Fermilab Library
0 1160 0038469 7

MURA 93

MURA TO-4

Internal

COMMENTS ON SEVERAL MODIFICATIONS OF THE MAGNETIC FIELD OF SPIRAL SECTOR MAGNETS.

Tihiro Ohkawa*

University of Illinois and MURA**

Several variations of magnetic fields are examined searching for better focussing properties and for the improvement of design of a c - w cyclotron. They are "cross ridges", "alternating ridges" and "phase shifted ridges".

A) Cross Ridges

In Mark V the frequencies of the betatron oscillation are given in the smooth approximation by

$$\nu_x^2 = k + 1$$
$$\nu_y^2 = -k + 2\,(A.G.) \qquad (1)$$

These relations help the design of a c - w cyclotron because of the presence of 2(A.G.) in ν_y to keep y-focussing. However, there are difficulties at the energies of proton where ν_x's cross integers due to change of k. The cross ridge type is tried to avoid this difficulty since this type is non-scaling type and presumably both ν_x and ν_y have the A. G. terms which vary with radius. It turns out, however, that it is difficult to compromise

---

* On leave from University of Tokyo

** Supported by N. S. F.

the requirement for a c - w cyclotron and the cross ridge focussing.

The magnetic field in the median plane is given by

$$B_z = B_0\left(\frac{r}{r_0}\right)^k \left\{ 1 + f \sin\frac{\ln\frac{r}{r_0}}{w} \cos N\theta \right\}$$

or (2)

$$= B_0 \frac{1}{2}\left[\left(\frac{r}{r_0}\right)^k \left\{ 1 + f \sin\left(\frac{\ln\frac{r}{r_0}}{w} - N\theta\right)\right\} + \left(\frac{r}{r_0}\right)^k \left\{ 1 + f \sin\left(\frac{\ln\frac{r}{r_0}}{w} + N\theta\right)\right\}\right]$$

Since this field is obtained by superposing the spiral ridge fields with opposite direction of spirals, the computation is carried by Laslett's procedure with the same notations in MURA LJL-5 and LJL-5 Sequel.

The coefficients of the vector potentials are given by

$$D_1 = -1 - f \sin\varphi \cos N\theta$$

$$D_2 = -\frac{k-2}{2} + f\left\{ -\frac{1}{2w}\cos\varphi\cos N\theta - \frac{k-1}{2}\sin\varphi\cos N\theta \right\} \qquad (3)$$

$$D_3 = -\frac{k^2-2k+3}{6} + f\left[-\frac{k-1}{3w}\cos\varphi\cos N\theta - \left(-\frac{1}{6w^2} + \frac{k^2-2k+3}{6}\right)\sin\varphi\cos N\theta \right]$$

$$D_4 = -\frac{k^3-4k^2+7k-12}{24} + f\left[\frac{\frac{1}{w^2} - 3k^2 + 8k - 7}{24w}\cos\varphi\cos N\theta - \frac{1}{24}\left(\frac{-3k+4}{w^2} + k^3 - 4k^2 + 7k - 12\right)\sin\varphi\cos N\theta \right]$$

where

$$\varphi = \frac{\ln\frac{r_1}{r_0}}{w}$$

and the approximate Lagrangian is written as follows,

$$\mathcal{L} = \frac{x'^2 + y'^2}{2} + x + \frac{x^2}{2} + \left[E_1 x + E_2 x^2 + E_3 x^3 + F_0 y^2 + F_1 x y^2 \right] \qquad (4)$$

with

$$E_1 = -1 - f \sin\varphi \cos N\theta$$

$$E_2 = -\frac{k+2}{2} + f\left[-\frac{1}{2w}\cos\varphi\cos N\theta - \frac{k+2}{2}\sin\varphi\cos N\theta\right]$$

$$E_3 = -\frac{k^2+3k+2}{6} + f\left[\frac{-(2k+3)}{6w}\cos\varphi\cos N\theta - \frac{-\frac{1}{w^2}+k^2+5}{6}\sin\varphi\cos N\theta\right]$$

$$F_0 = \frac{k}{2} + f\left[\frac{1}{2w}\cos\varphi\cos N\theta + \frac{k}{2}\sin\varphi\cos N\theta\right]$$

$$F_1 = \frac{2k^2+k}{4} + f\left[\frac{4k+1}{4w}\cos\varphi\cos N\theta + \frac{-\frac{2}{w^2}+2k^2+k-N^2}{4}\sin\varphi\cos N\theta\right]$$

By putting $X = K_0 + K_1 \sin N\theta + K_2 \cos N\theta + u$ where K_0, K_1 and K_2 are chosen in such a way that the forced motion terms in the equation of motion are removed, we separate the forced motion and the betatron oscillation.

