MURA/KMT -2.
MURA 59

VERTICAL APERTURE AND FIELD FLUTTER WITH SLOTTED POLE PIECES

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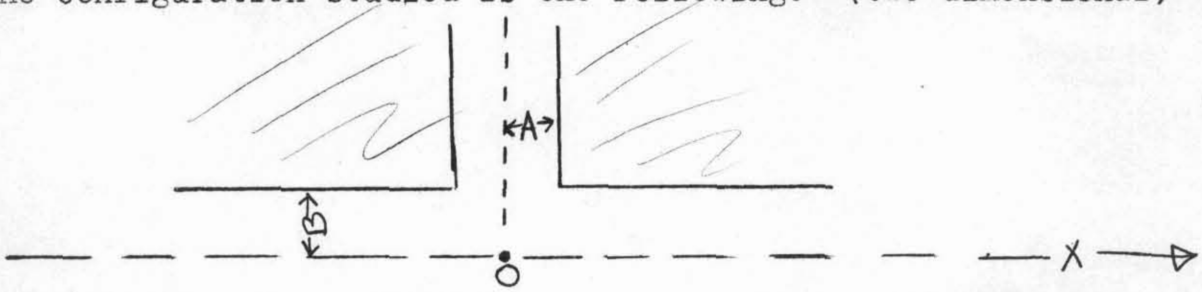
The configuration studied is the following: (two dimensional)



If the median plane field far away from the slot is H_o, then the close field is given by

$$H(v) = \frac{H_o}{\sqrt{1+\left(\frac{1-v^2}{\alpha^2}\right)}} \qquad \alpha = \frac{B}{A} = \frac{\text{vertical aperture}}{\text{slot width}}$$

where the position x of this field H(v) is

$$X = \frac{2A}{\pi}\left[\sin^{-1}\frac{v}{\sqrt{1+\alpha^2}} + \alpha \sinh^{-1}\left\{\frac{\alpha}{\sqrt{1+\alpha^2}}\;\frac{v}{\sqrt{1-v^2}}\right\}\right]$$

If $B \leq A$ then $H \cong H_o$ at $X = 2A$, so the computed fields will be satisfactory for a periodic structure of period $\lambda = 4A$. Then,

$\frac{H\ min}{H\ max} = \frac{\alpha}{\sqrt{H\alpha^2}}$. A field plot with $\frac{B}{A} = 3/4$ gave $H_{min} = .6\ H_{max}$ and $\overline{H} = .837\ H_{max}$, so $H = \overline{H} \begin{matrix}(1+.20)_{max} \\ (1-.28)_{min}\end{matrix}$, and the flutter appeared fairly sinusoidal.

Let us assume that the flutter field is exactly sinusoidal so $H = \overline{H} \begin{matrix}(1+f)_{max} \\ (1-f)_{min}\end{matrix}$ and $\frac{H_{max}}{H_{min}} = \frac{1+f}{1-f} = \frac{\sqrt{1+\alpha^2}}{\alpha}$. A smooth approximation analysis of the Mark V shows that if the number of betatron oscillations around the machine is to be kept constant, $\frac{f}{\lambda} \cong$ constant.

$$\left[2\ A.G. = \left(\frac{f}{\lambda/r}\;\frac{2\pi}{N}\right)^2\right]$$

Combining $\frac{f}{\lambda} = \beta$, with $\frac{1+f}{1-f} = \frac{\sqrt{1+\alpha^2}}{\alpha}$, and $\alpha = \frac{4B}{\lambda}$

One finds

$$B = \frac{\sqrt{\lambda}(1-\beta\lambda)}{8\sqrt{\beta}}$$

A Mark V machine with

$N = 42$ $\quad$ $r = 50$ meters

$k = 200$

$V_x = 14.1$

$V_f = 6.4$

has $2\ A.G. = V_x^2 + V_f^2 - 1 = 245$

so $\frac{f}{\lambda} = \frac{N}{2\pi r} \sqrt{2\ A.G.} = .021 = \beta$

for this machine: $B = \frac{\sqrt{\lambda}}{1.16} \; (1 - .021\lambda)$

units: cm.

λ	4.8	7.1	9.5	11.9	14.3	16.7	19.0	21.4
B	1.7	1.95	2.12	2.22	2.28	2.29	2.26	2.19
f	.10	.15	.20	.25	.30	.35	.40	.45

So the maximum semi-aperture for this machine is 2.3 cm at $f \sim 1/3$. The maximum is a broad one, with $f = 1/4$ giving B = 2.2 cm. To get larger spacing, the ripple must be enhanced with pole face windings.