MURA 91

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Internal

INDIRECT ACCELERATION

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In present circular accelerators particles are accelerated in the azimuthul direction. There are other types of accelerator in which particles gain energy rather indirectly by a sideways push. Several examples are shown in the following:

A) Subharmonic Cyclotron

In a homogeneous magnetic field, the equation of motion of an ion in the median plane starting from the origin is

$$\frac{d^2 r}{d t^2} = -\left(\frac{eH_0}{2mc}\right)^2 r \tag{1}$$

We apply a radially directed electric field which is given by

$$E_r = -\lambda r \cos \omega t \tag{2}$$

and is produced by a potential

$$V = \frac{\lambda}{2}\left(r^2 - 2z^2\right)\cos \omega t \tag{3}$$

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where $\omega r \ll c$.

Then the equation of motion becomes

$$\frac{d^2 r}{dt^2} + \left\{ \left(\frac{eH_0}{2mc}\right)^2 + \frac{e\lambda}{m}\cos\omega t \right\} r = 0 \tag{4}$$

Putting $\omega t = 2\tau$, we get the standard from of the Mathieu-equation,

$$\frac{d^2 r}{d\tau^2} + (p + q\cos 2\tau) r = 0 \tag{5}$$

$$p = \left(\frac{eH_0}{mc}\right)^2 \frac{1}{\omega^2}, \qquad q = \frac{4e\lambda}{m\omega^2}$$

The characteristic stable zones are shown in the Figure 1. If we choose parameters making the r-motion unstable, r increases with time. Since there is no azimuthul torque, $r^2\dot{\theta} - rA_\theta$ is a constant of the motion so that the increase of radius means that the particle is accelerated.

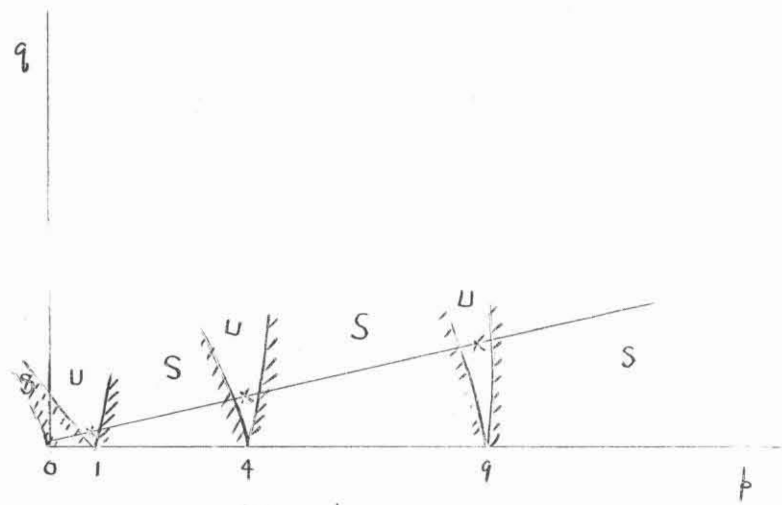

Fig - 1

With given magnetic field and electric field, considering ω as a parameter, the working point lies on a straight line through the origin in the Figure 1. The working point is chosen in an unstable region in order to get the increase of radius. $p = 1$ corresponds to ω being equal to the cyclotron frequency of the particle and the larger value of p, namely 4, 9,... are subharmonics of the cyclotron frequency. The width of the unstable region determines the variation of the parameters, for example, the allowable relativistic increase of mass. There are apparently an infinite number of subharmonics and we can accelerate particles with very low electric field frequency. However, the width of the unstable region repidly decreases as p increases.

The motion in axial direction is described by

$$\frac{d^2 z}{dt^2} - \frac{2e\lambda}{m} \cos\omega t \cdot z = 0 \tag{6}$$

and can be made stable with suitable parameters.

B) The "Surf-Board" accelerator

We choose for the guide field an FF(AG) which has linear energy compaction. A simple example is the type for which the magnetic field is given by

$$H_z = H_0 \left(\frac{r}{r_0}\right)^{-\frac{1}{2}} \tag{7}$$

The equation of motion of an ion in the median plane is

$$\frac{d^2 r}{dt^2} = -\frac{2}{9}\left(\frac{eH_0}{mc}\right)^2 + \frac{1}{3}\left(\frac{r_0}{r}\right)^{3/2}\left(\frac{eH_0}{mc}\right) C + \left(\frac{r_0}{r}\right)^3 C^2 \tag{8}$$

using $r^2 \dot{\theta} = rA_\theta + C$

where

$$C = r_0 v_0 - r_0 A_{\theta 0}$$

$$A_\theta = \frac{e}{mc} \frac{1}{r} \int H_z \, r \, dr$$

If the particle starts from the center, C = 0 and (8) becomes

$$\frac{d^2 r}{dt^2} = -\frac{2}{9}\left(\frac{eH_0}{mc}\right)^2 \tag{9}$$

We apply a radial electric field given by

$$E_r = E_0 \sin\left(\frac{r}{r_0} - at\right) \tag{10}$$

and (9) becomes

$$\frac{d^2 r}{dt^2} = -\frac{2}{9}\left(\frac{eH_0}{mc}\right)^2 + \frac{e}{m} E_0 \sin\left(\frac{r}{r_0} - at\right) \tag{11}$$

Putting $\frac{r}{r_0} - at = u$, we get

$$r_0^2 \frac{d^2 u}{dt^2} = -\frac{2}{9}\left(\frac{eH_0}{mc}\right)^2 + \frac{e}{m} E_0 \sin u \tag{12}$$

(12) is the same equation as the one for phase oscillations in the synchrotron. We can put $u = u_0 + x$, where u_0 is the equilibrium position relative to the electric field and is given by

$$-\frac{e}{m} E_0 \sin u_0 = \frac{2}{9}\left(\frac{eH_0}{mc}\right)^2 \tag{13}$$

with the condition

$$\frac{e}{m} E_0 > \frac{2}{9}\left(\frac{eH_0}{mC}\right)^2 \tag{13'}$$

The equation for x is

$$\frac{d^2X}{dt^2} = \frac{e}{m} E_0 \cos u_0 \cdot x \tag{14}$$

and x oscillates around the stable phase u_0, if $\cos u_0 < 0$. Since r is given by $r = r_0$ (u+ at) and u oscillates around u_0, r increases with time stably and the particles can be accelerated. The important property is that the condition of this acceleration does not depend on the radial velocity a of the electric field. If the particle starts from $r = r_0$, where $\frac{e}{c}(Hr)_{r=r_0} = m v_0$, the right hand side of equation (8) is zero at $r = r_0$ and increases to $-\frac{2}{9}\left(\frac{eH_0}{mC}\right)^2$ at $r = \infty$. So with parameters satisfying the condition (13), the equilibrium phase u_0 slowly changes with radius and the particle can follow this slow change.

The total energy gain is given by

$$E_0 \sin u_0 \cdot (r - r_0) \tag{15}$$

The axial oscillations might be made stable with spiral ridges. The above example is impractical, of course, because of the necessity for tremendous electric fields to keep the magnet size reasonable.

A type having the energy compaction in the axial direction looks less impractical. The focussing may be kept by FFAG focussing as in an electron FFAG cyclotron and the particles are pushed in the axial direction by an electric field $E_z = E_0 \sin\left(\frac{z}{r_0} - at\right)$. The length of the accelerator in the axial direction is the same as the corresponding linear accelerator. But there is no necessity for synchronism andthe particles can be pushed with arbitrary velocity a.