K_0, K_1 and K_2 are determined by

$$-K_0(k+1) - K_2\frac{f}{2}\left(\frac{1}{w}\cos\varphi + (k+2)\sin\varphi\right) - K_0^2\frac{k^2+3k+2}{2} - K_1 K_2 f\left(\frac{k+3}{2w}\cos\varphi + \frac{-\frac{1}{w^2}+k^2+5}{2}\right) - \frac{k^2+3k+2}{4}(K_1^2+K_2^2) = 0 \qquad (5)$$

$$N^2 K_1 - (k+1)K_1 - (k^2+3k+2)K_0 K_1 = 0$$

$$N^2 K_2 - f \sin\varphi - (k+1) K_2 - K_0 f \left(\frac{1}{w}\cos\varphi + (k+2)\sin\varphi\right)$$
$$- (k^2+3k+2) K_0 K_2 + K_0^2 f \left(-\frac{k+3}{2w}\cos\varphi - \frac{-\frac{1}{w^2}+k^2+5}{2}\sin\varphi\right)$$
$$+ \frac{f}{2}(K_1^2 + K_2^2)\left(-\frac{k+3}{2w}\cos\varphi - \frac{-\frac{1}{w^2}+k^2+5}{2}\sin\varphi\right) = 0$$

and are given by approximately

$$K_0 \doteq \frac{f\sin\varphi}{N^2 - (k+1) + \frac{f^2\alpha^2}{2(k+1)}} \quad \frac{-f\alpha}{2(k+1)}$$

$$K_1 \doteq 0 \tag{6}$$

$$K_2 = \frac{f\sin\varphi}{N^2 - (k+1) + \frac{f^2\alpha^2}{2(k+1)}}$$

where $\alpha = \frac{1}{w}\cos\varphi + (k+2)\sin\varphi$

The linear parts of the equations for betatron oscillation are

$$u'' = F_u u$$
$$y'' = F_y y \tag{7}$$

where

$$F_u = -(k+1) - K_0(k^2+3k+2) - \frac{f\beta}{2}K_2$$
$$-(f\alpha + f\beta K_0 + K_2(k^2+3k+2))\cos N\theta$$
$$- \frac{f\beta}{2} K_2 \cos 2N\theta$$
$$F_y = k + \frac{2k^2+k}{2}K_0 + \frac{K_2 f}{2}\gamma$$
$$+ \left\{f\alpha' + \frac{K_2}{2}(2k^2+k) + K_0 f\gamma\right\}\cos N\theta + \frac{K_2 f}{2}\gamma\cos 2N\theta$$

$$\beta = \frac{2k+3}{w}\cos\varphi + \left(-\frac{1}{w^2} + k^2 + 5\right)\sin\varphi$$

$$\gamma = \frac{4k+1}{2w}\cos\varphi + \frac{-\frac{2}{w^2} + 2k^2 + k - N^2}{2}\sin\varphi$$

$$\alpha' = \frac{1}{w}\cos\varphi + k\sin\varphi$$

Using the smooth approximation, we get ν's

$$\begin{aligned}\nu_x^2 &= (k+1) + K_0(k+1)(k+2) + \frac{f\beta}{2}K_2 \\ &\quad + \frac{1}{2N^2}\left(f\alpha + f\beta K_0 + K_2(k^2+3k+2)\right)^2 + \frac{1}{8N^2}\frac{f^2\beta^2 K_2^2}{4} \\ \nu_y^2 &= -k - \frac{2k^2+k}{2}K_0 - \frac{K_2}{2}f\gamma \\ &\quad + \frac{1}{2N^2}\left(f\alpha' + \frac{K_2}{2}(2k^2+k) + K_0 f\gamma\right)^2 + \frac{1}{8N^2}\frac{K_2^2 f^2\gamma^2}{4}\end{aligned} \tag{8}$$

