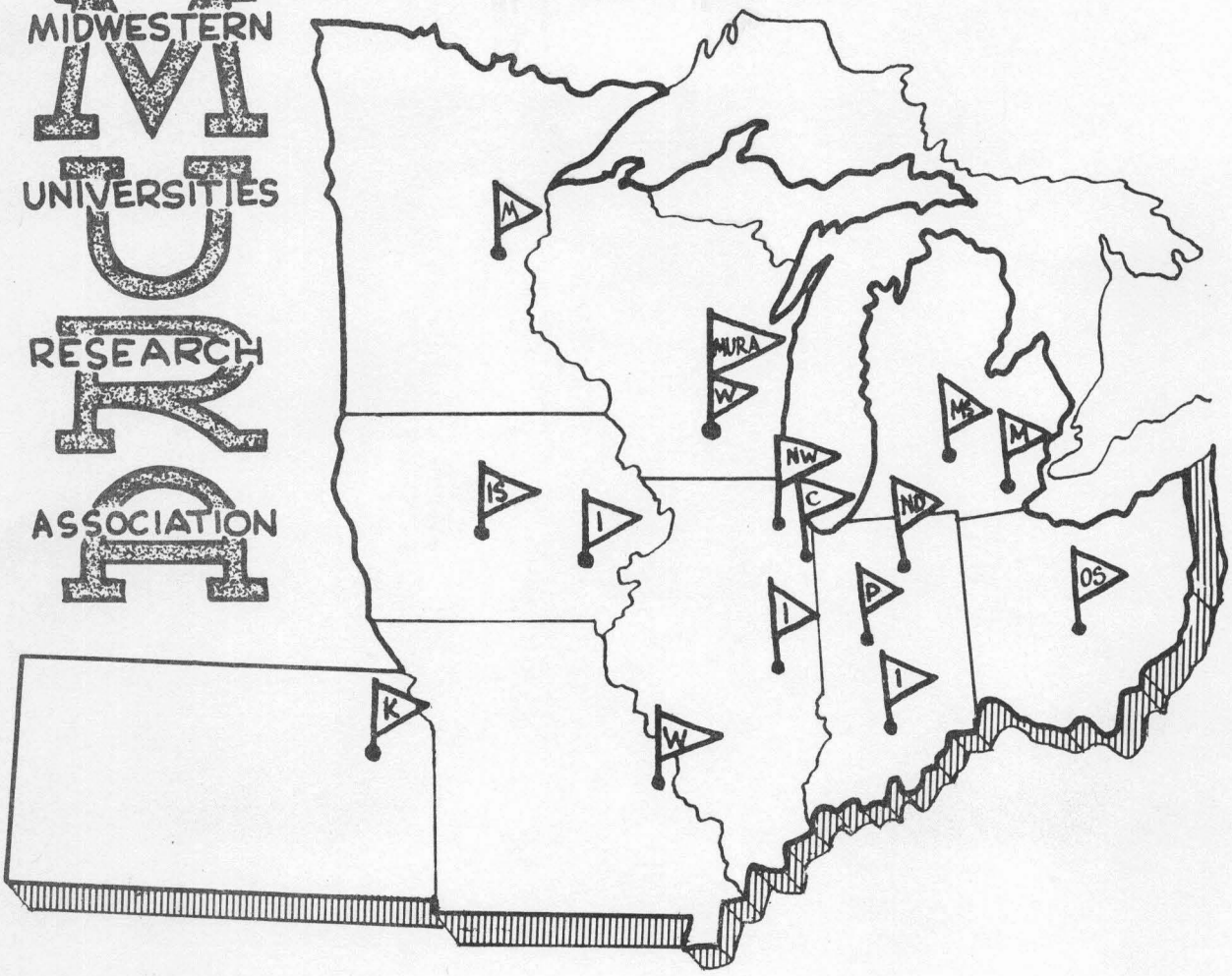
Fermilab Library
0 1160 0038433 3

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION



Injection Into the MARK Ib FFAG Model

REPORT NUMBER 74

MURA-LWJ-10
(Internal)

INJECTION INTO THE MARK Ib FFAG MODEL

Lawrence W. Jones

University of Michigan and

Midwest Universities Research Association

I. INTRODUCTION.

Radial oscillations following injection into the Michigan Model of Mark Ib are studied to find optimum conditions (amplitudes and angles of oscillation, value of ν, and position of injector) for survival of the beam in the absence of space charge effects. Vertical oscillations and vertical admittance for this model as well as a more complete description of the model parameters are given in reports by F. T. Cole.

