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Internal

DISTRIBUTIONS OF STRAIGHT SECTIONS IN MARK V

D. W. Kerst

University of Illinois and Midwestern Universities Research Association

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Using the method and example of a machine described in MURA-TBE/DWK-1, we can test the variation in σ for an arbitrary arrangement of sectors. We plot trace ΔM as a function of the position of a straight section in the accelerator and then neglecting terms in the total trace with coefficients θ^2 and higher higher powers of θ, we use

$$\text{trace } M = \text{trace } M_o + \sum \theta_k \text{ trace } \Delta M_k$$

where M_o is the matrix for no straight sections and θ_k is the length of the straight section in the k^{th} position in the sector and trace ΔM_k is read off of the graph. This computation of the trace gives different answers depending upon where the array of straight sections lies in the sector . By trial we must find the maximum and the minimum total trace to determine the difference in σ or in ν for particles of different radii in the same Mark V; because at different radii, the array of straight sections occurs at different positions relative to the position of the positive or negative field gradient.

As shown in MURA-TBE/DWK 1 a straight section occuring in a focussing sector produces

$$\text{Trace} \quad \Delta M_4 = e\, C_{1-F}\, a_F + f C_{1-F}\, C_F + g\, d_{1-F}\, a_F + h_{1-F}\, C_F$$

which is from

$$M_4 = \begin{pmatrix} 1 & \theta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_{1-F} & b_{1-F} \\ C_{1-F} & d_{1-F} \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} \begin{pmatrix} a_F & b_F \\ C_F & d_F \end{pmatrix}$$

and F is the fraction of a focussing segment between the straight section and the end of the focussing segment.

For radial motion:

$e = \cosh \Psi_-$ with $\Psi = \frac{\sqrt{n}\,\pi}{N}$

$f = \frac{1}{\sqrt{n_-}} \sinh \Psi_-$

$g = \sqrt{n_-} \sinh \Psi_-$

$h = \cosh \Psi_-$

and

$a_F = \cos F \Psi_+$

$C_F = -\sqrt{n_+} \sin F \Psi_+$

$C_{1-F} = -\sqrt{n_+} \sin (1-F) \Psi_+$

$d_{1-F} = \cos (1-F) \Psi_+$

For the axial, or Z, motion interchange the numbers used for Ψ_+ and Ψ_- in the formulae and interchange the numbers used for n_- and n_+ .

Straight sections occuring in a defocussing sector produce

$$\text{Trace } \Delta M_3 = a\, g_{1-F}\, e_F + b g_{1-F}\, g_f + C h_{1-F}\, e_F + d h_{1-F}\, g_F$$

which is from

$$M_3 = \begin{pmatrix} 1 & \theta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e_{1-F} & f_{1-F} \\ g_{1-F} & h_{1-F} \end{pmatrix} \begin{pmatrix} a & b \\ C & d \end{pmatrix} \begin{pmatrix} e_F & f_F \\ g_F & h_F \end{pmatrix}$$

and for radial motion

$$a = \cos \psi_+$$
$$b = \frac{1}{\sqrt{n_+}} \sin \psi_+$$
$$C = -\sqrt{n_+} \sin \psi_+$$
$$d = \cos \psi_+$$

and

$$g_{1-F} = \sqrt{n-}\ \sinh (1-F)\, \psi_-$$
$$h_{1-F} = \cosh (1-F)\, \psi_-$$
$$e_F = \cosh F\, \psi_-$$
$$g_F = \sqrt{n-}\ \sinh F\, \psi_-$$

with the previous interchanges for calculating axial motion with these formulae.

The graphs show the variation of these increments of traces for different locations of a straight section in the sector. It is merely necessary to add up the trace Δ M's for the straight sections. For example:

If we want to know how much the number of betatron oscillations around the accelerator varies with the position of the straight section, we note that since trace M = 2 cosσ ,

$\Delta \nu_{(F)} = - \frac{N \theta}{4 \pi \sin \sigma}$ trace $\Delta M_{(F)}$; and since $\Delta \nu(F)$ depends upon F, ν will vary by the maximum variation in $\Delta \nu$ (F).

For one straight section, the radial trace Δ M = - 48 for the largest value and for the smallest value traceΔ M=-3.5 thus $\delta \nu_{max} = \frac{N \theta}{4 \pi \sin \sigma}(44.5)$ is the variation in ν_r .

For our example of 37 = N sectors and σ=119°, and A 20 cm straight section

$$\delta \nu_{max} = .0067\ (44.5) \cong .30 \text{ waves.}$$

If we space three straight sections uniformly in a sector,

$\sum_k$ tr ΔM_k = -48 -27 -7.5 = - 82.5 for one of them between focussing and defocussing magnets and

$\sum_k$ trΔM_k= -3.5 - 2(35) = -73.5 for two of the three symmetrically placed in the defocussing sector.

Thus

$\delta \nu_{max}$ = .0067 (9.0) = .06 waves which is a great improvement over one straight section.

Testing four straight sections and also two straight sections a quarter of a <u>sector</u> apart both give

$\delta \nu_r \sim .33$ which is much worse than three straight sections.

The Z motion for three straight sections has $\delta_z = .077$ waves. One might safely go to 50 cm straight sections three per sector with $\delta \nu$ no worse than .2. For such long straight sections, the terms in θ^2 might become important.

σ's or ν's could be kept much more constant if the separation of straight sections is slightly less than the length of a sector, in this case another periodicity would be introduced into the machine and the question of how bad the stop bands would become would have to be examined.