or by putting

$$K_0 \sim \frac{-f^2\sin\varphi}{N^2}\,\frac{\alpha}{2(k+1)}$$

$$K_2 \sim \frac{f}{N^2}\sin\varphi$$

we get a rough estimate of ν's

$$\nu_x^2 \sim k+1 + \frac{f^2}{2N^2}\left\{\frac{k^2+5}{2} + \sin 2\varphi \frac{3k+5}{2w} + \cos 2\varphi\left(\frac{1}{w^2} - \frac{k^2}{2} - \frac{5}{2}\right)\right\}$$

$$\nu_y^2 \sim -k + \frac{f^2}{2N^2}\left\{\left(\frac{1}{w^2} + \frac{N^2}{4} + \frac{k^2}{2}\right) + \sin 2\varphi \frac{k^2+k-1}{2w(k+2)} - \cos 2\varphi\left(\frac{N^2}{4} + \frac{k^2}{2}\right)\right\} \tag{9}$$

or

$$\nu_x^2 \sim (k+1) + \frac{f^2}{2N^2}\frac{k^2+5}{2} + \left\{\left(\frac{2k+5}{2w}\right)^2 + \left(\frac{1}{w^2} - \frac{k^2}{2} - \frac{5}{2}\right)^2\right\}^{1/2} \sin(2\varphi + \psi_1)$$

$$\nu_y^2 \sim -k + \frac{f^2}{2N^2}\left(\frac{1}{w^2} + \frac{N^2}{4} + \frac{k^2}{2}\right) + \left\{\left(\frac{k^2+k-1}{2w(k+2)}\right)^2 + \left(\frac{N^2}{4} + \frac{k^2}{2}\right)^2\right\}^{1/2} \sin(2\varphi + \psi_2) \tag{10}$$

where

$$\tan\psi_1 = \left(\frac{1}{w^2} - \frac{k^2}{2} - \frac{5}{2}\right) \Big/ \left(\frac{2k+5}{2w}\right)$$

$$\tan\psi_2 = \left(\frac{N^2}{4} + \frac{k^2}{2}\right) \Big/ \left(\frac{k^2+k-1}{2w(k+2)}\right)$$

At a radius where $2\varphi = 0$, i.e. absence of scalloped motion (9) is

$$\nu_x^2 \sim (k+1) + \frac{f^2}{2N^2}\frac{1}{W^2}$$

$$\nu_y^2 \sim -k + \frac{f^2}{2N^2}\frac{1}{W^2} \tag{11}$$

and both ν's have the same amount of the (A.G.) term. On the other hand, the full size scalloped motion, i.e. $2\varphi = \pi$ gives the following ν's

$$\nu_x^2 \sim (k+1) + \frac{f^2}{2N^2}\left(-\frac{1}{w^2} + (k^2+5)\right)$$

$$\nu_y^2 \sim -k + \frac{f^2}{2N^2}\left(\frac{1}{w^2} + \frac{N^2}{2} + k^2\right) \qquad (12)$$

The A.G. term in ν_x has the negative value, if $\frac{1}{w^2} > k^2+5$ while the A.G. term in ν_y is always positive.

If 1/w is large, like in the ordinary design of Mark V, ν_x oscillates with large amplitude and ν_y is nearly constant. In case of the c - w cyclotron, since we want to use the narrow range of 2ϕ where the A.G. term of ν_x decreases and of ν_y increases with radius, the range of 2ϕ must be less than π .

$$2\,\frac{\ln\frac{r_1}{r_0}}{w} < \pi$$

And also the phase difference $\psi_2 - \psi_1$ must be about π to get the opposite gradient of ν_x and ν_y. If we choose the large value of 1/w to obtain a large $\psi_2 - \psi_1$, $\frac{r_1}{r_0}$ must be very small and this means the narrow energy range of the accelerator. It might be possible to obtain desired change of ν's with small 1/w and N. But in this case $\frac{1}{N^4}$ terms can not be neglected and it might be more convenient to compute numerically.

B) Alternating Ridges

A unit of magnet structure consists of two sectors which have the opposite direction of the spiral ridges. The magnetic field in median plane is given by

$$B_z = B_0\left(\frac{r}{r_0}\right)^k \left\{ 1 + f \sin\left(\frac{\ln\frac{r}{r_0}}{w} - N\theta + \varphi_1\right) \right\} \qquad 0 < N\theta < \pi$$

$$= B_0\left(\frac{r}{r_0}\right)^k \left\{ 1 + f \sin\left(\frac{\ln\frac{r}{r_0}}{w} + N\theta + \varphi_2\right) \right\} \qquad \pi < N\theta < 2\pi \tag{21}$$

Neglecting the effects of fringing field and of the straight section, the magnetic field is expanded in Fourier series of N θ. The same method as in (A) is used to obtain the equations of the betatron oscillation.