II. MODEL PARAMETERS.

The Mark Ib model is an 8 sector machine with betatron acceleration. The number of radial betatron oscillations per turn, ν_r, is between 2.5 and 3.0. Injection will be at 25,000 volts, while the betatron acceleration will provide only 40 volts per turn. The spiralling of the equilibrium orbit per turn may be found as follows:

$$\frac{dp}{p} = (k+1)\frac{dr}{r},$$

$$\frac{dp}{p} = \frac{1}{2}\frac{dE}{E}, \quad dr = \frac{r}{E}\frac{dE}{2(k+1)}.$$

For injection, r is about 35 cm, κ is about 3.5. The spiralling per turn, d, is then

$$d = \frac{35 \times 40}{25{,}000 \times 9} \cong .006 \text{ cm.}$$

Since a static injector structure may have dimensions between the mean radius of the emitted electron and the radius of the maximum extent of the injector of the order of a millimeter, the spiralling of the equilibrium orbit is not sufficient to miss the injector on the first revolution.

III. INJECTION WITH FINITE OSCILLATIONS.

It is assumed that there are no space charge effects, that the azimuthal extent of the injector structure is negligible, and that adiabatic damping is not present. We define the following:

D: the radial separation between the electron "source" and the maximum extent of the injector structure.

x_i: the amplitude of radial oscillation at injection.

d: the radial spiralling of the equilibrium orbit per turn due to acceleration.

In linear theory, the amplitude of oscillation, x, is given by

$$x = f(\theta) e^{i\sigma}.$$

For a given reference point in a machine, θ_0,

$$x = f(\theta_0) e^{iN\sigma} = x_0 \cos N\sigma = x_0 \cos 2\pi\nu,$$

where x_0 is the maximum amplitude of oscillation of the particle seen at θ_0.

We define a phase of oscillation, ϕ, such that

$$\phi = N\sigma = 2\pi\nu.$$

Then

$$x_i = x_0 \cos \phi_i ,$$

and on successive turns,

$$x_M = x_0 \cos (\phi_i + M\phi).$$

Since the injector is located inside the equilibrium orbit in all cases of interest, let x be positive for r less than r_0 of the equilibrium orbit.

The radial separation between the electron source and the position of the electron trajectory at θ_0 on the Mth turn, Δ_M, is then given by

$$\Delta_M = x_i - (x_M - Md),$$

where Δ_M is taken as positive for r greater than r of the injector. Δ_M may be found explicitly from

$$\Delta_M = x_0 [\cos \phi_i - \cos (\phi_i + M\phi)] + Md.$$

In the special case where $\phi_i = 0$ (corresponding to injection at the center of a sector along a line parallel to the equilibrium orbit,)

$$\Delta M = x_i (1 - \cos M\phi) + Md.$$

In order to miss an injector, it must be true that

$$\Delta M \geq D \text{ for all } M > 0.$$

Curves were plotted of the minimum values of Δ versus ν for six sets of conditions corresponding to $x_i = 1.00$, $d = 0.01$, $\phi_i = 0, +14.4°, -14.4°$ and $x_i = 2.00$, $d = 0.01$, $\phi_i = 0, +7.2°, -7.2°$. These are presented in figures 1 through 6. The circled numbers on the $\phi_i = 0$ curves give the M values corresponding to the minimum values of Δ. These numbers may also be considered as the order of resonances corresponding to values of ν for the

minima (e.g., ν = 2.66 is a 1/3 integral and ν = 2.75 is a 1/4 integral resonance.)

From these graphs it appears that it is desirable to inject with large oscillation amplitudes. On the $\phi_i = 0$ curves, there appear several regions of ν between 2.50 and 3.00 where the minimum Δ has a value greater than .10 (for x = 1.0) or .15 (for x = 2.0.) The largest range of ν where Δ min. is greater than 0.10 is near ν = 2.93; another region about half as broad in ν occurs near ν = 2.54. By comparing with the graphs for finite ϕ_i, it appears that each of these regions allow about an equal admittance, or spread in angle ϕ_i for a given injector. It may be noted that if ν lies just below a resonance, the injector is more easily missed by aiming the beam away from the equilibrium orbit; if ν is just greater than a resonance, the beam would be aimed toward the equilibrium orbit. (This is obvious from the precession of the oscillation maxima.)

From the graphs, some typical admittances may be read. These are given below, where D and x are in units such that d is 0.01.

x_i	D	ν	$\Delta\phi_i$
1.0	.13	2.945	~ 1°
1.0	.10	2.93	14.4°
1.0	.065	2.89	28.8°
2.0	.17	2.95	~1°
2.0	.15	2.94	7.2°
2.0	.11	2.91	14.4°

In the Michigan Model, d = .006 cm, therefore if at the injector the equilibrium orbit is 6 mm from the gun, the gun structure must not extend more than 0.8 mm beyond the electron source. If the equilibrium orbit is 12 mm from the gun, a 1.0 mm gun structure is a maximum.

