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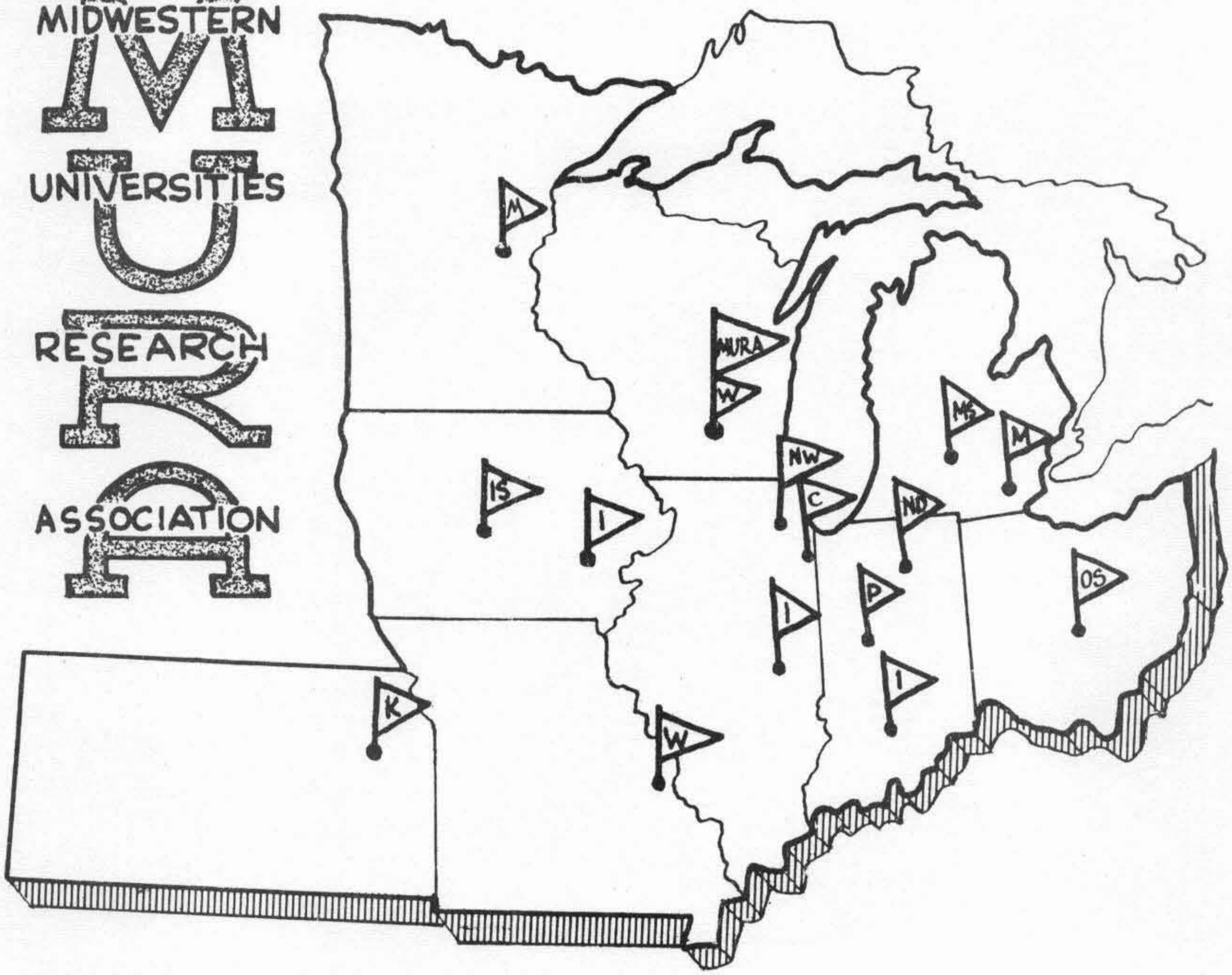

A SPECIAL CASE OF MARK II WITH NO REVERSE FIELD

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A Special Case of Mark II with No Reverse Field