The coefficients of Lagrangian are given by

$$E_1 = -1 + f\left[-\sin\varphi\cos(N\theta+\varphi_0) + \frac{2}{\pi}\sin\varphi_0\cos\varphi \sum{}' \frac{2j'}{j'^2-1}\sin j'N\theta - \frac{2}{\pi}\cos\varphi_0\cos\varphi \sum{}' \frac{2}{j'^2-1}\cos j'N\theta \right] \tag{22}$$

$$E_2 = -\frac{k+2}{2} + f\left[-\frac{1}{2w}\cos\varphi\cos(N\theta+\varphi_0) - \frac{k+2}{2}\sin\varphi\cos(N\theta+\varphi_0 + \frac{2}{\pi}\left(-\frac{1}{2w}\sin\varphi_0\sin\varphi + \frac{k+2}{2}\sin\varphi_0\cos\varphi\right)\sum{}' \frac{2j'}{j'^2-1}\sin j'N\theta + \frac{2}{\pi}\left(\frac{1}{2w}\cos\varphi_0\sin\varphi - \frac{k+2}{2}\cos\varphi_0\cos\varphi\right)\sum{}' \frac{2}{j'^2-1}\cos j'N\theta \right]$$

$$E_3 = -\frac{k^2+3k+2}{6} + f\Big[-\frac{2k+3}{6w}\cos\varphi\cos(N\theta+\varphi_0) - \Big(-\frac{1}{6w^2} + \frac{k^2+3k+2}{6}\Big)\sin\varphi\cos(N\theta+\varphi)$$

$$+\frac{2}{\pi}\Big\{-\frac{2k+3}{6w}\sin\varphi_0\sin\varphi + \Big(-\frac{1}{6w^2} + \frac{k^2+3k+2}{6}\Big)\sin\varphi_0\cos\varphi\Big\}\sum{}'\frac{2j}{j^2-1}\sin jN\theta$$

$$+\frac{2}{\pi}\Big\{\frac{2k+3}{6w}\cos\varphi_0\sin\varphi - \Big(-\frac{1}{6w^2} + \frac{k^2+3k+2}{6}\Big)\cos\varphi_0\cos\varphi\Big\}\sum{}'\frac{2}{j^2-1}\cos jN\theta\Big]$$

$$F_0 = \frac{k}{2} + f\Big[\frac{1}{2w}\cos\varphi\cos(N\theta+\varphi_0) + \frac{k}{2}\sin\varphi\cos(N\theta+\varphi_0)$$

$$+\frac{2}{\pi}\Big\{\frac{1}{2w}\sin\varphi_0\sin\varphi - \frac{k}{2}\sin\varphi_0\cos\varphi\Big\}\sum{}'\frac{2j}{j^2-1}\sin jN\theta$$

$$+\frac{2}{\pi}\Big\{-\frac{1}{2w}\cos\varphi_0\sin\varphi + \frac{k}{2}\cos\varphi_0\cos\varphi\Big\}\sum{}'\frac{2}{j^2-1}\cos jN\theta\Big]$$

$$F_1 = \frac{2k^2+k}{4} + f\Big[\frac{4k+1}{4w}\cos\varphi\cos(N\theta+\varphi_0) - \Big(\frac{1}{2w^2} - \frac{2k^2+k-N^2}{4}\Big)\sin\varphi\cos(N\theta+\varphi_0)$$

$$+\frac{2}{\pi}\Big\{\frac{4k+1}{4w}\sin\varphi_0\sin\varphi + \Big(\frac{1}{2w^2} - \frac{2k^2+k-N^2}{4}\Big)\sin\varphi_0\cos\varphi\Big\}\sum{}'\frac{2j}{j^2-1}\sin jN\theta$$

$$+\frac{2}{\pi}\Big\{-\frac{4k+1}{4w}\cos\varphi_0\sin\varphi - \Big(\frac{1}{2w^2} - \frac{2k^2+k-N^2}{4}\Big)\cos\varphi_0\cos\varphi\Big\}\sum{}'\frac{2}{j^2-1}\cos jN\theta\Big]$$

where $\varphi = \frac{\ln\frac{r_1}{r_0}}{w} + \frac{\varphi_2+\varphi_1}{2}$, $\varphi_0 = \frac{\varphi_2-\varphi_1}{2}$