To relate the injection oscillation phase shift, ϕ_i, to the physical angle from the injector, consider only injector locations at points of symmetry in the machine. In this special case,

$$x_0^2 = x_i^2 + \beta_i^2 x_i'^2 .$$

where β_i is the parameter of the transfer matrix at the injector position and x' is the radius of curvature, ρ, times the physical angle δ :

$$x' = \rho \frac{dx}{ds} = \rho \delta .$$

At injection also,

$$x_i = x_0 \cos \phi_i \cong x_0 \left(1 - \frac{\phi_i^2}{2}\right)$$

for small ϕ_i. But for small $\beta x'$,

$$x_i \cong x_0 \left(1 - \frac{\beta_i^2 x_i'^2}{2 x_0^2}\right).$$

Therefore,

$$\phi_i \cong \frac{\beta_i \rho \delta}{x_0} , \quad \delta \cong \frac{\phi_i x_0}{\beta_i \rho} .$$

In the present model, β is about 2.4 at the center of focusing sectors, 0.45 at the center of defocusing sectors and ρ is about 14 cm. Therefore in the case where x_i is 1.2 cm and $\Delta \phi_i$ is 7.2°, $\Delta \delta$ is about 1.4°, the angular spread accepted from the injector. This is at the center of a defocusing sector. At the center of a focusing sector, the admittance would be only about one-sixth as great. Terwilliger has pointed out that the maximum amplitude of oscillation in a focusing sector is $\sqrt{\beta \max / \beta \min}$ times its maximum in a defocusing sector. For this model, an oscillation of 1 cm at the center of a defocusing sector has a maximum of about 2.5 cm at the center of a focusing sector.

IV. APPROXIMATE FORMULATION.

For the case $\phi_i = 0$, we may see quickly the approximate maximum value of D which can be tolerated in a given machine by noting the intersection of the locus of higher order resonant minima with the cosine curve for the first turn (ν just under an integral resonance.)

The locus of the minima of higher order resonances is a hyperbola given by

$$\Delta = M d,$$

$$M = \frac{1}{|I-\nu|} = \frac{1}{\delta\nu}, \text{ where I is the integer nearest } \nu \text{ (3 in this case,)}$$

$$\Delta = \frac{d}{\delta\nu}.$$

The first turn curve is, for $d \ll x_i$,

$$\Delta = x_i(1-\cos\phi) = x_i[1-\cos(2\pi\delta\nu)] \cong x_i\frac{(2\pi\delta\nu)^2}{2}$$

At the intersection of these two curves,

$$(\delta\nu)^3 = \frac{d}{2\pi^2 x_i},$$

$$\Delta = (2\pi^2 d^2 x_i)^{1/3}.$$

For the two cases above, this expression gives $\Delta = .126$ for $x_i = 1.0$ and $\Delta = .157$ for $x_i = 2.0$.

V. CONCLUSIONS.

It is possible to predict whether any injected ray will miss an injector, although for a given ν value and injector size it is not easy to give an exact expression for admittance into the machine. However several features of the radial injection problem are indicated: it is desirable to inject in a defocusing (radially) sector with large amplitudes of oscillation. For the model,

a D.C. injector in the absence of space charge should probably not have an extent greater than 1 millimeter beyond the electron source. With such an injector, a spread of a degree or more from the injector may be accepted. There are several regions of ν where an injector could be missed equally well. However, near an integral resonance the broadest band of ν occurs.