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I. Introduction

Some time ago it was suggested that perhaps the circumference of Mark I and Mark II FFAG types might be reduced by eliminating the reversed field magnets and relying on edge focusing entirely for one sign of focusing. With strong edge focusing, the slope of the edges might be so great as to cause the edges of adjacent sectors to meet at some radius and even cross (producing a local triangle-shaped region of twice the guide field). Considering only the region of such a machine where the sectors do not cross, that is where the guide field is alternately constant and zero, the linear equations for the σ 's may be easily derived. A strong disadvantage of this design is that k is very small, since it arises as a different effect. Also, the σ values are very sensitive to n and to edge angle. We conclude that this is not an attractive design for very high energy synchrotrons.

II. Equations for σ.

Using the notation and method of previous MURA reports,[1] the matrix elements for transformations through a magnet, an edge, a straight section, and an edge may be multiplied, and cosine σ equated to half the trace of the product matrix. For radial motion,

$$M = \begin{pmatrix} \cos\phi & \frac{1}{K_1}\sin\phi \\ -K_1\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix}\begin{pmatrix} 1 & \delta \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix},$$

[1] MURA-KRS-6
MURA-LWJ/DWK/LJL/KRS/KMT-2
MURA-LWJ-7
MURA-LWJ-8

$$\cos \sigma_x = (1 + t\delta)\cos\phi + \left(\frac{t^2\delta + 2t}{2K_1} - \frac{K_1\delta}{2}\right)\sin\phi ,$$

where, refering to Figure 1

$$\phi = K_1 \beta$$

$$K_1 = \sqrt{n+1}$$

$$t = \tan \alpha$$

$$\delta = \ell/\rho .$$

Similarly, for vertical oscillations,

$$M = \begin{pmatrix} \cosh \psi & \frac{1}{K}\sinh\psi \\ k\sinh\psi & \cosh\psi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -t & 1 \end{pmatrix} \begin{pmatrix} 1 & \delta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -t & 1 \end{pmatrix} ,$$

$$\cos \sigma_z = (1 - t\delta)\cosh\psi + \left(\frac{t^2\delta - 2t}{2K} + \frac{K\delta}{2}\right)\sinh\psi$$