$\sum'$: summation with j = even

The forced motion is separated by putting

$$\begin{aligned} x &= x_f + u \\ &= K_0 + K_1\cos(N\theta+\varphi_0) + \sum{}'(a_j\cos jN\theta + b_j\sin jN\theta) + u \end{aligned} \tag{23}$$

K_o and K_1 are given approximately by

$$K_1 \doteqdot \frac{f \sin\varphi}{N^2 - (k+1)} \tag{24}$$

$$K_o \doteqdot \frac{-K_1 f}{k+1}\left(\frac{1}{2w}\cos\varphi + \frac{k+2}{2}\sin\varphi\right)$$

Since this type is non-scaling, ν's change with radius and first of all we must examine the amount of change of ν's. Though all coefficients contain the higher harmonics such as $\sin j'N\theta$ and $\cos j'N\theta$ these terms contribute to ν only by order of $\left(\frac{1}{j'N}\right)^2$. Actually, even if all higher harmonics work in phase, their contribution might not exceed $\sum' \frac{4(j'^2+1)}{j^2(j'^2-1)}\left(\frac{2}{\pi}\right)^2 \frac{1}{N^2}$ and this is of the order of 25% of the fundamental. Hence we can consider only the fundamentals to look at the change of ν. Then all coefficients of Lagrangian are equivalent to the type (A) and consequently ν's are given by (8).

The above consideration shows this type has the same characteristics as (A) except the contribution of the higher harmonics.

C) Phase Shifted Ridges

The magnetic field in the median plane is given by

$$B_z = B_0\left(\frac{r}{r_0}\right)^k \left\{ 1 + f \sin\left(\frac{\ln\frac{r}{r_0}}{w} - N\theta + \varphi_1\right)\right\} \qquad 0 < N\theta < \pi$$

$$= B_0\left(\frac{r}{r_0}\right)^k \left\{ 1 + f \sin\left(\frac{\ln\frac{r}{r_0}}{w} - N\theta + \varphi_2\right)\right\} \qquad \pi < N\theta < 2\pi$$

The coefficients of Lagrangian are evaluated as in (B) and it turns out that the fundamental parts of the coefficients are all multiplied by factor cos $\frac{\varphi_2-\varphi_1}{2}$ and all higher harmonic parts are multiplied by factor sin $\frac{\varphi_2-\varphi_1}{2}$.

For example

$$E_2 = -\frac{k+2}{2} + f\Big[\Big\{-\frac{1}{2w}\cos(N\theta-\varphi) + \frac{k+2}{2}\sin(N\theta-\varphi)\Big\}\cos\frac{\varphi_2-\varphi_1}{2}$$
$$+ \sin\frac{\varphi_2-\varphi_1}{2}\Big\{\frac{2}{\pi}\Big(-\frac{1}{2w}\sin\varphi + \frac{k+2}{2}\cos\varphi\Big)\sum{}'\frac{2j}{j^2-1}\sin jN\theta$$
$$+ \frac{2}{\pi}\Big(-\frac{1}{2w}\cos\varphi + \frac{k+2}{2}\sin\varphi\Big)\sum{}'\frac{2}{j^2-1}\cos jN\theta\Big\}\Big]$$

where $\varphi = \frac{\ln\frac{r_1}{r_0}}{w} + \frac{\varphi_2+\varphi_1}{2}$

If the phase shift is small, the harmonic parts are small and the machine is nearly scaled. And when the phase shift reaches π, the fundamentals vanish. However, all coefficients of Lagrangian have the same factor sin $\frac{\varphi_2-\varphi_1}{2}$ and cos $\frac{\varphi_2-\varphi_1}{2}$ in each

harmonic and very little is gained by introducing the phase shift between ridges.

For instance, if we want to reduce the non-linear terms and introduce $\frac{\varphi_2 - \varphi_1}{2}$, linear terms are also reduced.

The above three types do not give the desirable features unfortunately. But with small l/w and N the above approximation is extremely rough. A combination of small l/w and N might give some improvements of the design of a c - w cyclotron. There are also the other possibilities by inserting the phase shift or changing the direction of ridges oftener.