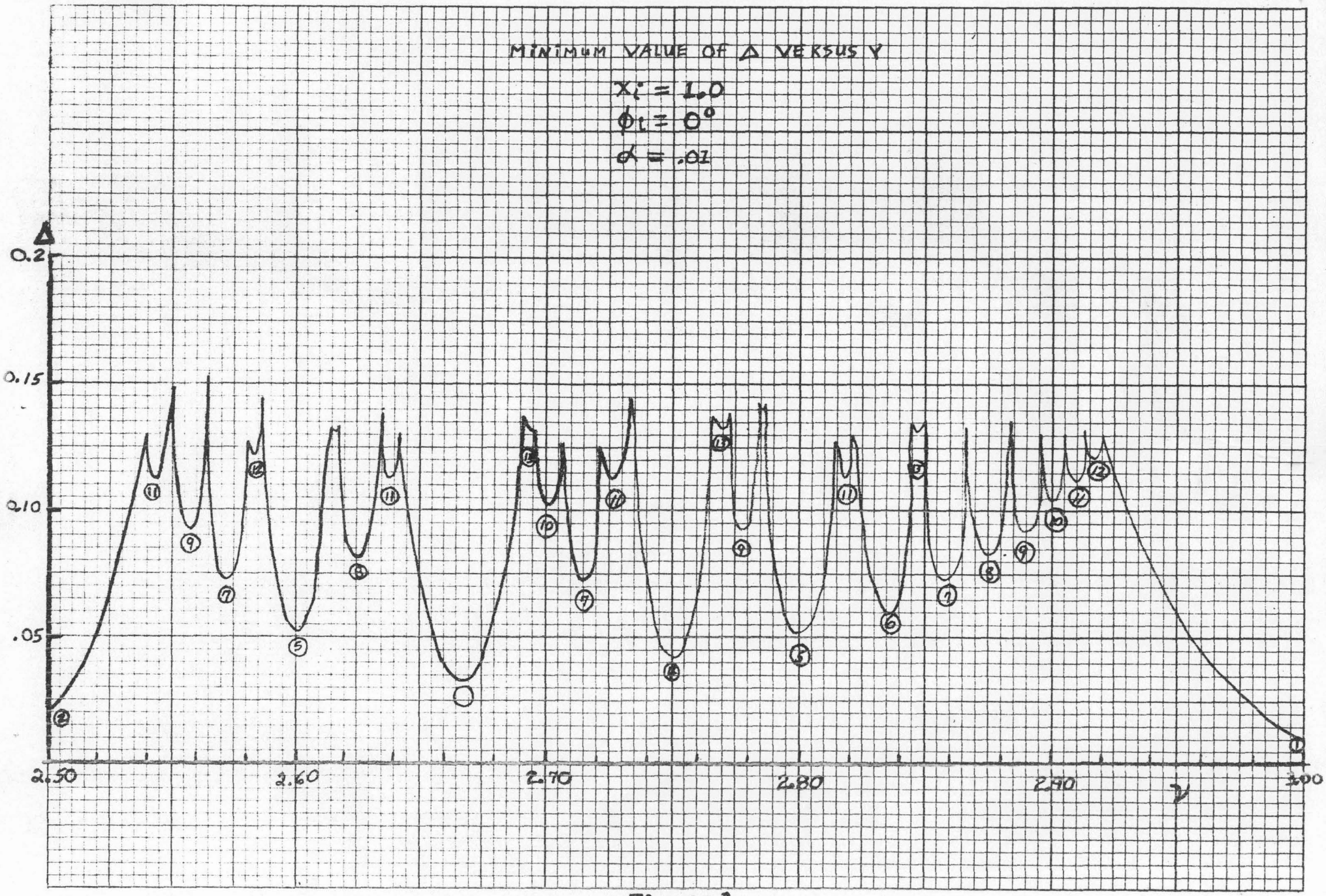

MINIMUM VALUE OF Δ VERSUS ν
$X_i = 1.0$
$\phi_i = 0°$
$\alpha = .01$
Δ
0.2
0.15
0.10
.05
2.50
2.60
2.70
2.80
2.90
3.00
ν