where

$$\psi = K\beta$$

$$K = \sqrt{n} .$$

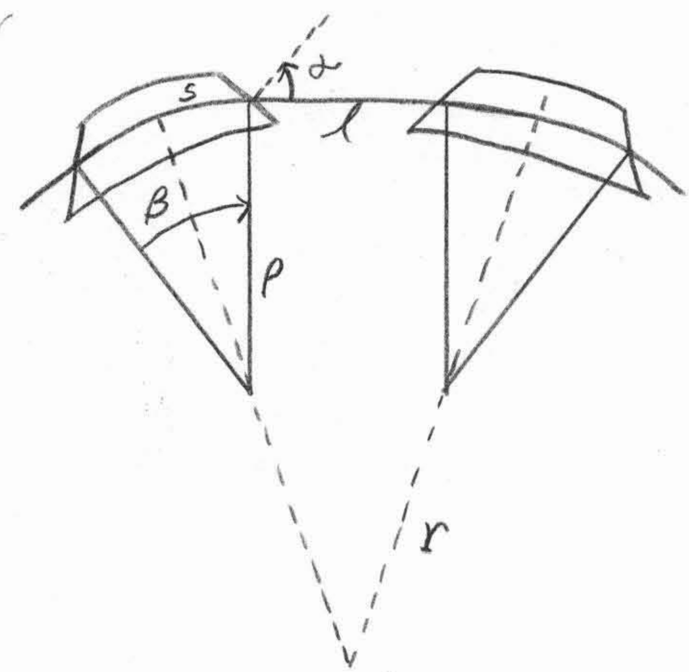

Figure 1

For FFAG accelerators in general,

$$\bar{H} = \bar{H}_0 \left(\frac{r}{r_0}\right)^K$$

$$P = P_0 \left(\frac{r}{r_0}\right)^{K+1}$$

$$n = \frac{\rho}{H}\frac{dH}{dr}$$

$$K = \frac{r\bar{H}'}{\bar{H}}$$

Here $\bar{H} = H \frac{s}{s+\ell} = \frac{H}{C}$

$$K = nC - \left(\frac{N(s+l)\tan\alpha}{\pi s}\right),$$

or $$K = \frac{s+\ell}{s}\left[n - \frac{N}{\pi}\tan\alpha\right].$$

A curious feature of this design is that, although n may be everywhere positive, k may be positive, zero, or negative.

III. Particular Example. N = 50.

In very high energy machines, only large values of N need be considered. When N is low, accelerators of the reverse field type become more practical due to orbit scalloping, however low N leads to low k and a radial aperture too large to be practical for a multi-Bev accelerator.

In order to gain an understanding of the behavior of this design, several machines with N = 50 and $s = \ell$ were computed for a range of values of n and edge angle α. The values of cos σ_x and cos σ_z are tabulated in Table I for various n and α values. These values correspond to a particular point ($s = \ell$) in different machines.

Table I

$\sqrt{n}$	$\tan\delta$	3	4	5	6	7	8	9	10
7	$\cos\sigma_x$	.90							
	$\cos\sigma_z$	1.01							
8	$\cos\sigma_x$		* .91	1.14					
	$\cos\sigma_z$		.93	.67					
9	$\cos\sigma_x$		.68	.81					
	$\cos\sigma_z$		1.26	.96					
10	$\cos\sigma_x$	.17 *	.39	.54	* .73	.89			
	$\cos\sigma_z$	1.4	1.44	1.17	.88	.54			
11	$\cos\sigma_x$			.25	.06	.64			
	$\cos\sigma_z$			1.4	1.15	.78			
12	$\cos\sigma_x$				.05	.20	.38	* .60	
	$\cos\sigma_z$				1.4	1.18	.88	.70	
13	$\cos\sigma_x$					−.29	.14	.01	.63
	$\cos\sigma_z$					1.91	1.32	1.23	.94

* $k \cong 0$. for given $\sqrt{n}$, $k < 0$ for greater δ
k 0 for smaller δ

It may be observed in this example that there is only a small positive region of k which is vertically stable. Where n is positive and k is negative, the high energy orbits lie in weaker fields than the low energy orbits. This is probably not a case of practical interest.

To extend a particular machine to other radii, the point $\tan\delta = 8$, $n = 144$ from Table I was computed for different ratios s/ℓ. Rather than hold σ_x and σ_z constant, δ and n were held constant and values of σ computed. Since at a given s/ℓ ($s = \ell$ in table I) it appeared that σ was very sensitive to both δ and n, it was surprising to see that σ is not very sensitive to variation in s/ℓ. The results of this example are given in Table II.

Table II

$N = 50$, $n = 144$, $\tan\alpha = 8$

s/ℓ	$C = \frac{s+\ell}{s}$	$\cos\sigma_x$	$\cos\sigma_z$
1	2	.38	.88
2	1.5	.55	.92
4	1.25	.64	1.01
∞ *	1.00	.73	.93

*This point is physically impossible since α has no meaning at $\ell = 0$.

IV. Conclusions.

In the numerical example above, k ranged between 16 and 32. 50 to 60 sector Mark Ib or Mark V designs have k values of 100 to 200. This alone makes this Mark II unattractive. With $\tan\alpha = 8$ and $N = 50$, the straight section goes from the magnet length, s, to zero in 1/250 of the radius. For $k \cong 30$, this change in radius corresponds to only a 10% change in momentum. Conceivably a much greater number of sectors or much higher n value would make this design more attractive, but the experience with Mark II indicates the converse. Therefore at this time none of the reverse field types of FFAG appear more practical than the original Mark Ib. The spiral field still offers the best and only way of achieving a small circumference factor for high energy accelerators.