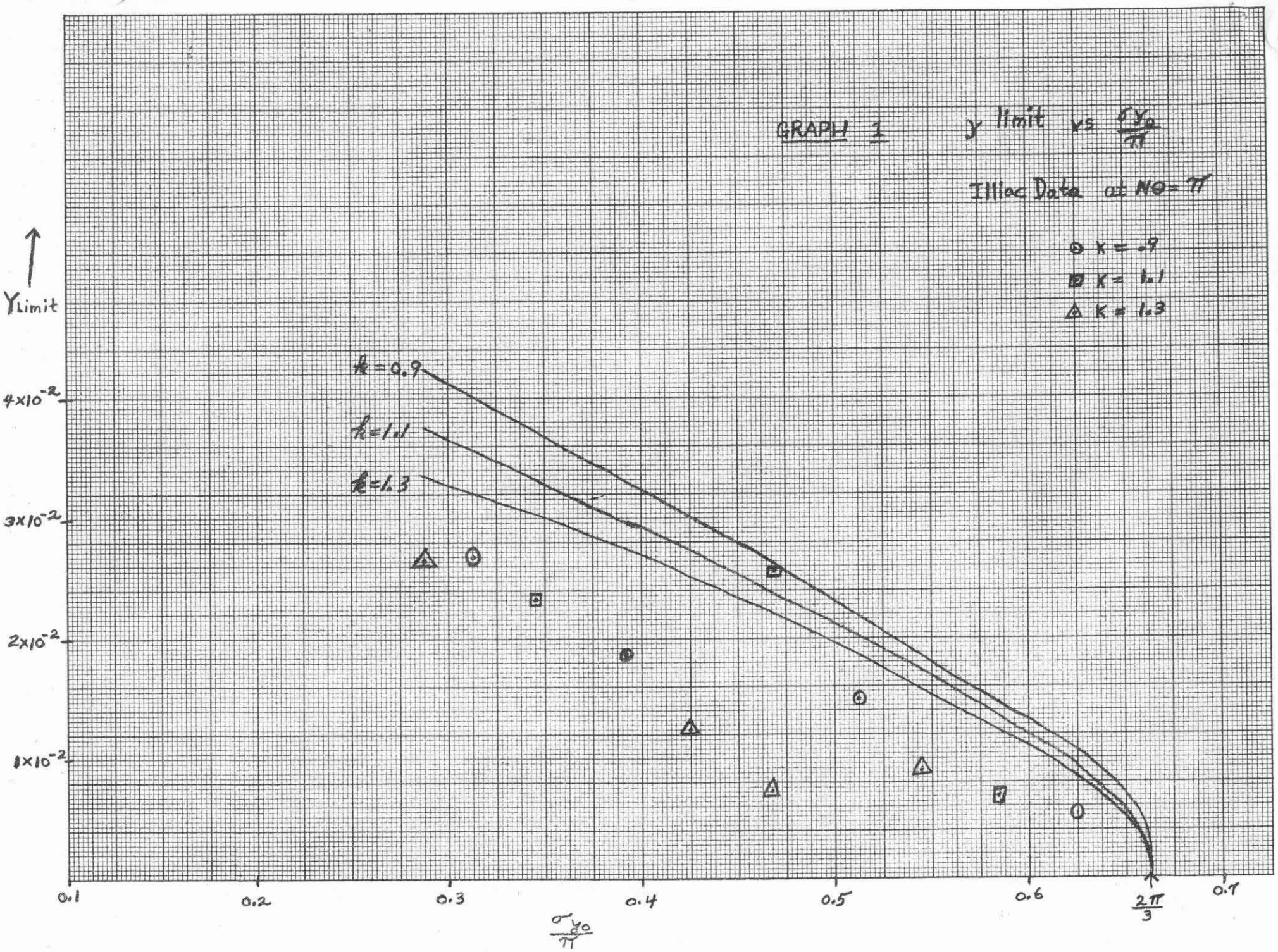
$$\text{y limit sep. sector} = \left[\frac{\sum_n f_n^2}{\sum_n f_n^2 n^2}\right]^{1/2} \text{y limit Mark V} \tag{10}$$

In general, the reduction in stability limit is not very large, even for fields with quite high harmonic content, 50% of second harmonic only reducing the limit by 20%.

One cannot help but observe that as y amplitude increases the effective flutter <u>always</u> increases, thus always increasing the y-tune. Thus if one believes that $\sigma = 2\pi/3$ is the most important limiting tune, operating with $\sigma_{y_0} > 2\pi/3$ should increase the y stability limit by a large factor--it probably now being limited by $\sigma = \pi$.

k	$1/w$	$\frac{{}^{o}y_o}{\pi}$	10^2 y limit	10^2 observed limit at $N\theta = \pi$
.9	33.6	.623	.94	.52
	30.0	.507	2.1	1.4
	26.4	.394	3.2	1.8
	24.0	.317	4.0	2.6
1.1	33.6	.585	1.3	.66
	30.0	.466	2.3	2.5
	26.4	.346	3.3	2.3
1.3	33.6	.547	1.5	0.9
	31.2	.464	2.1	0.72
	30.0	.422	2.4	1.2
	26.4	.289	3.3	2.6

TABLE I Values of y limit and Illiac results for Model Size Machines $f = 1/4$, $N = 5$.


GRAPH 1
Y limit vs $\frac{\sigma_{y_0}}{\pi}$
Illiac Data at $N\theta = \pi$
⊙ K = .9
⊡ K = 1.1
△ K = 1.3
k = 0.9
k = 1.1
k = 1.3
Y Limit
4×10⁻²
3×10⁻²
2×10⁻²
1×10⁻²
0.1
0.2
0.3
0.4
0.5
0.6
$\frac{2\pi}{3}$
0.7
$\frac{\sigma_{y_0}}{\pi}$