Figure 1

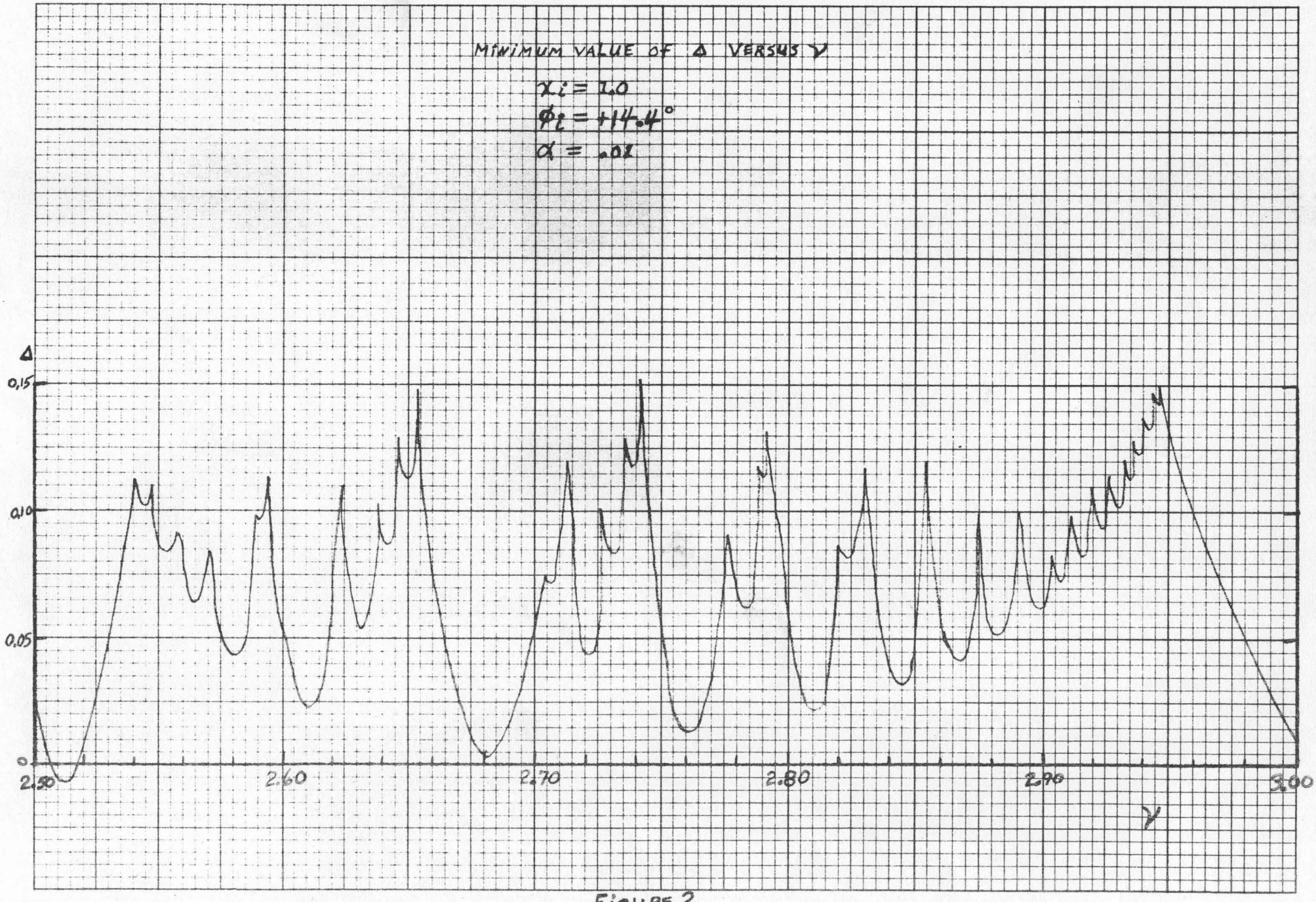

MINIMUM VALUE OF Δ VERSUS ν
$x_i = 1.0$
$\phi_i = +14.4°$
$\alpha = .01$
Δ
0.15
0.10
0.05
0
2.50
2.60
2.70
2.80
2.90
3.00
ν

Figure 2

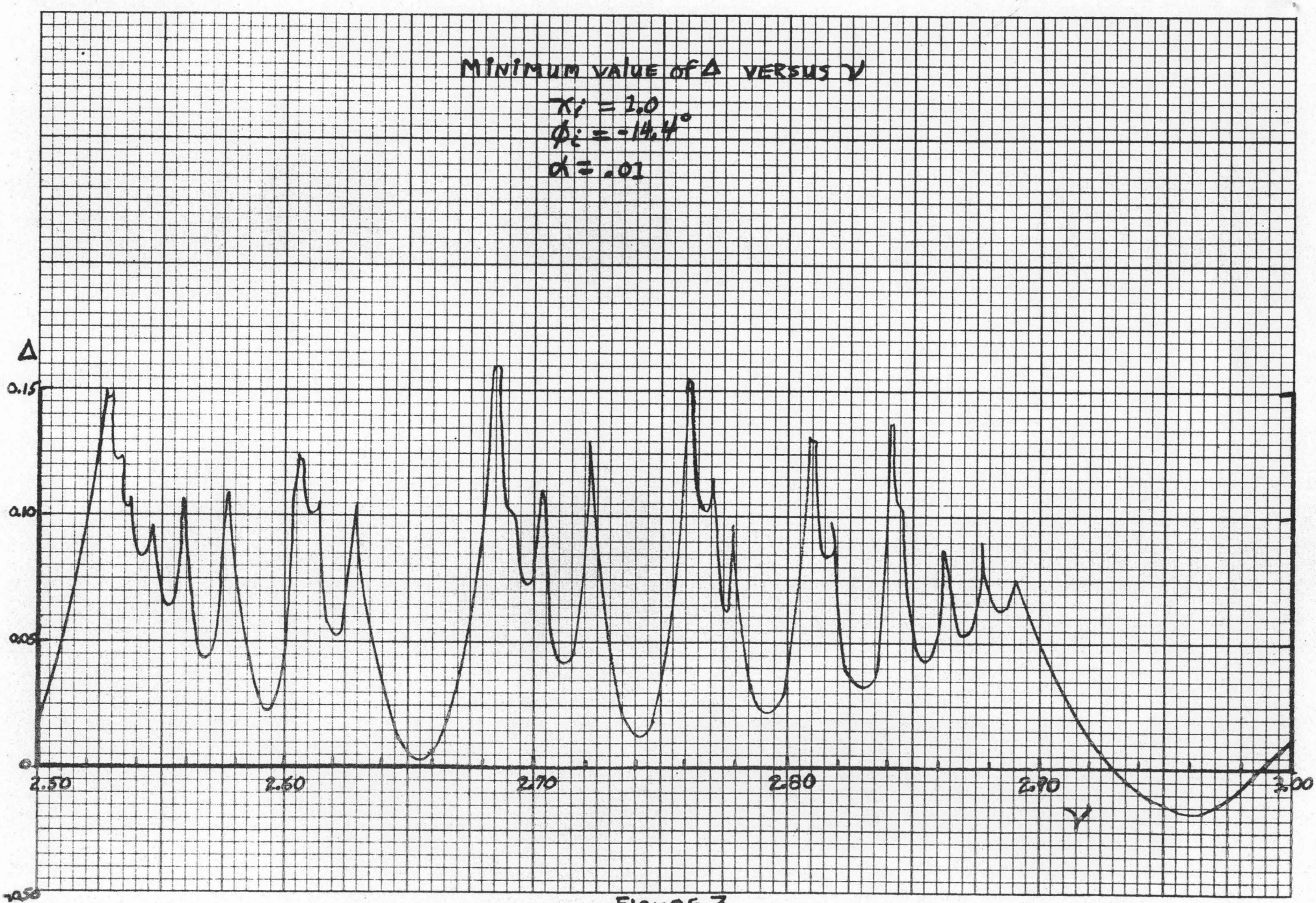

MINIMUM VALUE OF Δ VERSUS ν
$x_i = 1.0$
$\phi_i = -14.4°$
$\alpha = .01$
Δ
0.15
0.10
0.05
0
-0.50
2.50
2.60
2.70
2.80
2.90
3.00
ν

FIGURE 3

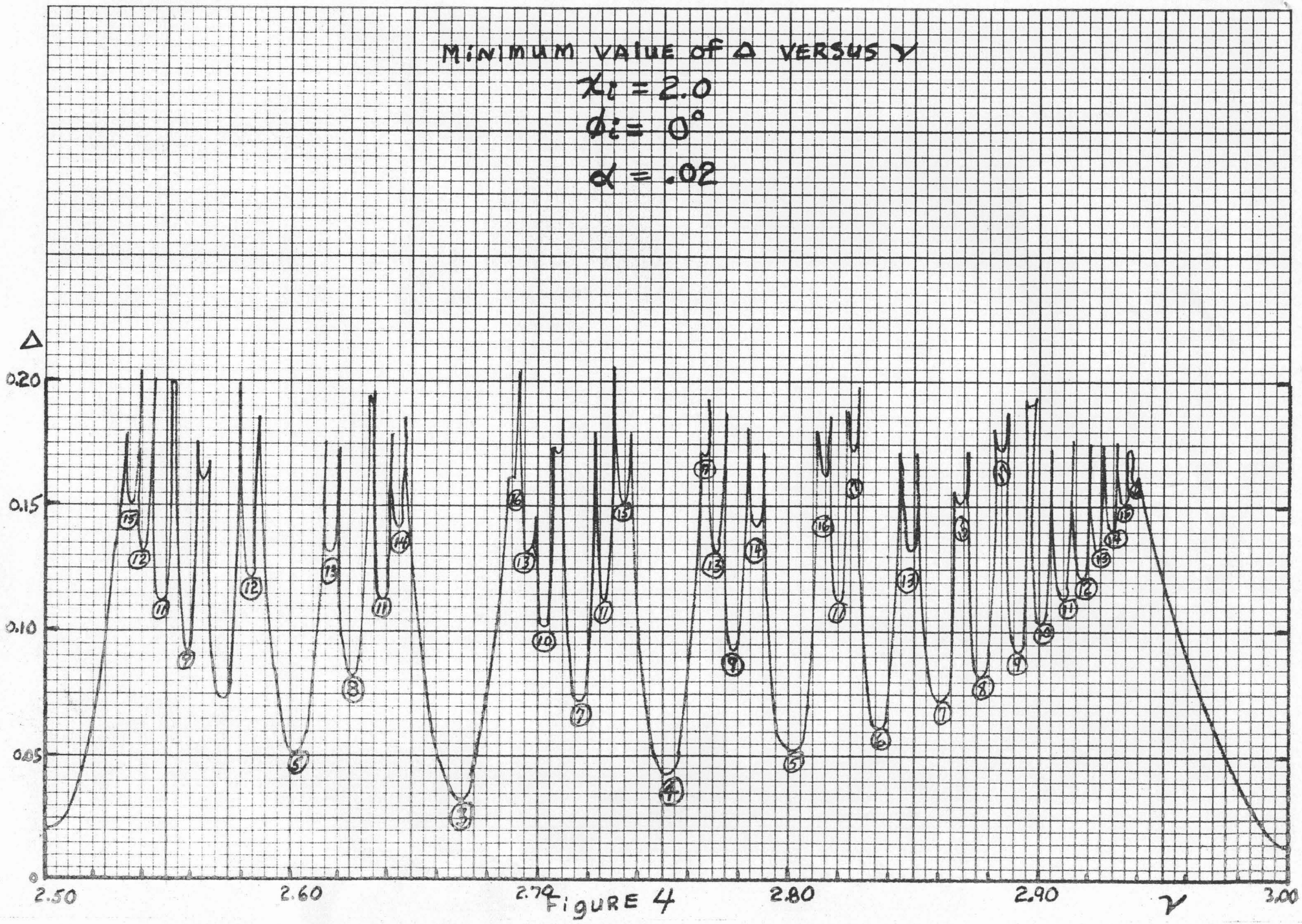

MINIMUM VALUE OF Δ VERSUS ν
$x_i = 2.0$
$\phi_i = 0°$
$\alpha = .02$
Δ
0.20
0.15
0.10
0.05
0
2.50
2.60
2.70
2.80
2.90
3.00
ν

FIGURE 4

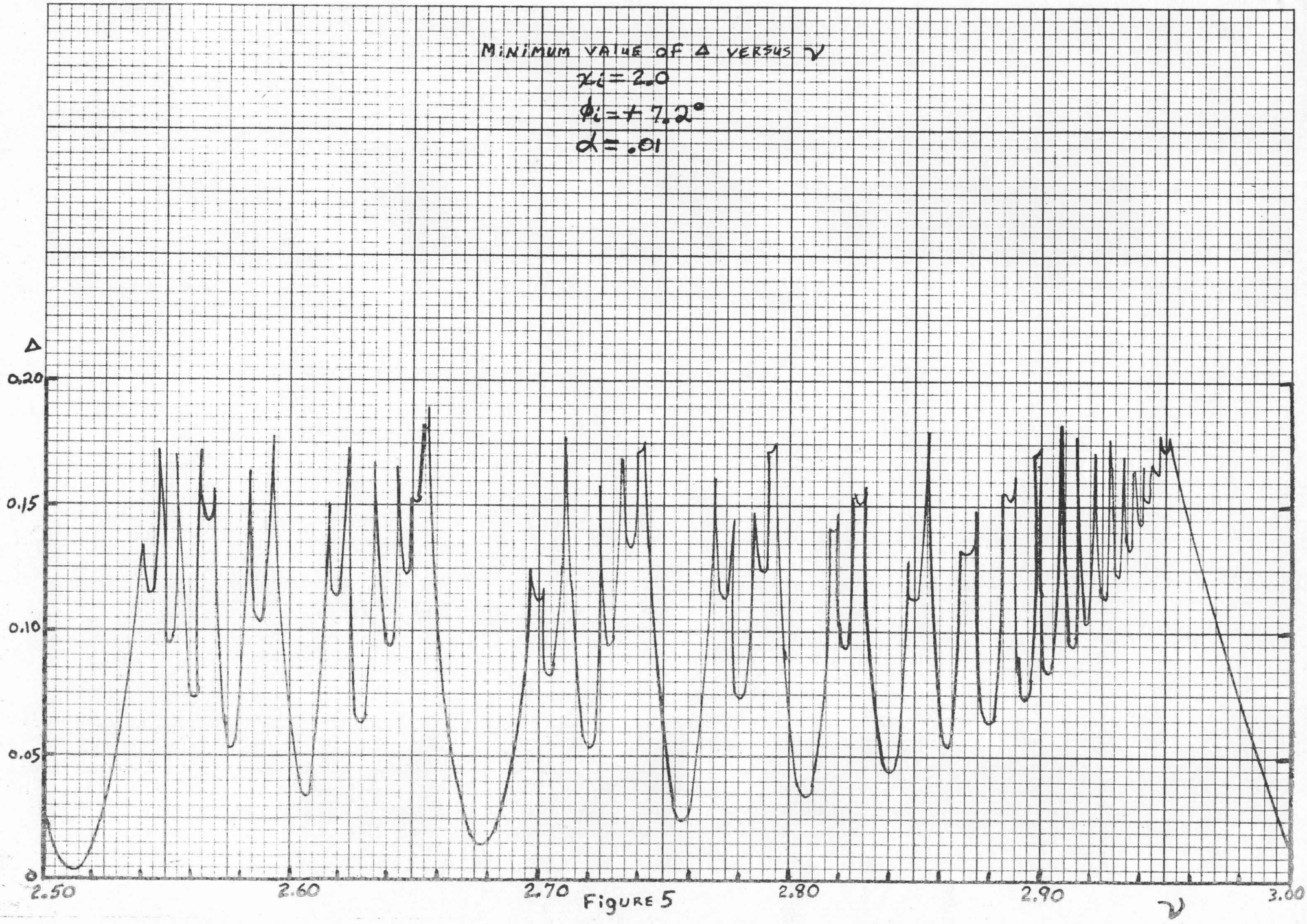

MINIMUM VALUE OF Δ VERSUS ν
$x_i = 2.0$
$\phi_i = +7.2°$
$\alpha = .01$
Δ
0.20
0.15
0.10
0.05
0
2.50
2.60
2.70
2.80
2.90
3.00
ν

Figure 5

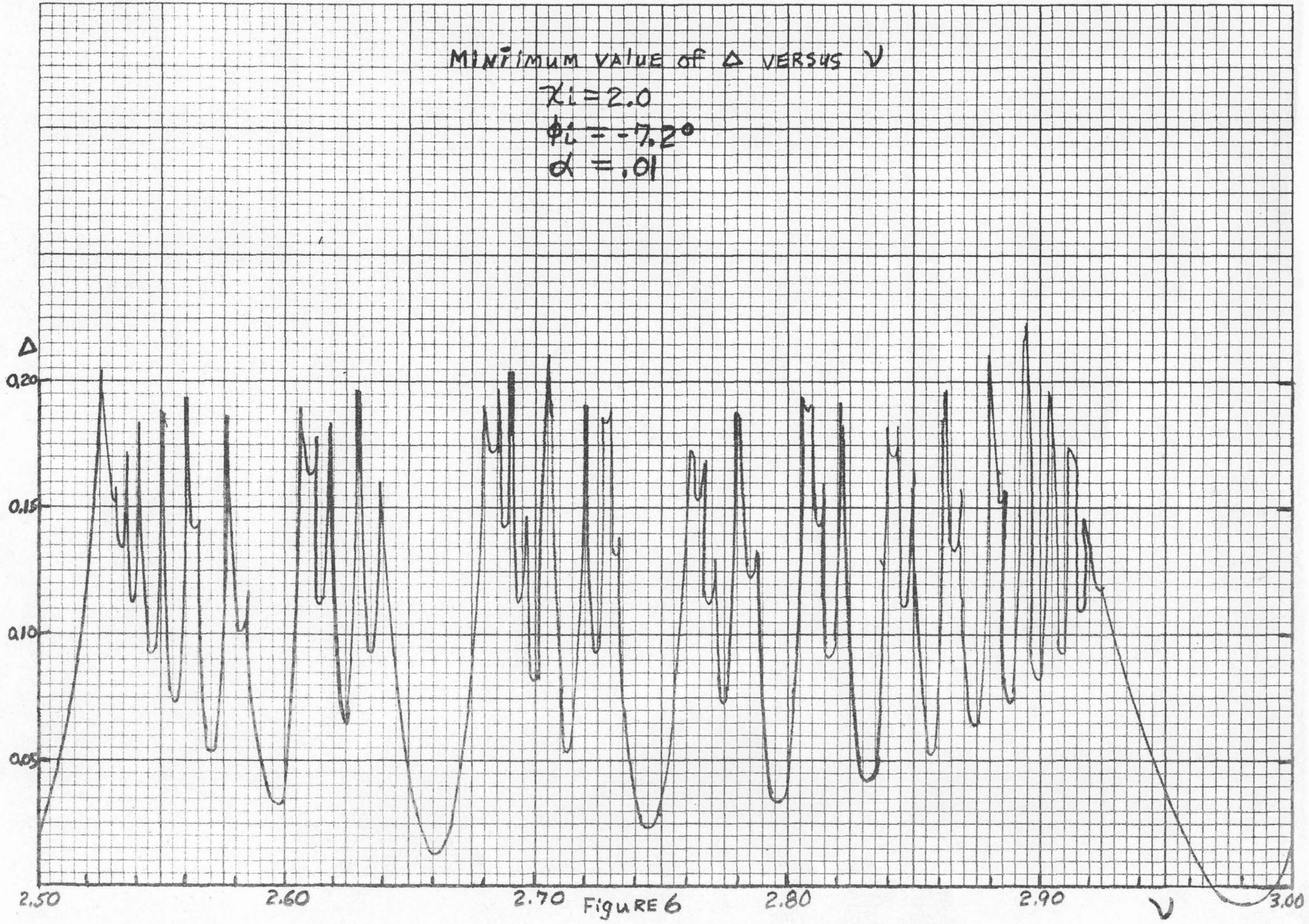

MINIMUM VALUE OF Δ VERSUS ν
$\chi_i = 2.0$
$\phi_i = -7.2°$
$\alpha = .01$
Δ
0.20
0.15
0.10
0.05
2.50
2.60
2.70
2.80
2.90
3.00
ν

FIGURE 6